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Fatigue Failure

Fatigue is the lowering of strength or failure of a material due to repetitive stress which may be above or below the yield strength. It is a common phenomenon in load-bearing components in cars and airplanes, turbine blades, springs, crankshafts and other machinery, biomedical implants, and **consumer products**, such as shoes or springs, that are subjected constantly to repetitive stresses in the form of tension, compression, bending, vibration, thermal expansion and contraction, or other stresses. These stresses are often below the yield strength of the material! However, when the stress occurs a sufficient (enough) number of times, it causes failure by fatigue. Fatigue can occur even if the components are subjected to stress above the yield strength. A component is often subjected to the repeated application of a stress below the yield strength of the material.

Fatigue failures typically occur in three stages. **Firstly**, a tiny crack initiates or nucleates typically at the surface, often at a time well after loading begins. Normally, nucleation sites are at or near the surface, where the stress is at a maximum, and include surface defects such as scratches or pits, sharp corners due to poor design or manufacture, inclusions, grain boundaries, or dislocation concentrations. **Next**, the crack gradually propagates as the load continues to cycle. **Finally**, a sudden fracture of the material occurs when the remaining cross-section of the material is too small to support the applied load. Thus, components fail by fatigue because even though the overall applied stress may remain below the yield stress, at a local length scale the stress intensity exceeds the yield strength. For fatigue to occur, at least part of the stress in the material has to be tensile. We are normally concerned with fatigue of metallic and polymeric materials.

In Ceramics, we normally do not consider fatigue since ceramics typically fail because of their low fracture toughness. Any fatigue cracks that may form will lower the useful life of the ceramic since it will cause the lowering of the fracture toughness. In general, we design ceramics for static (and not cyclic) loading.

Polymeric materials also show fatigue failure. The mechanism of fatigue in polymers is different than that in metallic materials. In polymers, as the materials are subjected to repetitive stresses, considerable heating can occur near the crack tips and the inter-relationships between fatigue and another mechanism, known as **creep**, affect the overall behavior.

Fatigue is also important in dealing with **Composites**. As fibers or other reinforcing phases begin to degrade (damaged or destroyed) as a result of fatigue, the overall elastic modulus of the composite decreases and this weakening will be seen before the fracture due to fatigue.

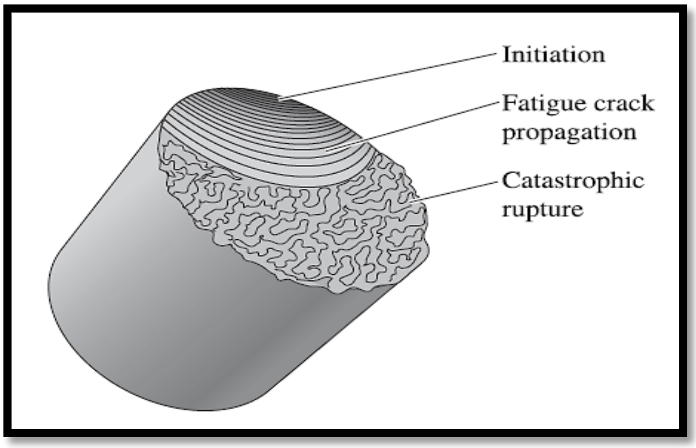

Figure 1. Schematic representation of a fatigue fracture surface in a steel shaft.

In figure 1, fatigue failures are often easy to identify. The fracture surface particularly near the origin is typically **smooth**. The surface becomes **rougher** as the original crack increases in size and may be **fibrous** during final crack propagation.

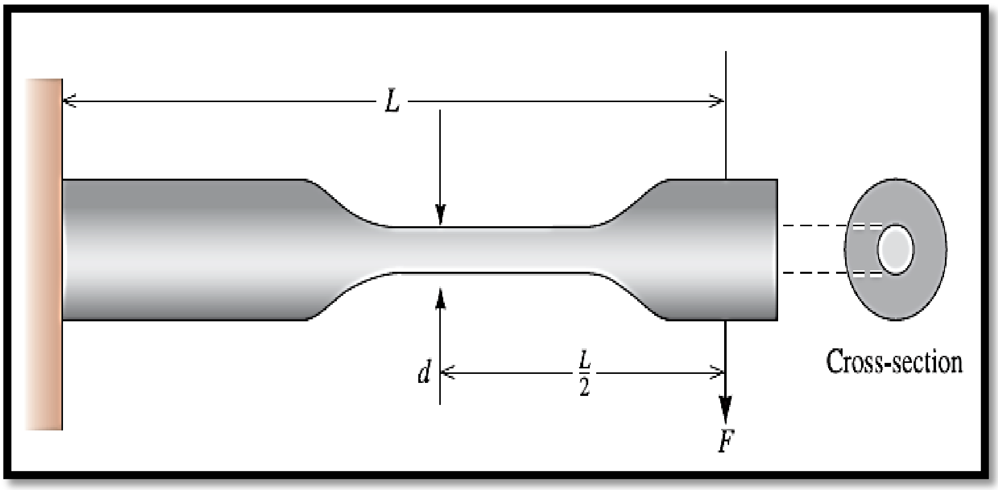

Figure 2. Geometry for the rotating cantilever beam specimen setup.

Application of Fatigue Testing

Material components are often subjected to loading conditions that do not give equal stresses in tension and compression (Figure 3). For example, the maximum stress during compression may be less than the maximum tensile stress. In other cases, the loading may be between a maximum and a minimum tensile stress; here the **(S-N)** curve is presented as the stress amplitude versus number of cycles to failure. Stress amplitude **(σ_a)** is defined as half of the difference between the maximum and minimum stresses; mean stress **(σ_m)** is defined as the average between the maximum and minimum stresses:

Maximum stress, σ_{max}

Minimum stress, σ_{min}

Stress range

$$\Delta\sigma \text{ or } \sigma_r = \sigma_{max} - \sigma_{min} \qquad \text{Eq.1}$$

Alternating stress

$$\sigma_a = \frac{\Delta\sigma}{2} = \frac{\sigma_{max} - \sigma_{min}}{2} \qquad \text{Eq.2}$$

Mean stress

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \qquad \text{Eq.3}$$

Stress ratio

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

Amplitude ratio

$$A = \frac{\sigma_a}{\sigma_m} = \frac{1-R}{1+R}$$

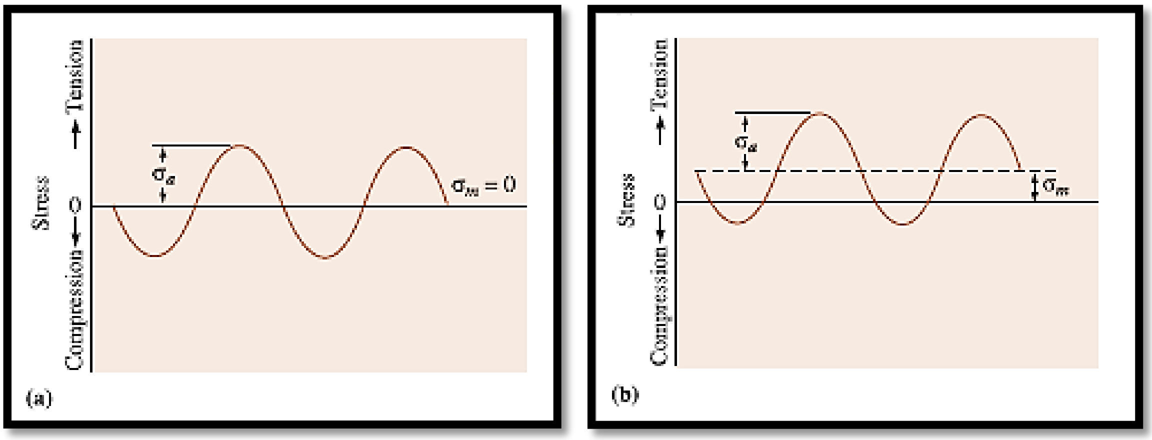

Figure 3. Examples of stress cycles. (a) Equal stress in tension and compression, (b) greater tensile stress than compressive stress.

A compressive stress is considered a negative stress. Thus, if the maximum tensile stress is 50,000 psi and the minimum stress is a 10,000 psi compressive stress, using Equations (1, 2, 3), the stress amplitude is 20,000 psi and the mean stress is 30,000 psi.

As the mean stress increases, the stress amplitude must decrease in order for the material to withstand the applied stresses. This condition can be summarized by the Goodman relationship:

$$\sigma_a = \sigma_{fs}\left[1 - \left(\frac{\sigma_m}{\sigma_{TS}}\right)\right]$$

where $\boldsymbol{\sigma_{fs}}$ is the desired fatigue strength for zero mean stress and $\boldsymbol{\sigma_{TS}}$ is the tensile strength of the material. Therefore, in a typical rotating cantilever beam fatigue test, where the mean stress is zero, a relatively large stress amplitude can be tolerated without fatigue. If, however, an airplane wing is loaded near its yield strength, vibrations of even a small amplitude may cause a fatigue crack to initiate and grow.

Crack Growth Rate: In many cases, a component may not be in danger of failure even when a crack is present. To estimate when failure might occur, the rate of propagation of a crack becomes important. Figure 4 shows the crack growth rate versus the range of the stress-intensity factor **ΔK**, which characterizes crack geometry and the stress amplitude. Below a threshold **ΔK**, a crack does not grow; for somewhat higher stress-intensities, cracks grow slowly; and at still higher stress-intensities.

A crack grows at a rate given by:

$$\frac{da}{dN} = C(\Delta K)^n$$

In above equation, **C** and **n** are empirical constants that depend upon the material. Finally, when **ΔK** is still higher, cracks grow in a rapid and unstable manner until fracture occurs.

The rate of crack growth increases as a crack increases in size, as predicted from the stress intensity factor (Equation):

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$$\Delta K = K_{max} - K_{min} = f\sigma_{max}\sqrt{\pi a} - f\sigma_{min}\sqrt{\pi a} = f\Delta\sigma\sqrt{\pi a}$$

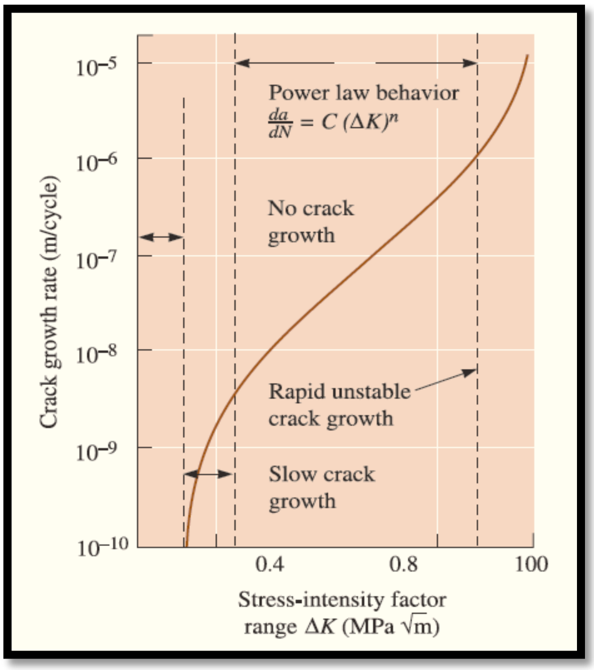

Figure 4. Crack growth rate versus stress intensity factor range for a high strength steel. For this steel, C = 1.62×10^{-12} and n = 3.2 for the units shown.

If the cyclical stress **$\Delta\sigma = (\sigma_{max} - \sigma_{min})$** is not changed, then as crack length (**a**) increases, **ΔK** and the crack growth rate **da/dN** increase. In using this expression, however, one should note that a crack will not propagate during compression. Therefore, if **σ_{min}** is compressive, or less than zero, then **σ_{min}** should be set equal to zero.

Knowledge of crack growth rate is of assistance (help) in designing components and in nondestructive evaluation to determine if a crack poses imminent danger to the structure. One approach to this problem is to estimate the number of cycles required before failure occurs. By rearranging Equation and substituting for **ΔK**:

$$dN = \frac{1}{Cf^n \Delta\sigma^n \pi^{n/2}} \frac{da}{a^{n/2}}$$

If we integrate this expression between the initial size of a crack and the crack size required for fracture to occur, we find that

$$N = \frac{2[(a_c)^{(2-n)/2} - (a_i)^{(2-n)/2}]}{(2-n)Cf^n \Delta\sigma^n \pi^{n/2}}$$

where $\mathbf{a_i}$ is the initial flaw size and $\mathbf{a_c}$ is the flaw size required for fracture. If we know the material constants **n** and **C** in Equation, we can estimate the number of cycles required for failure for a given cyclical stress.

Effect of Temperature: As the material's temperature increases, both fatigue life and endurance limit decrease. Furthermore, a cyclical temperature change encourages failure by thermal fatigue; when the material heats in a non-uniform manner, some parts of the structure expand more than others. This non-uniform expansion introduces a stress within the material, and when the structure later cools and contracts, stresses of the opposite sign are imposed (applied). As a consequence (importance) of the thermally induced stresses and strains, fatigue may finally occur. The frequency with which the stress is applied also influences fatigue behavior. In particular, high-frequency stresses may cause polymer materials to heat; at increased temperature, polymers fail more quickly. Chemical effects of temperature (e.g., oxidation) must also be considered.

EXAMPLE: Design of a Fatigue-Resistant Plate.

A high-strength steel plate (Figure 4), which has a plane strain fracture toughness of **80 MPa √m**, is alternately loaded in tension to **500 MPa** and in compression to **60 MPa**. The plate is to survive for **10** years, with the stress being applied at a frequency of once every **5** minutes. Design a manufacturing and testing procedure that assures that the component will serve as intended.

SOLUTION

$$K_{Ic} = f\sigma\sqrt{\pi a_c}$$

$$80 \text{ MPa}\sqrt{\text{m}} = (1)(500 \text{ MPa})\sqrt{\pi a_c}$$

$$a_c = 0.0081 \text{ m} = 8.1 \text{ mm}$$

The maximum stress is 500 MPa; however, the minimum stress is zero, not 60 MPa in compression, because cracks do not propagate in compression. Thus, $\Delta\sigma$ is

$$\Delta\sigma = \sigma_{\text{max}} - \sigma_{\text{min}} = 500 - 0 = 500 \text{ MPa}$$

We need to determine the minimum number of cycles that the plate must withstand:

$$N = (1 \text{ cycle}/5 \text{ min})(60 \text{ min/h})(24 \text{ h/d})(365 \text{ d/y})(10 \text{ y})$$

$$N = 1{,}051{,}200 \text{ cycles}$$

If we assume that $f = 1$ for all crack lengths and note that $C = 1.62 \times 10^{-12}$ and $n = 3.2$ in Equation , then

$$1{,}051{,}200 = \frac{2[(0.008)^{(2-3.2)/2} - (a_i)^{(2-3.2)/2}]}{(2-3.2)(1.62 \times 10^{-12})(1)^{3.2}(500)^{3.2}\pi^{3.2/2}}$$

$$1{,}051{,}200 = \frac{2[18 - a_i^{-0.6}]}{(-1.2)(1.62 \times 10^{-12})(1)(4.332 \times 10^{8})(6.244)}$$

$$a_i^{-0.6} = 18 + 2764 = 2782$$

$$a_i = 1.82 \times 10^{-6} \text{ m} = 0.00182 \text{ mm for surface flaws}$$

$$2a_i = 0.00364 \text{ mm for internal flaws}$$

The manufacturing process must produce surface flaws smaller than 0.00182 mm in length. We can conduct a similar calculation for specifying a limit on edge cracks. In addition, nondestructive tests must be available to assure that cracks exceeding this length are not present.