Class: 2nd
Subject: Strength of Materials
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Strength of Materials

Second Stage

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Stresses in Beams

Bending Stress

Beam-Deformation Terminology

To simplify this study of beams, we initially:

- consider only straight beams that have a **longitudinal plane of symmetry** (LPS), and
- For which the loading and support are symmetric with respect to this plane, as illustrated in Figure (5.1).

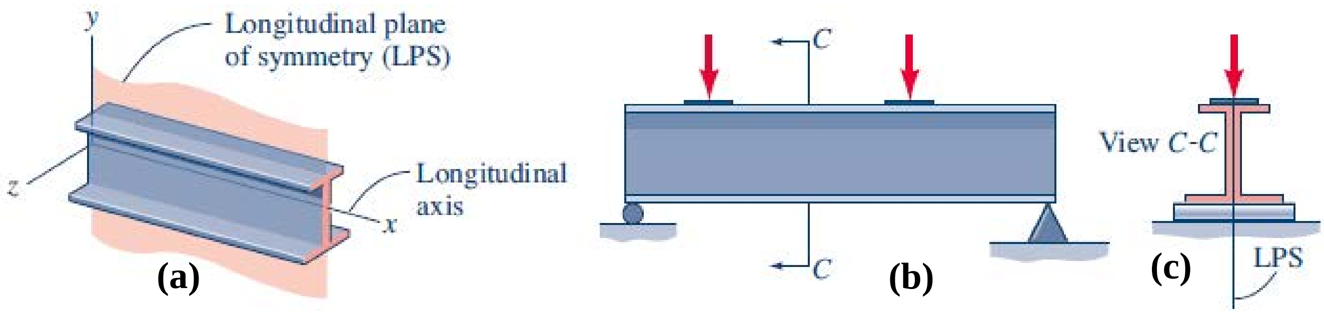


Figure (5.1)

- Under these conditions, this longitudinal plane of symmetry is the **plane of bending.**
- As indicated in Figure (5.1a), **coordinate axes** will be assigned as follows:
- The plane of bending is labeled the ***xy*** plane;
- The **longitudinal axis** of the beam is labeled the ***x*** axis, with the positive ***x*** axis directed to the right;
- The positive ***y*** axis points upward; and, finally,
- The ***z*** axis forms a right-handed coordinate system with the other two axes.
- To investigate the distribution of stresses in a beam, it is convenient to imagine the beam to be a bundle of many ***longitudinal fibers*** parallel to the ***x*** axis. Figure (5.2) depicts a few of these imaginary "fibers."

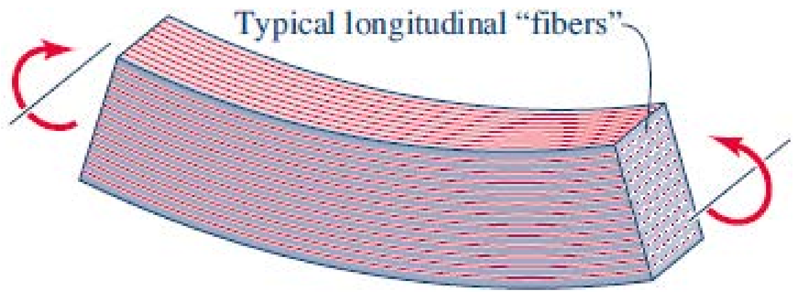


Figure (5.2): A beam represented as a "bundle" of longitudinal "fibers."

- Under the action of an applied bending moment ***M***, there is a shortening of the upper fibers and a stretching of the lower fibers, causing the beam segment to be curved upward. But some longitudinal fibers retain their original length. These are said to form the **neutral surface** (NS), which is identified in Figure (5.3).

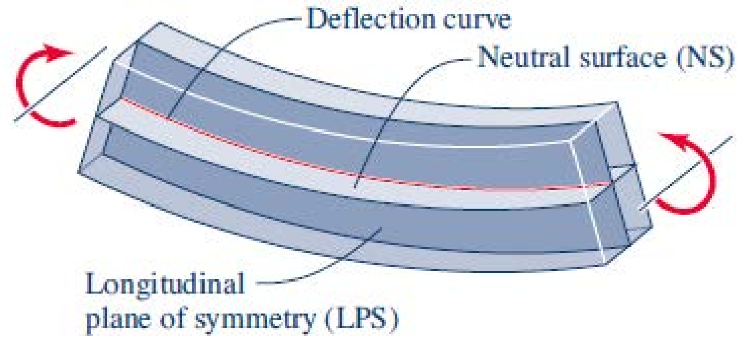


Figure (5.3)

- If equal couples M_0 are applied to the ends of an otherwise unloaded segment of beam, as in Figure (5.4), the moment is constant along the segment and the segment is said to be in **pure bending.**

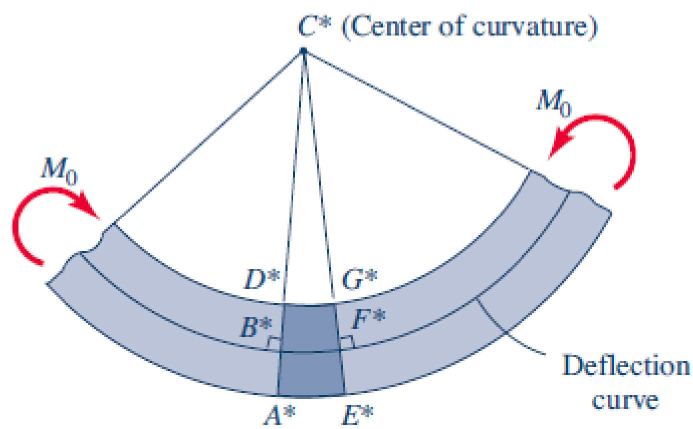


Figure (5.4)

Assumptions in the Theory of Simple Bending

The following assumptions are made in the theory of simple bending:

- ❑ The material of the beam is perfectly homogeneous (i.e. of the same kind throughout) and isotropic (i.e. of equal elastic properties in all directions).
- ❑ The beam material is stressed within its elastic limit and thus, obeys Hooke’s law.
- ❑ The transverse sections, which were plane before bending, remains plane after bending also.

- ❑ Each layer of the beam is free to expand or contract, independently, of the layer above or below it.
- ❑ The value of **E** (Young's modulus of elasticity) is the same in tension and compression.
- ❑ The beam is in equilibrium.
- ❑ The strain (ε) of a layer is proportional to its distance from the neutral axis (y).

$$\boldsymbol{\varepsilon = \frac{y}{\rho}}$$

Where $\boldsymbol{\rho}$ is the radius of curvature (Figure 5.5).

- ❑ Also knowing that the bending stress:

$$\boldsymbol{\sigma_b = strain \times Elasticity = \varepsilon E}$$

$$\boldsymbol{\therefore \sigma_b = \frac{y}{\rho} E = y \frac{E}{\rho}}$$

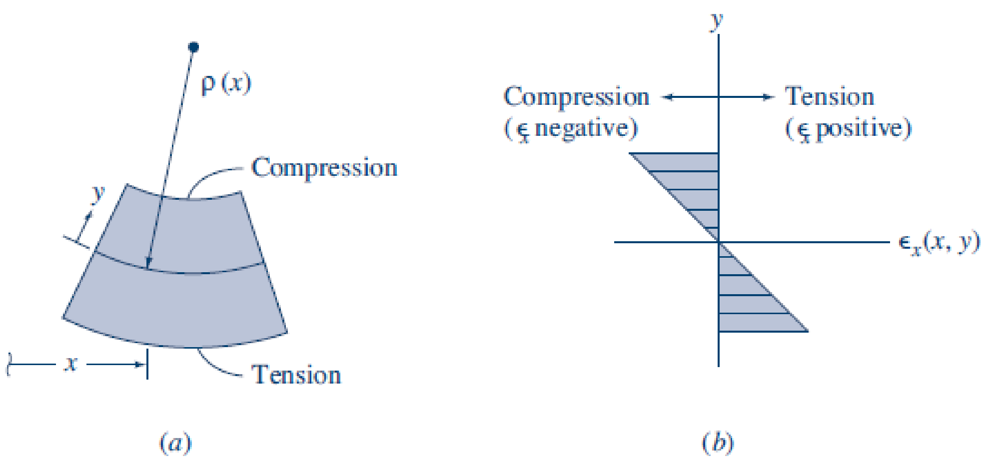


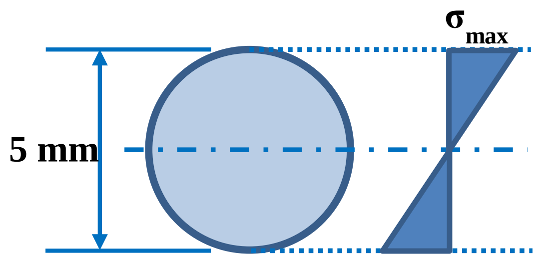
Figure (5.5): The strain distribution at a cross section where (x) is positive.

- ▶ Since $\boldsymbol{E}$ and $\boldsymbol{\rho}$ are constants in this expression, therefore the stress at any point is directly proportional to (y), i.e., the distance of the point from the neutral axis.

❖ **Note. Since the bending stress is inversely proportional to the radius (ρ), therefore for maximum stress the radius be minimum and vice versa.**

Examples

Example (5.1): A steel wire of (5mm) diameter is bent into a circular shape of (5m) radius. Determine the maximum stress induced in the wire. Take E=200 GPa.



Solution:

$$y = \frac{5}{2} = 2.5\ mm$$

$$\sigma_{max} = y\frac{E}{\rho} = (2.5)\frac{200 \times 10^3}{5 \times 10^3} = 100\ MPa$$

Example (5.2): A metallic rod of (10mm) diameter is bent into a circular form of radius (6m). If the maximum bending stress developed in the rod is (125 MPa), find the value of Young's modulus for the rod material.

Solution:

$$y = \frac{10}{2} = 5\ mm$$

$$E = \frac{\sigma_{max}}{y}\rho = \frac{125}{5}(6 \times 10^3) = 150 \times 10^3\ MPa$$

Moment of Resistance

- Consider a section of the beam as shown in Figure (5.6). Let N.A be the neutral axis of this section.

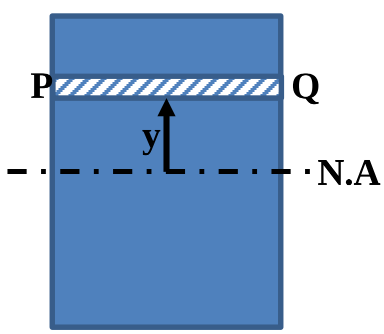


Figure (5.6)

- Now consider a small layer ***PQ*** of the beam section at a distance (y) from the neutral axis.
- Let ***dA***=area of the layer ***PQ***
- The intensity of stress in the layer ***PQ*** is:

$$\sigma = y\frac{E}{\rho}$$

and the total force in the layer ***PQ*** is:

$$dF = y\frac{E}{\rho}(dA)$$

and the moment of this total force about the neutral axis is:

$$dM = y\frac{E}{\rho}(dA)(y) = \frac{E}{\rho}y^2dA$$

- The algebraic sum of all such moments about the neutral axis must equal to M. Therefore,

$$M = \sum \frac{E}{\rho}y^2dA$$

- The expression ($\sum y^2dA$) represents the moment of inertia (***I***) of the whole section about the neutral axis. Therefore,

$$M = \frac{E}{\rho}\times I \ \rightarrow \frac{M}{I} = \frac{E}{\rho}$$

$$\nabla \quad \sigma = \frac{E}{\rho}y \quad or \quad \frac{\sigma}{y} = \frac{E}{\rho}$$

$$\therefore \frac{M}{I} = \frac{\sigma}{y}$$

$$\rightarrow \quad \boldsymbol{\sigma} = \frac{\boldsymbol{My}}{\boldsymbol{I}}$$

Example (5.3): Find the maximum tensile and compressive stresses for the beam shown in Figure (5.6).

Solution:

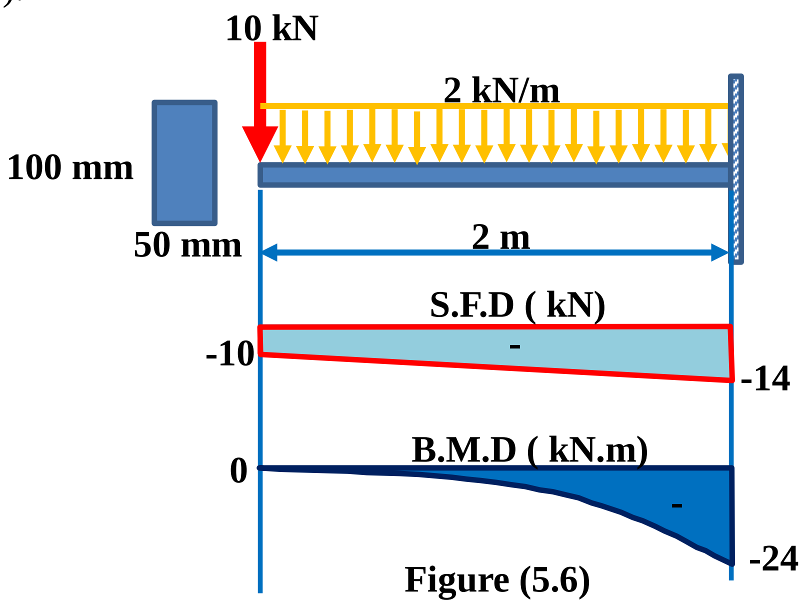


Figure (5.6)

$$\sum F_y = 0 \ \rightarrow R = 10 + 2(2) = 14\ kN\ \uparrow$$

$$\sum M = 0 \ \rightarrow \ \ M = 10(2) + 2(2)(1) \rightarrow M = 24\ kN.m$$

$$\bar{y} = \frac{\sum a_i y_i}{\sum a_i} \quad , here \qquad \bar{y} = 50mm$$

$$I = \frac{bh^3}{12} = \frac{50(100)^3}{12} = 4.167 \times 10^6 mm^4$$

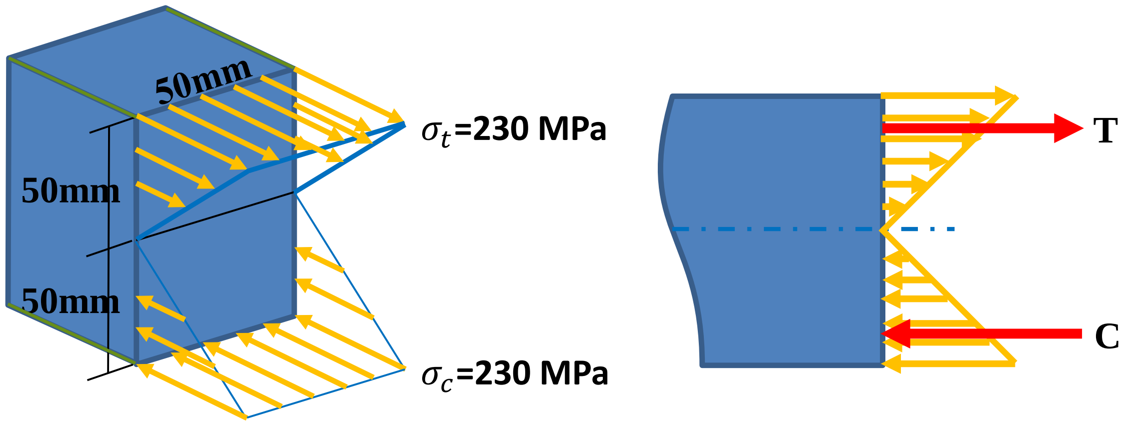
$$\sigma = \frac{My}{I}$$

$$\sigma_t = \frac{24 \times 10^6(50)}{4.167 \times 10^6} = 230\ MPa$$

$$\sigma_c = \frac{24 \times 10^6(50)}{4.167 \times 10^6} = 230\ MPa$$

Find the tensile and compressive forces:

$$T = C = \frac{1}{2}(230)(50)(50) = 287.9\ kN$$



Example (5.4): Find the dimension (**b**) for the beam shown in Figure (5.7), if the flexural stress is not to exceed (10 MPa).

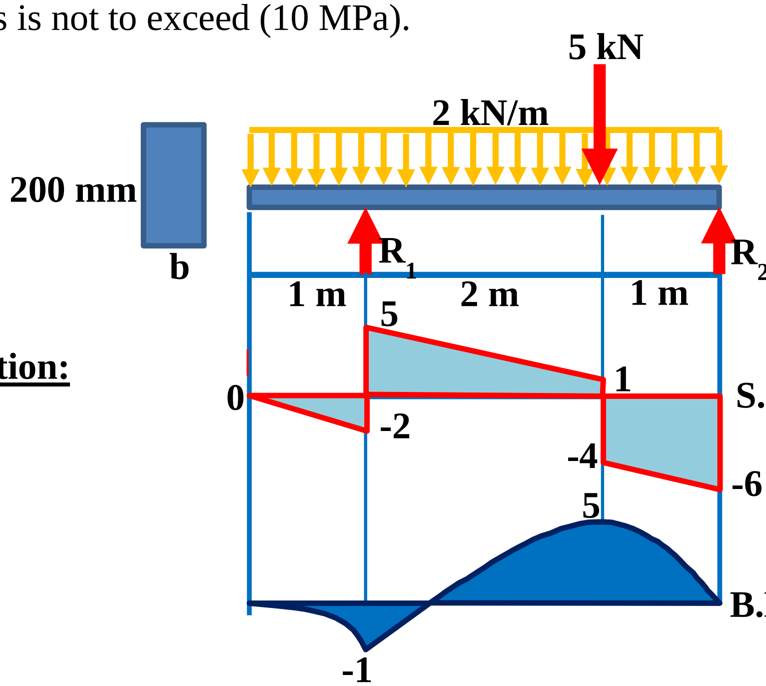


Figure (5.7)

Solution:

$$\sum M_{R2} = 0 \rightarrow$$

$$R_1(3) = 2(4)(2) + 5(1)$$

$$\rightarrow R_1 = 7\ kN \uparrow$$

$$\sum Fy = 0 \rightarrow R_2 = 6\ kN \uparrow$$

▶ From B.M.D, the max. bending moment is: **5 kN.m**

$$I = \frac{bh^3}{12} = \frac{b(200)^3}{12} = 6.67 \times 10^5 b\ mm^4$$

$$\sigma = \frac{My}{I} \rightarrow 10 = \frac{5 \times 10^6(100)}{6.67 \times 10^5 b}$$

$$\rightarrow b = 75mm$$

Example (5.5): A circular bar (20mm) in diameter, bent into a semicircle the **a** mean radius of (600mm) as shown in Figure (5.8), if (P=2000N) and (F=1000N), compute the maximum flexural stress developed at section (a –a). Neglect the deformation of the bar.

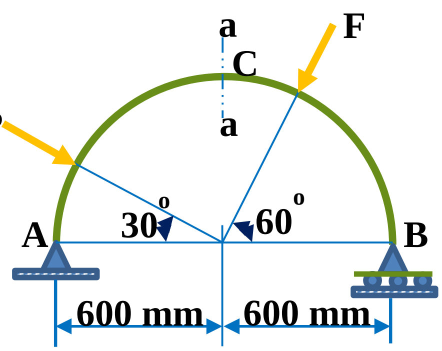


Figure (5.8)

Solution:

$$\sum M_B = 0 \rightarrow$$

$$A_y(1200) - (2000sin30 + 1000sin60)(600) = 0$$

$$\rightarrow A_y = 933\ N \uparrow$$

$$\sum Fy = 0 \rightarrow B_y = 933\ N \uparrow$$

$$\sum Fx = 0 \rightarrow A_x + 2000cos30 - 1000cos60 = 0$$

$$\rightarrow A_x = -1232\ N \rightarrow A_x = 1232\ N \leftarrow$$

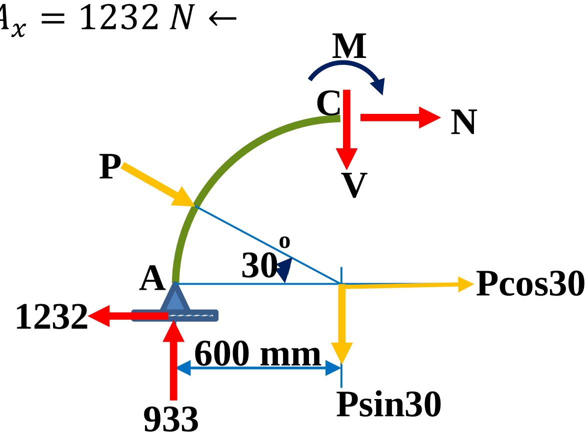
Section (a-a):

$$\sum M_C = 0 \rightarrow$$

$$933(600) + 1232(600) -$$

$$2000cos30(600) = M$$

$$\rightarrow M = 259800\ N.m$$



$$I = \frac{\pi}{32}(D^4) = \frac{\pi}{32}(20^4) = 15708\ mm^4$$

$$C = r = 10mm$$

$$\sigma = \frac{My}{I} = \frac{259800(10)}{15708} = 331\ MPa$$

Example (5.6): A wooden beam (150mm) wide by (300 mm) deep is loaded as shown in Figure (5.9). If the maximum flexural stress is (8 MN/m^2), find the maximum values of (***w***) and (***P***) that can be applied simultaneously.

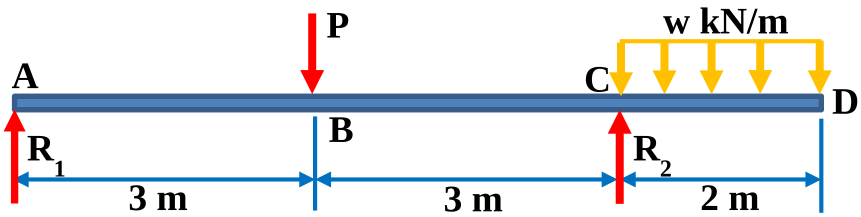
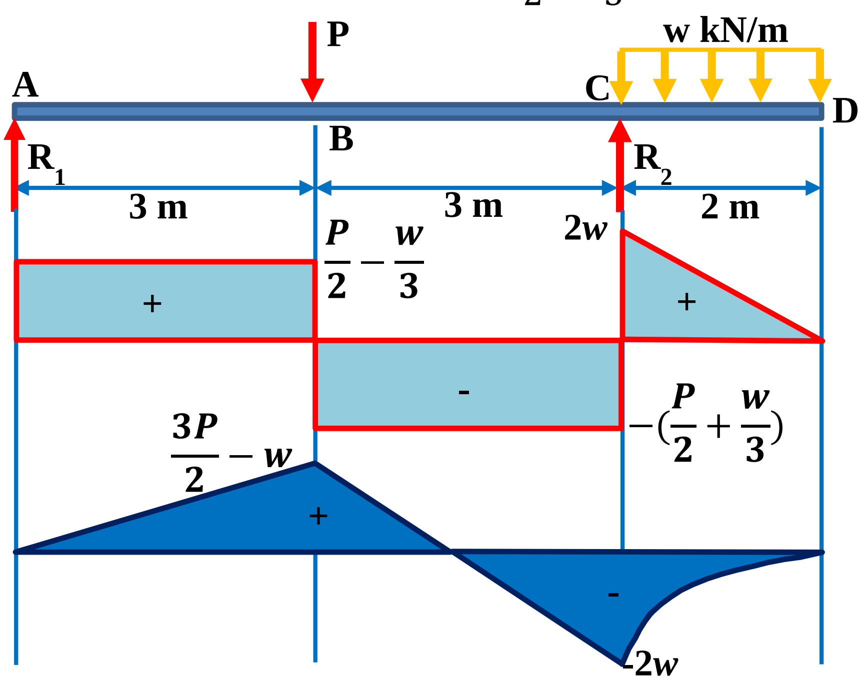


Figure (5.9)

Solution:

$$\sum M_{R2} = 0 \ \rightarrow R_1(6) + 2w(1) - 3P = 0 \rightarrow R_1 = \frac{P}{2} - \frac{w}{3}$$

$$\sum Fy = 0 \ \rightarrow R_2 = \frac{P}{2} + \frac{7w}{3}$$



After drawing B.M.D, get:

$$M_B = \frac{3P}{2} - w \quad and \quad M_C = -2w$$

$$I = \frac{bh^3}{12} = \frac{150(300)^3}{12} = 3.375 \times 10^8 \; mm^4$$

$$\sigma = \frac{My}{I} \rightarrow 8 \times 10^6 = \frac{2w(150 \times 10^{-3})}{3.375 \times 10^8}$$

$$\rightarrow w = 9 \; kN/m$$

$$\sigma = \frac{My}{I} \rightarrow 8 \times 10^6 = \frac{(\frac{3P}{2} - w)(150 \times 10^{-3})}{3.375 \times 10^8}$$

$$\rightarrow P = 18 \; kN$$

<u>Example (5.7):</u> The cross section of a beam is a T with the dimensions shown in Figure (5.10). The moment at the section is ***M*= 4 kip.ft**. Determine:

(a) The location of the neutral axis of the cross section,

(b) The moment of inertia with respect to the neutral axis, and

(c) The maximum tensile stress and the maximum compressive stress on the cross section.

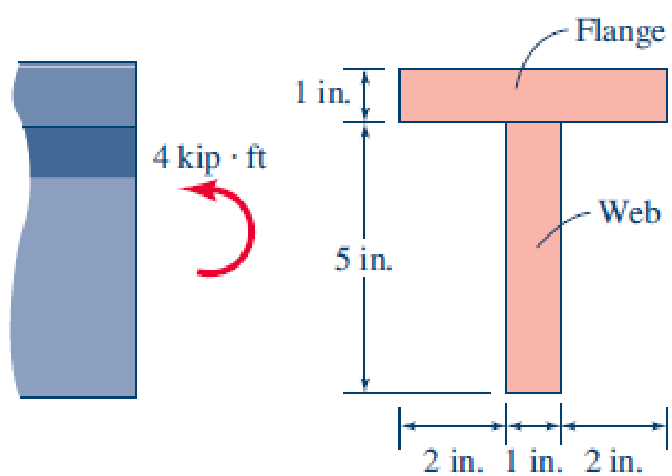
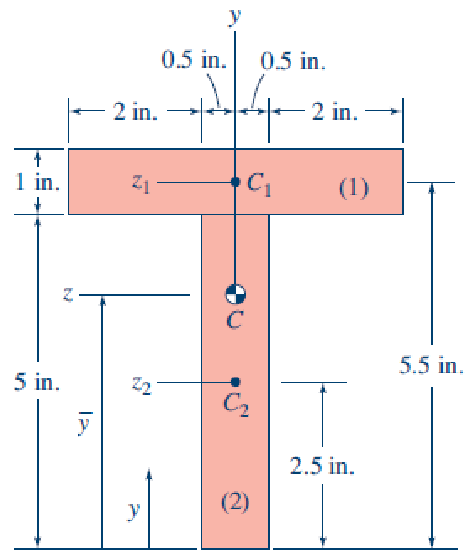


Figure (5.10)

Solution:

(a) Locate the neutral axis:



$$\bar{y} = \frac{\sum a_i y_i}{\sum a_i} = \frac{5(1)(5.5) + 1(5)(2.5)}{5(1) + 1(5)}$$

$$\bar{y} = 4\ in.$$

(b) Determine the moment of inertia with respect to the neutral axis:

$$I_{\bar{c}} = \sum(I_c + Ad^2)$$

$$I_{\bar{c}} = \frac{5(1)^3}{12} + 5(1)(5.5-4)^2 + \frac{1(5)^3}{12} + 1(5)(4-2.5)^2$$

$$I_{\bar{c}} = 33.3\ in^3$$

(c) Compute $\sigma_T)_{max.}$ and $\sigma_C)_{max.}$:

The maximum compression occurs at the top of the beam, and the maximum tension occurs at the bottom of the beam. From the flexure formula,

$$\sigma = \frac{My}{I} \rightarrow$$

$$\sigma_T)_{max.} = \frac{(4\text{ kip.ft})(12\text{ in./ft})(4\text{ in.})}{33.3\ in^3} = 5.76\ ksi$$

$$\sigma_C)_{max.} = \frac{(4\text{ kip.ft})(12\text{ in./ft})(2\text{ in.})}{33.3\ in^3} = 2.88\ ksi$$

▶ **For the beam of Example 5.7:-**

(a) Determine the resultant compressive force F_C, the resultant tensile force F_T, and the distance that separates them, as indicated in Figure (5.11); and

(b) Show that F_C and F_T form a couple of magnitude ***M*=4 kip.ft.**

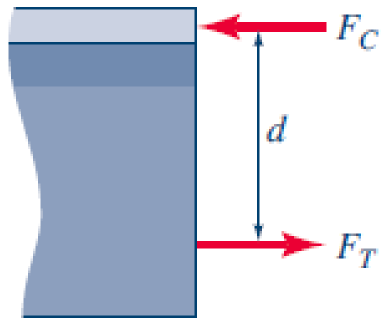


Figure (5.11)

Solution:

(a) Determine the resultant compressive force, the resultant tensile force, and the distance between them.

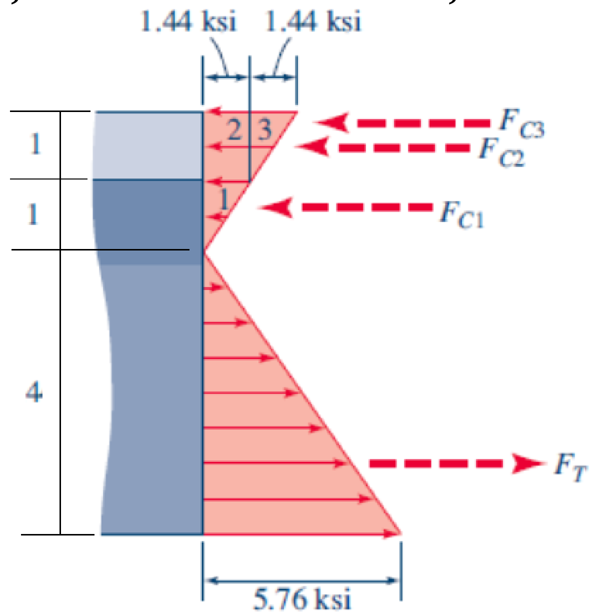
$$F_{C1} = \frac{1}{2}(1.44)(1)(1) = 0.72\ kips$$

$$F_{C2} = (1.44)(5)(1) = 7.2\ kips$$

$$F_{C3} = \frac{1}{2}(1.44)(5)(1) = 3.6\ kips$$

$$F_C = F_{C1} + F_{C2} + F_{C3} = 11.52\ kips$$

$$F_T = \frac{1}{2}(5.76)(1)(4) = 11.52\ kips$$



Since F_C is the resultant of F_{C1}, F_{C2}, and F_{C3}, we determine its location by computing first moments using the dimensions shown in Figure (5.12):

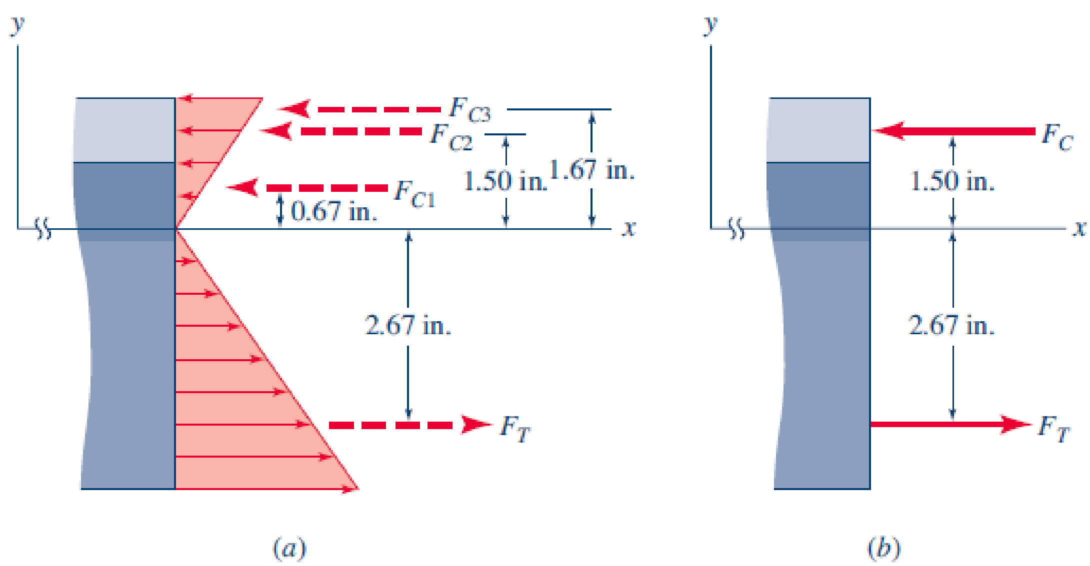


Figure (5.12)

$$F_{C1}(d_{C1}) + F_{C2}(d_{C2}) + F_{C3}(d_{C3}) = F_C(d_C)$$

$$d_C = \frac{F_{C1}(d_{C1}) + F_{C2}(d_{C2}) + F_{C3}(d_{C3})}{F_C}$$

$$d_C = \frac{0.72(0.67) + 7.2(1.5) + 3.6(1.67)}{11.52} = 1.5\ in.$$

As indicated in Figure (5.12*b*).Therefore,

$$d = d_C + d_T$$

$$d = 1.5 + 2.67 = 4.17\ in.$$

$$F_C = F_T = 11.52\ kips$$

(b) Determine the magnitude of the couple formed by F_C and F_T.

Since $\boldsymbol{F_C}$, $\boldsymbol{F_T}$, the resultant force on the cross section is zero. The couple formed by F_C and F_T is given by:

$$M = F_C(d) = F_T(d) = 11.52(4.17)$$

$$M = 48\ kips.\ in$$

Or

$$M = 48\ kips.\ in \times \left(\frac{1}{12}\right) = 4\ kips.ft$$

Which is the value of the applied moment.