Murtadha Al-Masoudy

Al-Mustaqbal University College
Air Conditioning and Refrigeration Techniques Engineering Department

Strength of Materials

Second Stage

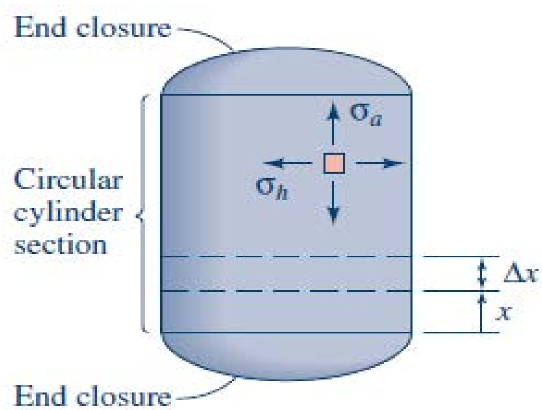
M.Sc Murtadha Mohsen Al-Masoudy

م.م. مرتضى المسعودي

Members Subjected to Axi-symmetric Loads Pressurized thin walled cylinder

Cylindrical Pressure Vessels.

- The diver's air tank in Figure (3.17a) is an example of a thin-wall cylindrical pressure vessel. Figure (3.17b) shows a circular-cylinder pressure vessel with end closures.



(a) (b)

Figure (3.17)

- In the cylindrical section the normal (tensile) stresses in the longitudinal, or axial, direction is called the ***axial stress*** ($\boldsymbol{\sigma_a}$ ***or*** $\boldsymbol{\sigma_L}$).
- In the circumferential, or hoop, direction is called the ***hoop stress*** ($\boldsymbol{\sigma_h}$).
- Assumed that the vessel contains a pressurized gas whose weight can be neglected.
- It is also assumed that $\boldsymbol{\sigma_a}$ and $\boldsymbol{\sigma_h}$ are constant through the thickness of the wall of the pressure vessel.

Axial (Longitudinal) Stress:

- To determine the axial stress, we can employ the free-body diagram in Figure (3.18), where a "cut" has been made through the shell wall and the gas at an arbitrary cross section x at some distance from the end closure. Summing the forces in the x direction, we get:

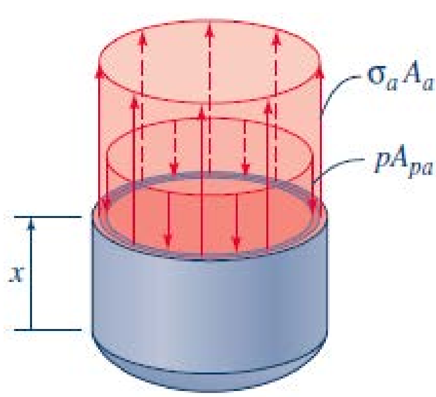


Figure (3.18)

$$\sum F_x = 0: \qquad \sigma_a A_a - pA_{pa} = 0$$

$$\sigma_a[\pi(r+t)^2 - \pi r^2] - p\pi r^2 = 0$$

But, since $t << r$, this can be approximated by:

$$\sigma_a[2\pi rt] - p\pi r^2 = 0$$

So, the axial stress for cylinder is given by:

$$\boldsymbol{\sigma_a = \frac{pr}{2t} \quad or \quad \frac{pD}{4t}}$$

Where

p : pressure inside the cylinder,

D, r: diameter and radius of the cylinder, and

t : thickness of cylinder's wall

Hoop Stress:

The free-body diagram in Figure (3.19b), which is based on the cross sectional and longitudinal cutting planes illustrated in Figure (3.19a), may be used in determining the hoop stress, $\boldsymbol{\sigma_h}$.

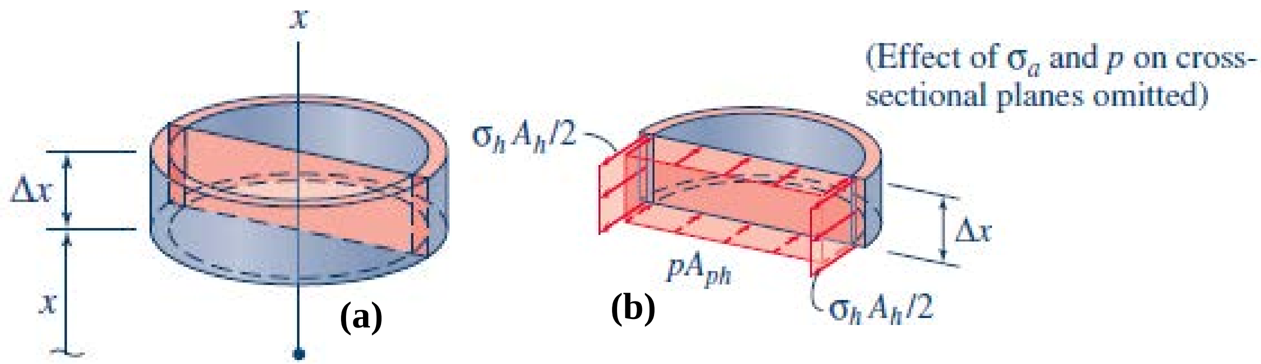


Figure (3.19)

By taking an arbitrary longitudinal cutting plane that cuts the vessel in half (i.e., it contains the longitudinal axis), we obtain two surfaces on which the stress is $\boldsymbol{\sigma_h}$. Also, the two $\boldsymbol{\sigma_h}$-forces in Figure (3.19b) are parallel. Thus, summing forces in the hoop (i.e., tangential to the circumference) direction, we get:

$$\sum F_x = 0: \qquad \sigma_h A_h - pA_{ph} = 0$$

The areas in Eq. above are A_h=2$t\Delta x$ and A_{ph}=2$r\Delta x$, so above Eq. gives the following expression for hoop stress in cylinder:

$$\boldsymbol{\sigma_h = \frac{pr}{t} \ \ or \ \ \frac{pD}{2t}}$$

- The fact that the hoop stress in a cylindrical shell has twice the magnitude of the axial stress accounts for the typical failure mode of cylindrical shells that is depicted in Figure (3.20).

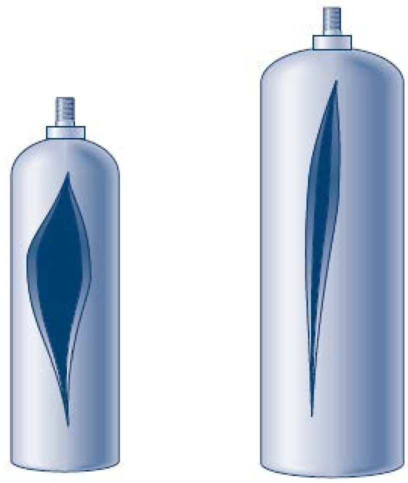


Figure (3.20): Typical appearance of failed cylindrical pressure vessels

Spherical Pressure Vessels.

- Consider now a thin-wall spherical pressure vessel of inner radius ***r*** and wall thickness t ($r/t \geq 10$). Because of the spherical symmetry, the normal stress will be the same in any direction in the shell, as indicated in Figure (3.21a). This stress, which is labeled $\boldsymbol{\sigma_s}$ to identify it with a spherical pressure vessel, can be determined by taking a free-body diagram that consists of half of the spherical vessel plus the pressurized gas occupying the half-sphere. By summing the forces on the free-body diagram in Figure (3.21b), we get:

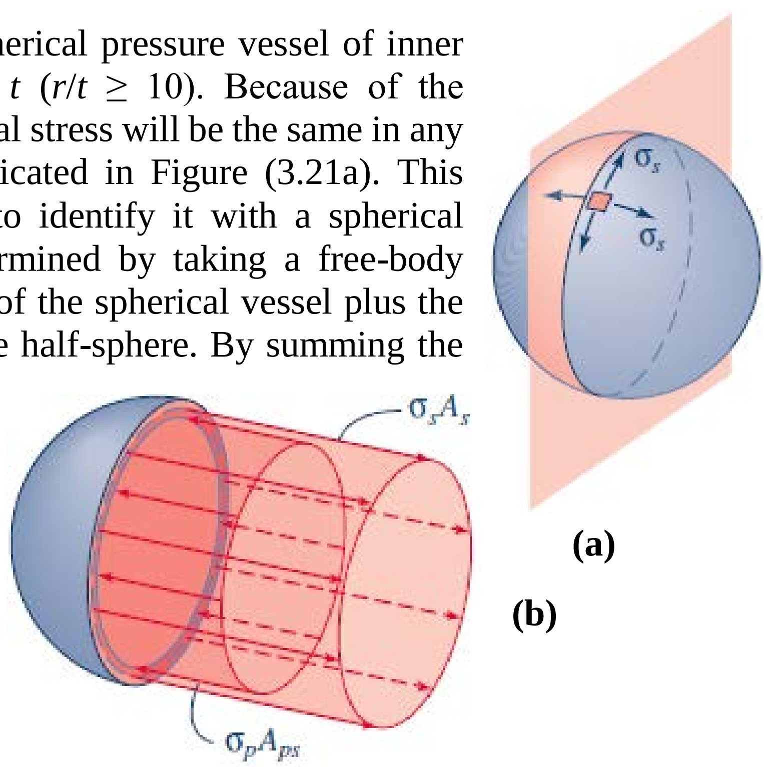


Figure (3.21): A spherical shell under internal pressure

$$\sum F_x = 0: \qquad \sigma_s A_s - pA_{ps} = 0$$

$$\sigma_s[\pi(r+t)^2 - \pi r^2] - p\pi r^2 = 0$$

But, since $t << r$, this can be approximated by:

$$\sigma_s[2\pi rt] - p\pi r^2 = 0$$

So, the normal stress for Sphere is given by:

$$\boldsymbol{\sigma_s = \frac{pr}{2t} \;\; or \;\; \frac{pD}{4t}}$$

- The stress in a spherical pressure vessel is equal to the longitudinal stress in a cylindrical pressure vessel having the same r/t ratio; and it is just half the value of the hoop stress in the cylinder. Thus, for a given pressure, a spherical pressure vessel can have a thinner wall than a cylindrical vessel can.

<u>Example 1:</u> Find the tangential and longitudinal stresses for the vessel shown in Figure (3.22).

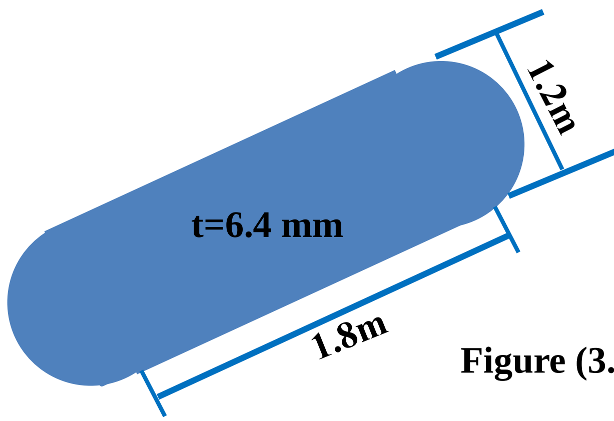
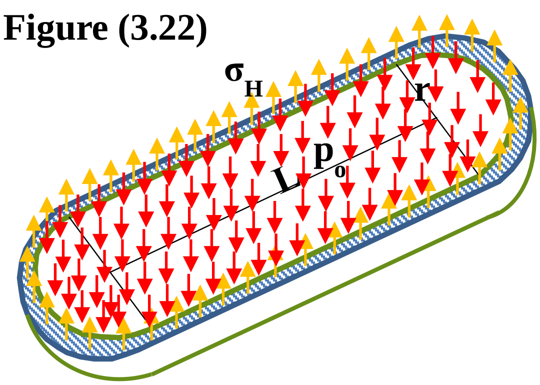


Figure (3.22)



<u>Solution:</u>

- **<u>Tangential stress (σ_H):</u>**

$$\sum F_y = 0$$

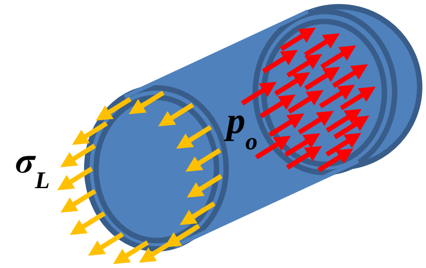
$$p_o[r^2\pi + 2rL] = \sigma_H[2\pi r + 2L]t$$

$$\sigma_H = \frac{p_o[r^2\pi + 2rL]}{[2\pi r + 2L]t}$$

- For $r = 0.6m,\ L = 1.8m,\ t = 6.4mm,\ p_o = 1\ MPa$

$$\sigma_H = \frac{1[(600)^2\pi + 2(600)(1800)]}{[2\pi(600) + 2(1800)]6.4} = 69.77\ MPa$$

- **Longitudinal stress (σ_L):**

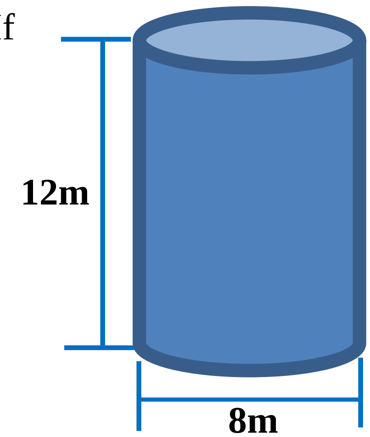


$$\sum F_x = 0$$

$$p_o[r^2\pi] = \sigma_L[2\pi r]t$$

$$\sigma_L = \frac{p_o r}{2t} = \frac{1(0.6)}{2(0.0064)} = 46.88\ MPa$$

Example 2: A water tank is (8m) in diameter and (12m) high. If the tank is to be completely filled, determine the minimum thickness of the tank plating if the stress is limited to 40 MPa.



Solution:

$$\rho_{water} = 1000\frac{kg}{m^3}\ \ or\ \ 1.0\frac{gm}{cm^3}$$

$$\gamma_{water} = 1000 \times 9.81 = 9800\ N/m^3$$

$$p_o = \gamma h = 9800(12)$$

$$\therefore p_o = 117600\ N/m^2 (0.1176\ MPa)$$

$$\sigma_H = \frac{p_o r}{t}\ \rightarrow 40 = \frac{0.1176(4000)}{t}$$

$$\rightarrow t = 11.76\ mm$$

$$\sigma_L = \frac{p_o r}{2t}\ \rightarrow 40 = \frac{0.1176(4000)}{2t}$$

$$\rightarrow t = 5.88\ mm$$

$$\therefore use\ t = 11.76\ mm \cong 12\ mm$$

Example 3: The tank shown in Figure (3.23) is fabricated from (10mm) steel plate. Determine the maximum longitudinal and circumferential stresses caused by an internal pressure of (1.2 MPa).

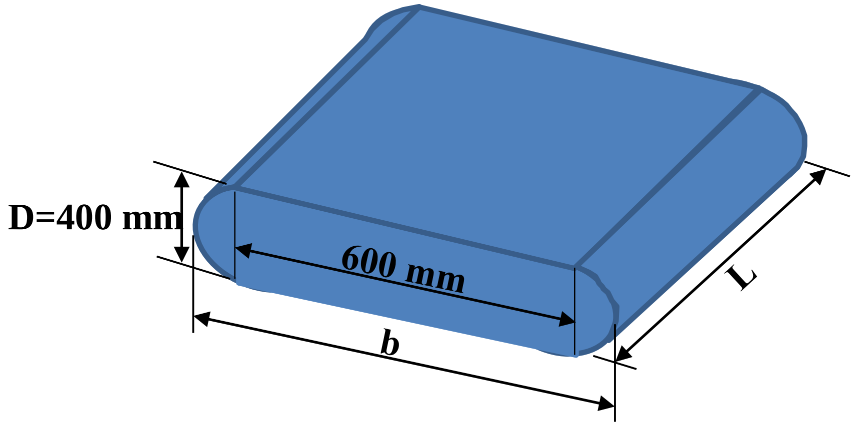
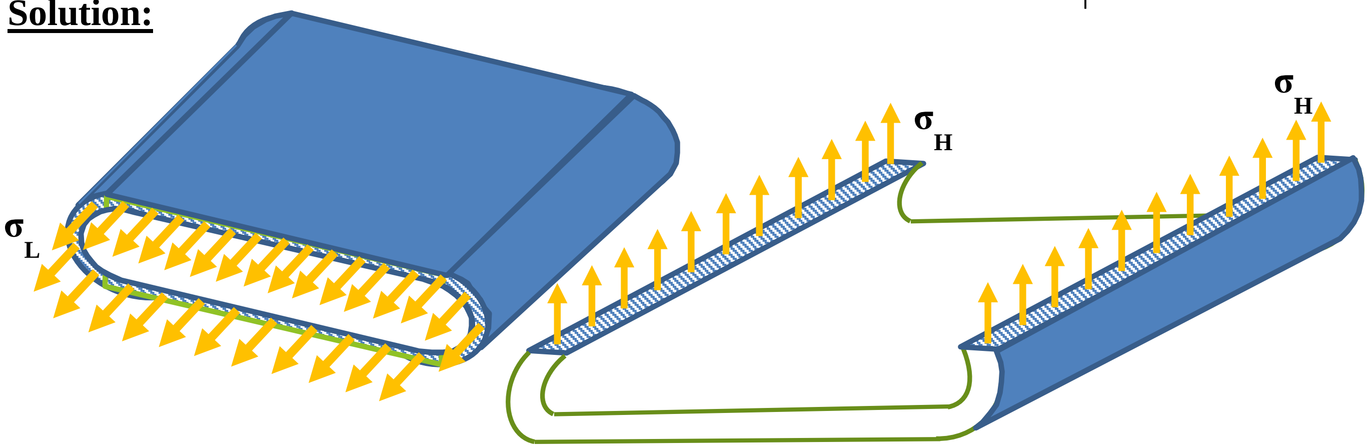


Figure (3.23)

Solution:



$$\sum F_y = 0$$

$$\sigma_H (L)(t)(2) = p_o (b)(L)$$

$$\sigma_H = \frac{p_o(b)}{2t} = \frac{1.2(600+400)}{2(10)} = 60\ MPa$$

$$\sum F_x = 0$$

$$\sigma_L (2 \times 600 + D\pi)t = p_o\left(D^2\frac{\pi}{4} + D \times 600\right)$$

$$\sigma_L = \frac{p_o\left(D^2\frac{\pi}{4} + D \times 600\right)}{(2 \times 600 + D\pi)t}$$

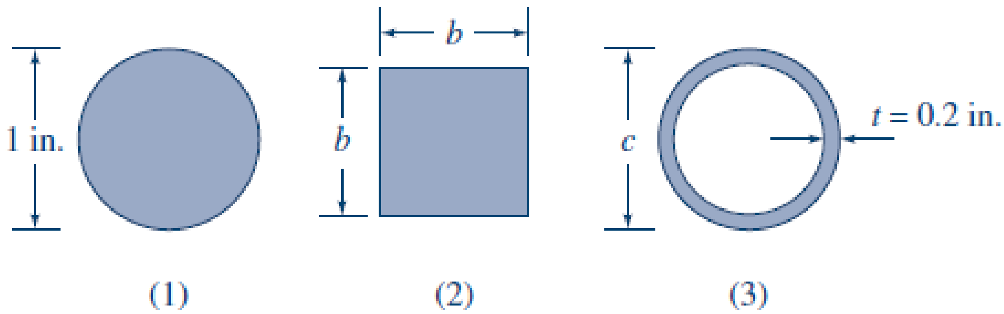
$$\sigma_L = \frac{1.2(400^2\frac{\pi}{4} + 600(400))}{(1200 + 400\pi)(10)} = 17.9\ MPa$$

Problems

Problem 1. A 1-in.-diameter solid bar (1), a square solid bar (2), and a circular tubular member with 0.2-in. wall thickness (3), each supports an axial tensile load of 5 kips.

(a) Determine the axial stress in bar (1).

(b) If the axial stress in each of the other bars is 6 ksi, what is the dimension, ***b***, of the square bar, and what is the outer diameter, ***c***, of the tubular member?



Problem 2. Consider the free-hanging rod shown in Figure (problem 3.2). The rod has the shape of a conical frustum, with radius R_0 at its top and radius R_L at its bottom, and it is made of material with mass density. The length of the rod is L. Determine an expression for the normal stress, (x), at an arbitrary cross section x $(0 \leq x \leq L)$, where x is measured downward from the top of the rod.

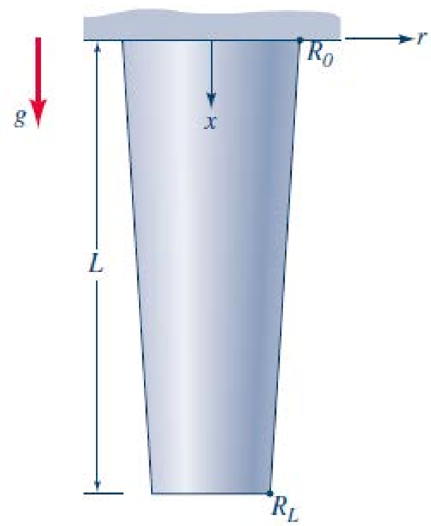


Figure (problem 2)

Problem 3. A solid brass rod *AB* and a solid aluminum rod *BC* are connected together stby a coupler at *B*, as shown in Figure (problem 3).The diameters of the two segmen are d_1=60 mm and d_2 =50 mm, respectively. Determine the axial stresses σ_1 (in rod *AB*) and σ_2 (in rod *BC*).

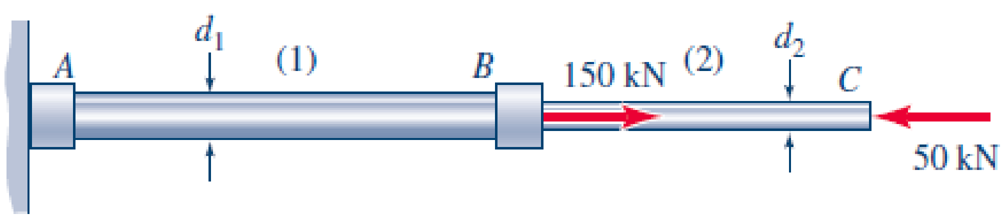


Figure (problem 3)

Problem 4. A column in a two-story building is fabricated from square structural tubing having the cross-sectional dimensions shown in Figure (Problem 4). Axial loads P_A =200 kN and P_B =350 kN are applied to the column at levels *A* and *B*, as shown in Figure (Problem 4a). Determine the axial stress $_1$ in segment *AB* of the column and the axial stress $_2$ in segment *BC* of the column. Neglect the weight of the column itself

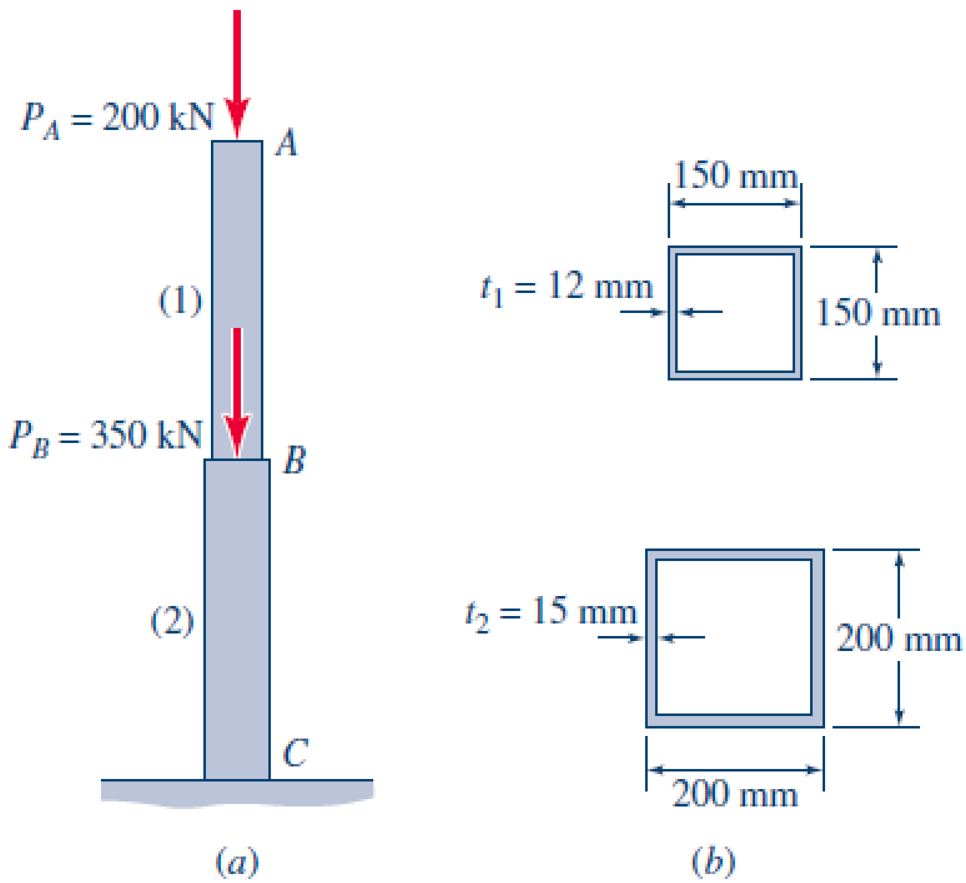


Figure (Problem 4)