Lecture One

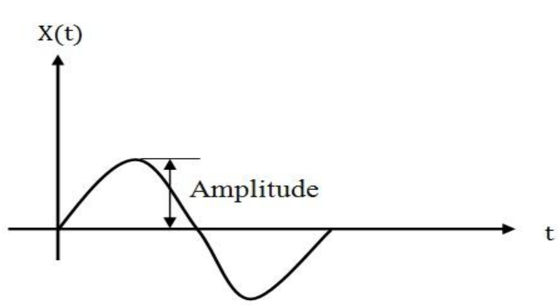
Background

1- Some Mathematical Relations

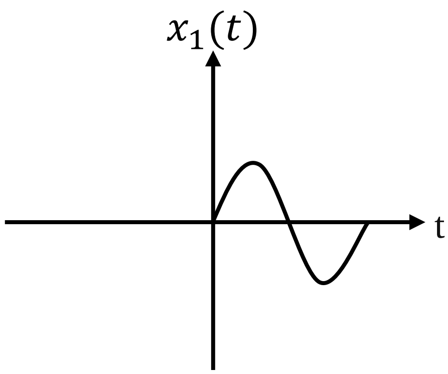
- $\sin at = \frac{e^{jat} - e^{-jat}}{2j}$
- $\cos at = \frac{e^{jat} + e^{-jat}}{2}$
- $sa(t) = \frac{\sin t}{t}$ (sinc function)
- $\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$
- $\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$
- $\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$
- $\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$
- $\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$
- $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$

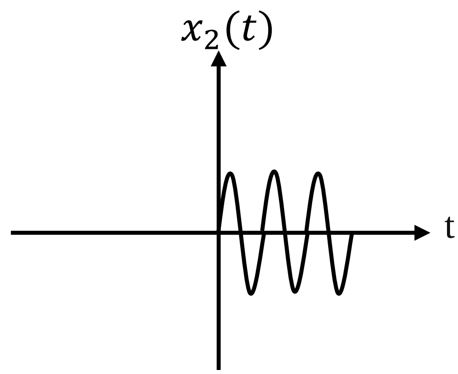
2- Signal Characteristics:

- **Amplitude** (A) is the maximum displacement of a particle in a wave from its equilibrium position. It is measured in meters (m).



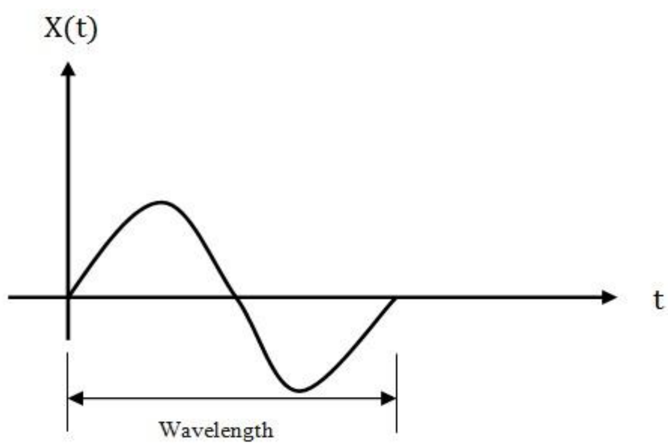
- **Frequency (f)** is the number of complete waves passing a point in one second. It is measured in hertz (Hz).





Note// $x_2(t)$ has higher frequency than $x_1(t)$.

- **Wavelength (λ)** is the distance between two identical points on a wave (i.e. one full wave). It is measured in meters (m).



- **Wave speed (c)** is measured in meters per second (m/s).

Wave speed (c), frequency (f) and wavelength (λ) are linked together in the following equation.

$c = f\lambda$

- c = wave speed (m/s)
- λ = wavelength (m)

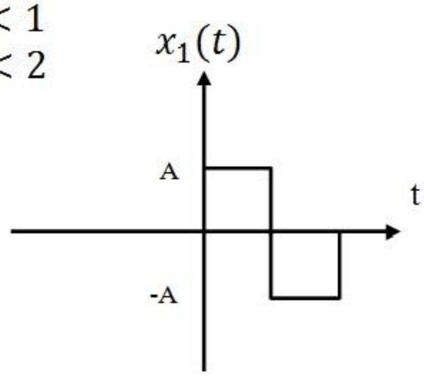
- **Phase**

Points on a wave which are always travelling in the same direction, rising a falling together, are **in phase** with each other.

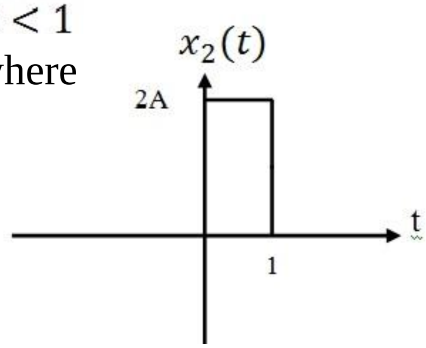
Points on a wave which are always traveling in opposite directions to each other, one is rising while the other is falling, are in **anti-phase** with each other.

3- **Mathematical Representation of Some Function:**

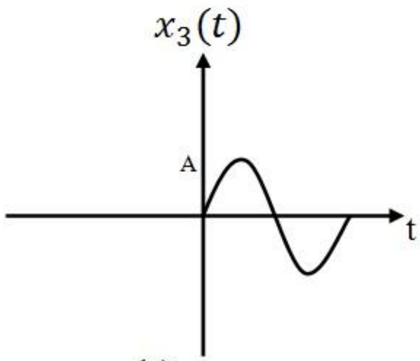
1- $x_1(t) = \begin{cases} A & 0 < t < 1 \\ -A & 1 < t < 2 \end{cases}$



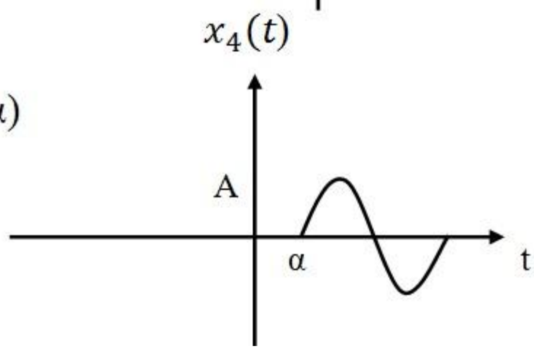
2- $x_2(t) = \begin{cases} 2A & 0 < t < 1 \\ 0 & \text{elsewhere} \end{cases}$



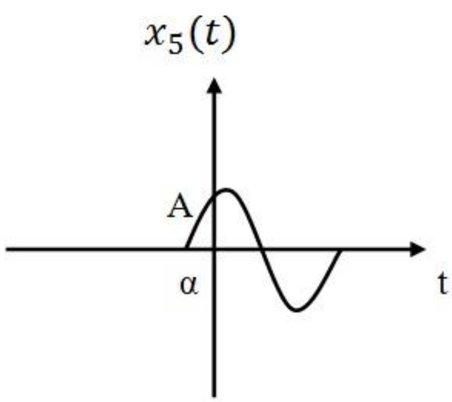
3- $x_3(t)$= A sin t



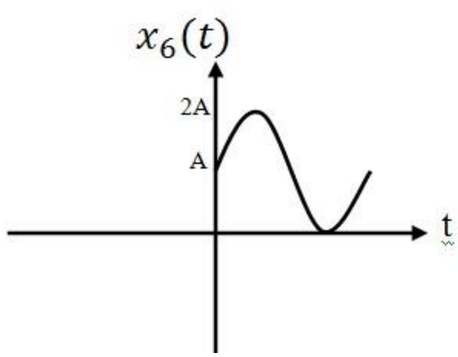
4- $x_4(t)$= A sin (t-α)

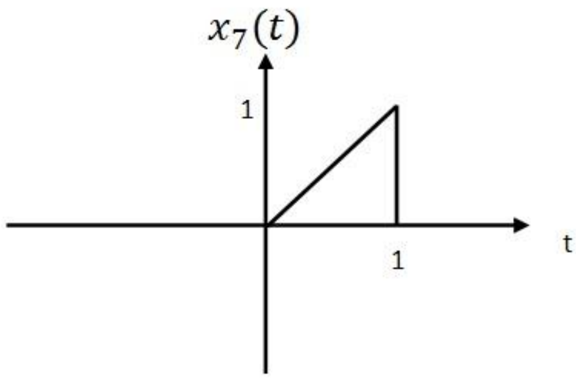


5- $x_5(t)$= A sin (t+α)



6- $x_6(t)$= A sin(t) +A





By using slope law:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$$

Or,

$$\frac{1-0}{1-0} = \frac{y-0}{x-0}$$

y = x

Or, $x_7(t) = t$ for $0 < t < 1$