



Basic Operation on Signals

An issue of major importance is the use of systems to process or manipulate signals. This issue involves a combination of some basic operations.

However, two classes of these operations can be identified that are:

① Operation of dependent variables

- A. Amplitude scaling (Amplitude shifting, Amplification): The scaled signal $ax(t)$ is $x(t)$ multiplied by the factor a where a is a constant real number, such as, the physical device that performs amplitude scaling is an electronic amplifier.

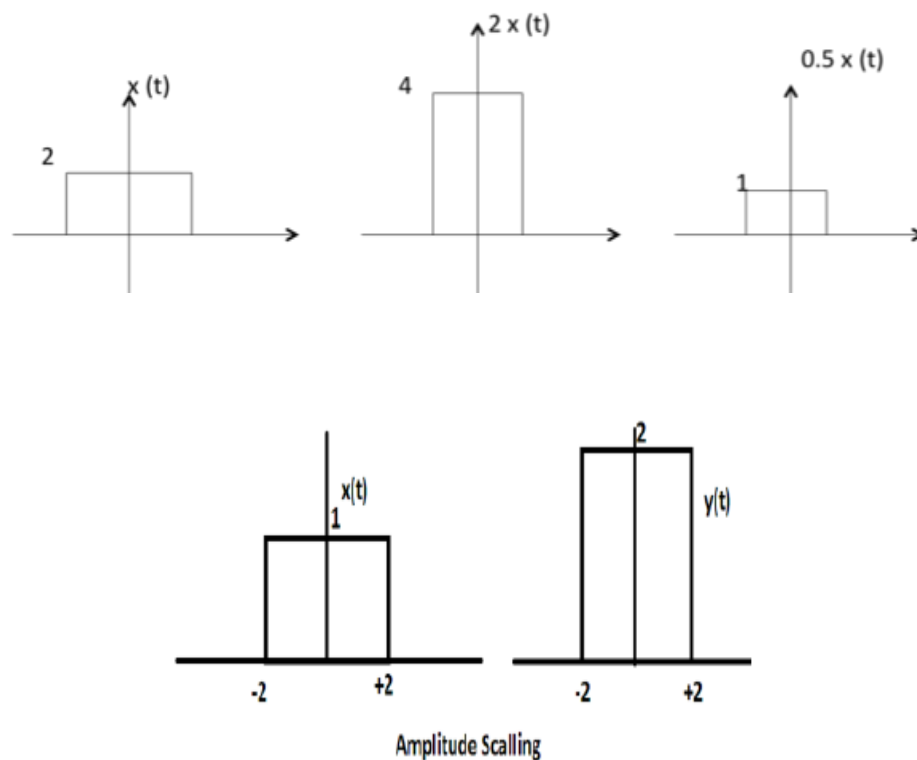


Figure 8: The amplitude scaling operation

In this case, only the values of y axis is changed since the amplitude is associated with this axis while, the values of x axis is constant.



B. Addition: If $x_1(t)$ and $x_2(t)$ denote a pair of CTSs. The signal $z(t)$ obtained by the addition of $x_1(t)$ and $x_2(t)$ is defined by:

$$z(t) = x_1(t) + x_2(t)$$

In the case of DTS, it written as:

$$z[n] = x_1[n] + x_2[n]$$

It can be noted that the addition of two signals is nothing but addition of their corresponding amplitudes. This can be best explained by using the following example:

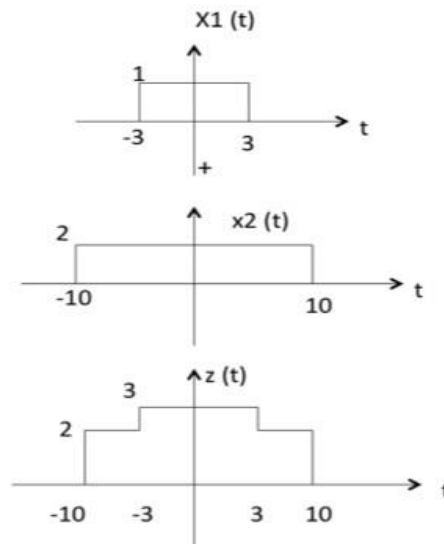


Figure 9: The addition operation

C. Subtraction: If $x_1(t)$ and $x_2(t)$ refer to a pair of CTSs. Then, the signal $z(t)$ obtained by the subtracting of $x_1(t)$ from $x_2(t)$ is defined by:

$$z(t) = x_1(t) - x_2(t)$$

In the case of DTS, it written as:

$$z[n] = x_1[n] - x_2[n]$$

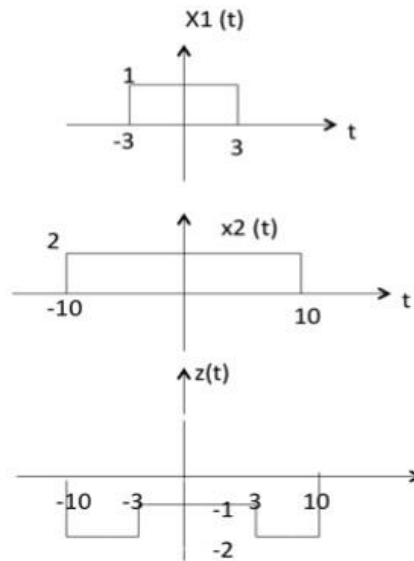


Figure 10: The subtraction operation

- D. Multiplication:** let $x_1(t)$ and $x_2(t)$ denote a pair of CTSs. The signal $z(t)$ resulting from the multiplication of $x_1(t)$ and $x_2(t)$ is defined by the following equation:

$$z(t) = x_1(t) * x_2(t)$$

That is, for each prescribed time (t) the value of $z(t)$ is given by the product of the corresponding values of $x_1(t)$ and $x_2(t)$.

For discrete-time signals we write:

$$z[n] = x_1[n] x_2[n]$$

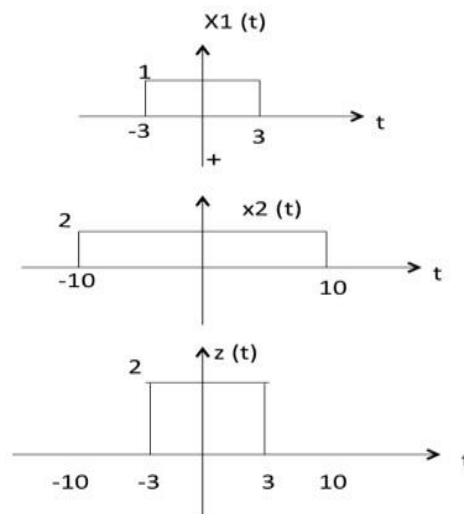


Figure 11: The Multiplication operation



② Operation of independent variables

A. Time shifting: Suppose that we have a signal $x(t)$ and we define a new signal by adding/subtracting a finite time value to/from it. We now have a new signal, $z(t)$. Graphically, this kind of signal operation results in a positive or negative “shift” of the signal along its time axis. However, note that while doing so, none of its characteristics are altered. This means that the time-shifting operation results in the change of just the positioning of the signal without affecting its amplitude.

If CTS is $x(t)$, then $z(t)=x(t-T)$ is the signal $x(t)$ shifted to the right by T units.

If CTS is $x(t)$, then $z(t)=x(t+T)$ is the signal $x(t)$ shifted to the left by T units.

When DTS is $x[n]$, then $z[n]=x[n-N]$ is the signal $x[n]$ shifted to the right by N samples.

When DTS is $x[n]$, then $z[n]=x[n+N]$ is the signal $x[n]$ shifted to the left by N samples.

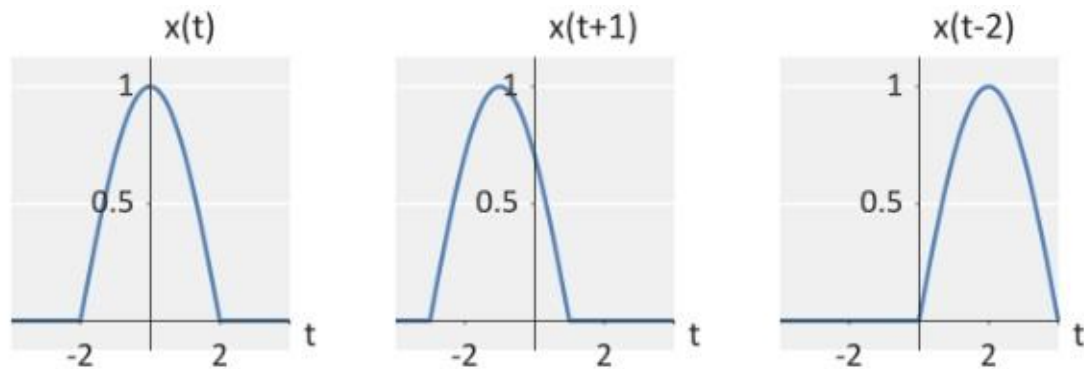


Figure 12: The time shifting operation

B. Time scaling: is a compression or expansion of a signal in time let $x(t)$ denote a CTS, the signal $z(t)$ obtained by scaling the independent variable, time (t), by a factor “ a ” is defined by two cases which are:

- If $a > 1$, the signal $z(t)$ is a compressed version of $x(t)$. In this case:

$$z(t) = x(at)$$



- If $a < 1$, the signal $z(t)$ is an expanded (stretched) version of $x(t)$. Thus, the resulted signal $z(t)$ is computed as:

$$z(t) = x(t/a)$$

Note: the factor “a” must not be equal to 0.

This mean that the signal $x(t)$ is scaled in time by multiplying the time variable by a positive constant (a), to produce $z(t)$.

All these cases are described in the below figure.

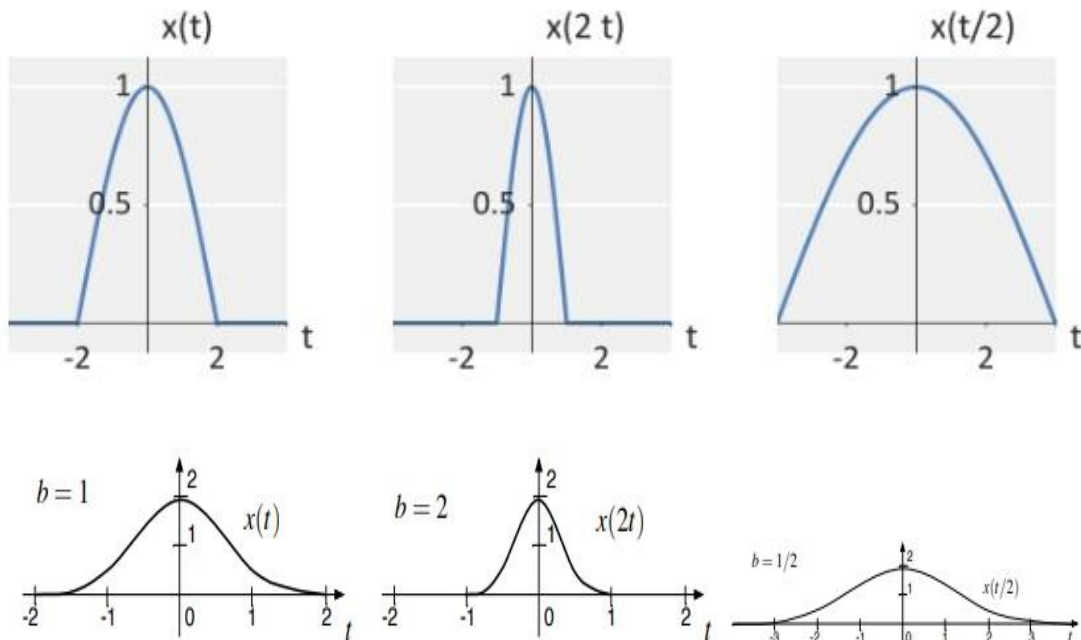


Figure 13: The time scaling operation

- C. Time reversal (time inversion, reflection): The signal $z(t)$ represents a reflected version of $x(t)$ about the amplitude axis. Let $x(t)$ denotes a CTS signal and $z(t)$ denotes the signal obtained by replacing time (t) with $(-t)$, as shown in the next two cases:

- The CTS is represented as:

$$y(t)=x(-t)$$

- While the DTS is as follows:

$$y[n]=x[-n]$$

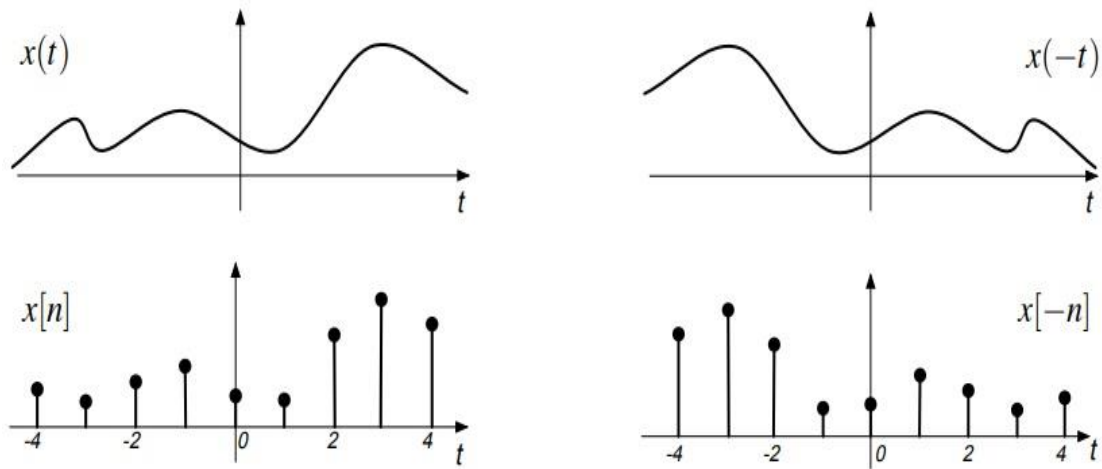


Figure 14: The time reversal operation