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MSc. Murtadha Al-Masoudy.
Mathematics-2-
2nd Year.

## Complex Numbers

A Complex number can be represented by an expression of the form $(a+ib)$ where:

a and b :- are real numbers.
i :- is a symbol with the property that $i^2=-1$ i.e $(i=\sqrt{-1})$ is called imaginary unit.

**Definitions:-**

(1) $Z=a+ib$ where Z: is a complex number
a: real part
b: imaginary part

$\bar{Z}=a-ib$ where $\bar{Z}$: is a Complex Conjugate to the Complex number (Z)

Ex:- If $Z=3+4i$ then $\bar{Z}=3-4i$

(2) The absolute value or modulus of $(a+ib)$ is defined as

$$|a+ib|=\sqrt{a^2+b^2}$$

(3) If $Z_1=a+ib$ and $Z_2=c+id$ are equal then
$a=c$ and $b=d$

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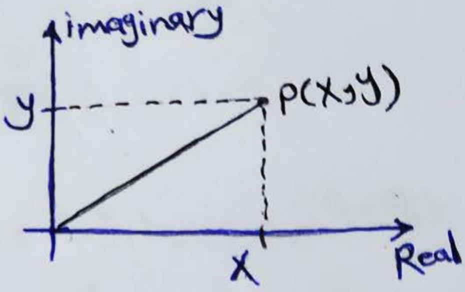
## Complex numbers representation :-

There are two geometric representations of the complex number ($z = x + iy$)

(1) The point $P(x, y)$ in $xy$-plane which is called Argand diagram or complex plane. where $x$-axis represents the real axis and $y$-axis represents the imaginary axis.




(2) The vector $\overrightarrow{OP}$ from the origin to $P(r, \theta)$ which is called polar form of complex numbers where $r$ and $\theta$ are called polar coordinates.

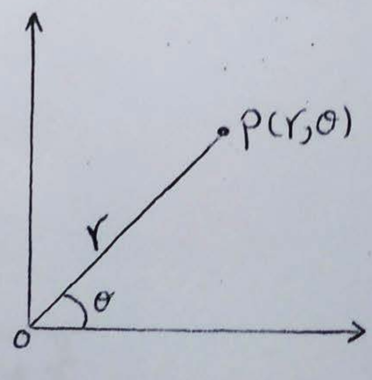
$$z = x + iy = r(\cos\theta + i\sin\theta) = re^{i\theta}$$

$$r = |z| = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\frac{y}{x}$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$




Where $\theta$ is the amplitude or argument angle with $x$-axis.

$\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$ $\leftrightarrow$ Euler's formula.

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## properties of Complex number :-

If $z_1 = x_1 + i y_1 = r_1 e^{i\theta_1}$

$z_2 = x_2 + i y_2 = r_2 e^{i\theta_2}$ then

1. $z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$

2. $z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$

3. $k \cdot z_1 = k(x_1 + i y_1) = k x_1 + k y_1 i$

4. $z_1 \cdot z_2 = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1) = r_1 r_2 e^{i(\theta_1 + \theta_2)}$

5. $z_1 \cdot \bar{z}_1 = |z_1|^2$ and $z_2 \cdot \bar{z}_2 = |z_2|^2$

6. $$\frac{z_1}{z_2} = \frac{z_1 \cdot \bar{z}_2}{z_2 \cdot \bar{z}_2} = \frac{(x_1 + i y_1)(x_2 - i y_2)}{(x_2 + i y_2)(x_2 - i y_2)} = \frac{(x_1 x_2 + y_1 y_2) + i(x_2 y_1 - x_1 y_2)}{x_2^2 + y_2^2}$$
$$= \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}$$

7. $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} = \frac{r_1}{r_2}$ $\quad r_2 \neq 0$

8. $\bar{\bar{z}}_1 = z_1$ and $\bar{\bar{z}}_2 = z_2$

9. $|z_1| = \sqrt{x_1^2 + y_1^2}$ and $|z_2| = \sqrt{x_2^2 + y_2^2}$

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**EX:-** let $Z_1 = 2+3i$ and $Z_2 = 4+i$ find

① $Z_1 + Z_2$ ② $Z_1 - Z_2$

③ $Z_1 \cdot Z_2$ ④ $Z_1 / Z_2$

**Solution:-**

① $Z_1 + Z_2 = (2+3i) + (4+i) = \boxed{6+4i}$

② $Z_1 - Z_2 = (2+3i) - (4+i) = \boxed{-2-2i}$

③ $Z_1 \cdot Z_2 = (2+3i) * (4+i) = 8+2i+12i+3i^2$
$= 8+14i-3 = \boxed{5+14i}$

$i^2 = -1$

④ $\dfrac{Z_1}{Z_2} = \dfrac{(2+3i)}{(4+i)} * \dfrac{(4-i)}{(4-i)} = \dfrac{8-2i+12i-3i^2}{(4)^2+(1)^2}$

$= \dfrac{8+10i+3}{17} = \boxed{\dfrac{11+10i}{17}}$

**EX:-** Put the complex number $(1-i\sqrt{3})$ in the Polar form.

**Solution:-**

$Z = 1 - i\sqrt{3} = x + iy$

$r = |Z| = \sqrt{(1)^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = \boxed{2}$

$\theta = \tan^{-1}\dfrac{y}{x} = \tan^{-1}\dfrac{-\sqrt{3}}{1} = \dfrac{-\pi}{3} = \boxed{-60}$

$Z = r(\cos\theta + i\sin\theta) = re^{i\theta}$

$= 2(\cos(\frac{-\pi}{3}) + i\sin(\frac{-\pi}{3})) = 2e^{-i\frac{\pi}{3}}$

$= 2(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}) = 2e^{-i\frac{\pi}{3}}$

Note:- $\ln re^{i\theta} = \ln r + i\theta$

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**Ex:-** Find $\ln(-2)$

**Solution:-**

$\ln(re^{i\theta}) = \ln r + i\theta$

$\ln(-2) = \ln(-2 + i0)$

$r = \sqrt{(-2)^2 + (0)^2} = \boxed{2}$

$\theta = \tan^{-1}\frac{0}{-2} \Rightarrow \boxed{\theta = \pi}$

$\therefore \ln(-2) = \ln 2 + i\pi$

**Ex:-** Find $x, y$ if $(3+4i)^2 - 2(x - iy) = x + iy$

**Solution:-**

$(3+4i)^2 - 2x + 2iy = x + iy$

$9 + 24i + 16i^2 - 2x + i2y = x + iy$

$i^2 = -1 \Rightarrow 9 + 24i - 16 - 2x + i2y = x + iy$

$-7 - 2x = x \Rightarrow -7 = x + 2x$

$3x = -7 \Rightarrow \boxed{x = -7/3}$

$24i + i2y = iy \quad ] \div i$

$24 = y - 2y \Rightarrow -y = 24$

$\therefore \boxed{y = -24}$