Block Diagram Reduction

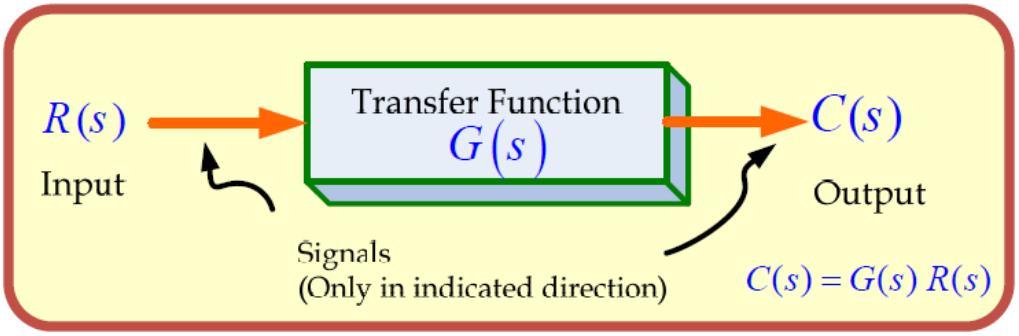


Figure 1: Single block diagram representation

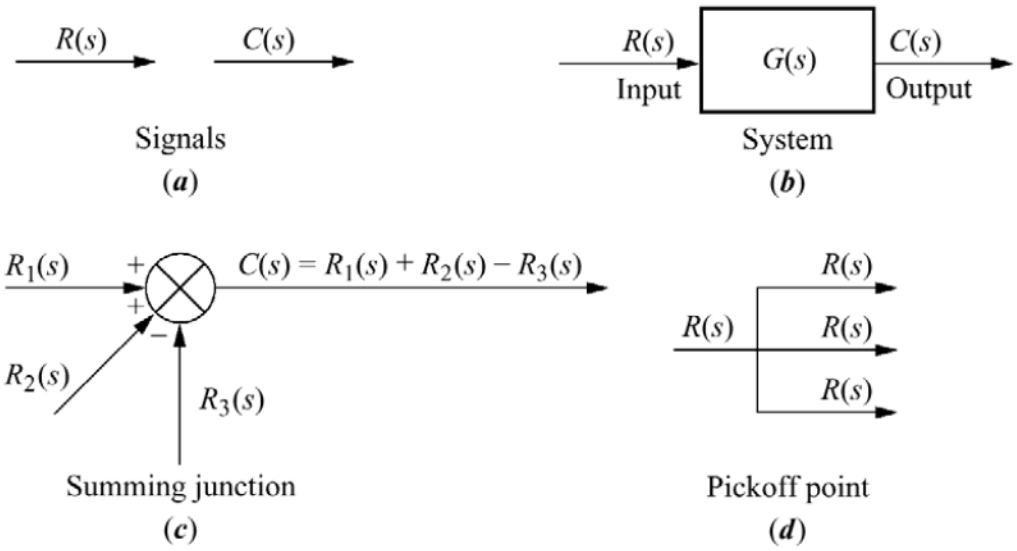


Figure 2: Components of Linear Time Invariant Systems (LTIS)

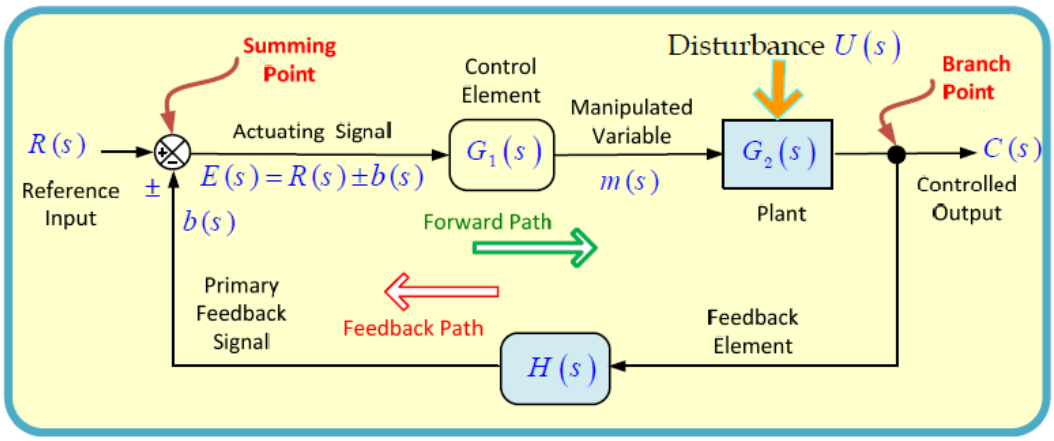


Figure 3: Block diagram components

Definitions

- $G(s)$ ≡ Direct transfer function = Forward transfer function.
- $H(s)$ ≡ Feedback transfer function.
- $G(s)H(s)$ ≡ Open-loop transfer function.
- $C(s)/R(s)$ ≡ Closed-loop transfer function = Control ratio
- $C(s)/E(s)$ ≡ Feed-forward transfer function.

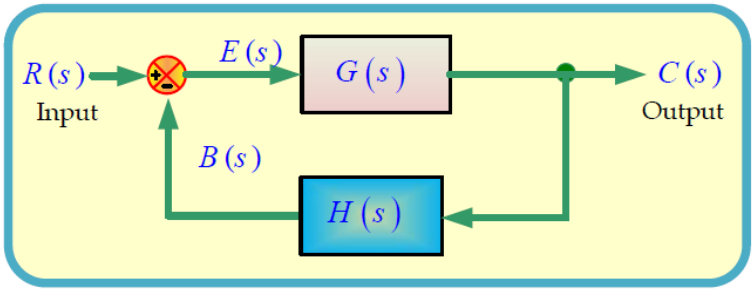


Figure 4: Block diagram of a closed-loop system with a feedback element

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

BLOCK DIAGRAM SIMPLIFICATIONS

Cascade (Series) Connections

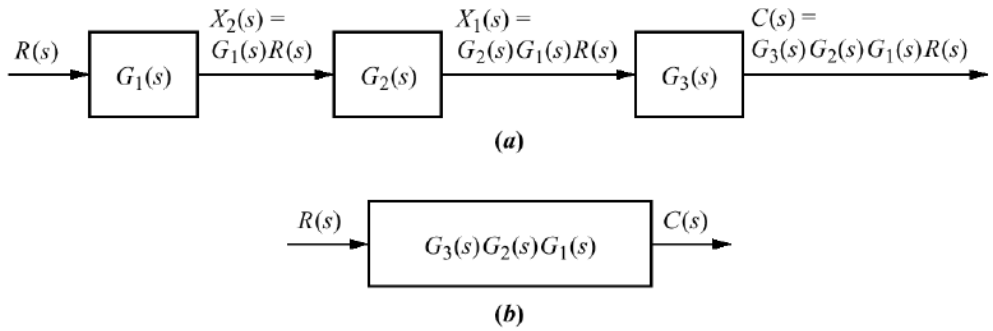


Figure 5: Cascade (Series) Connections

Parallel Connections

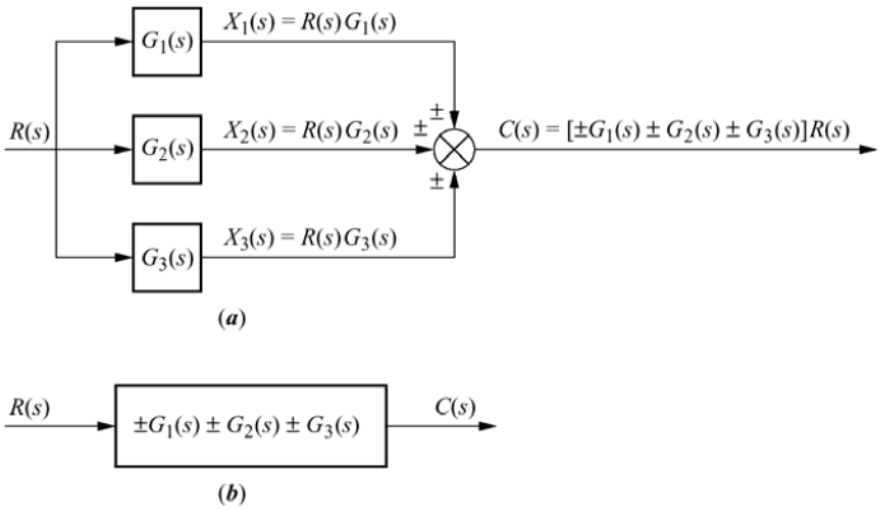
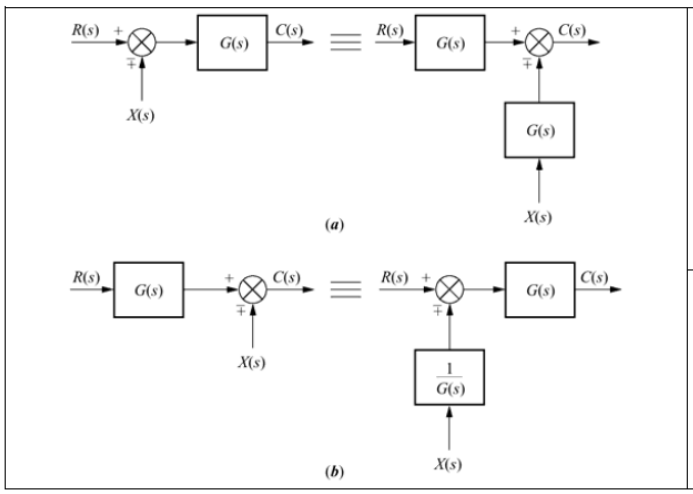


Figure 6: Parallel Connections

Block Diagram Algebra for Summing Junctions



$$C = G(+R \pm X) = +GR \pm GX$$

$$C = GR \pm X = G(+R \pm X/G)$$

Figure 7: Summing Junctions

Block Diagram Algebra for Branch Point

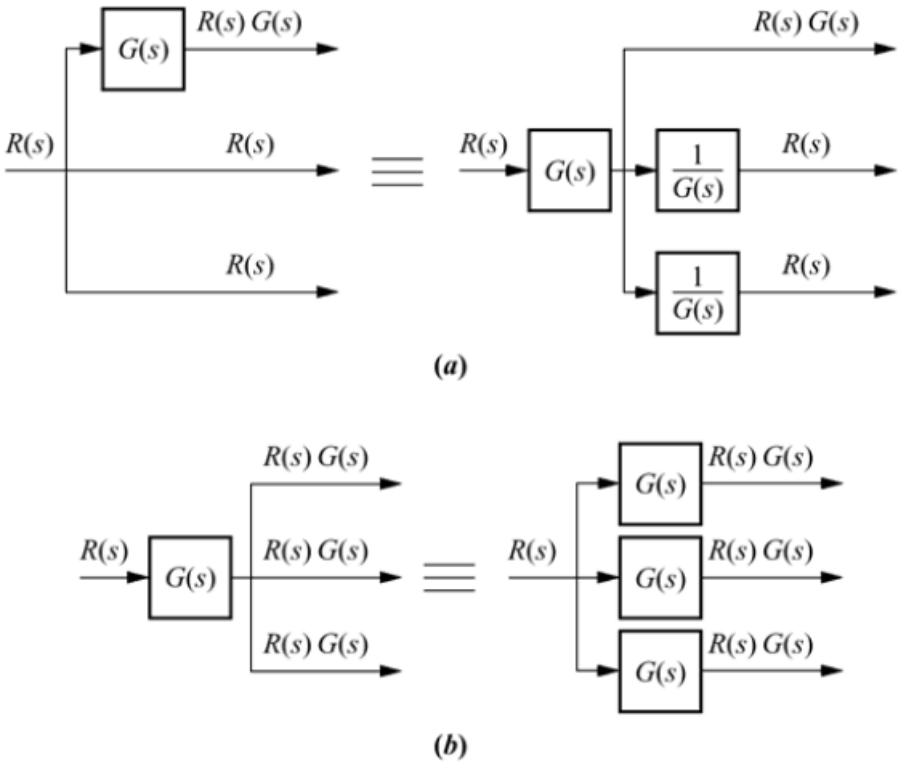


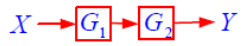
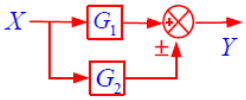
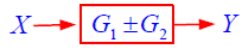
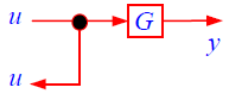
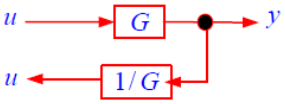
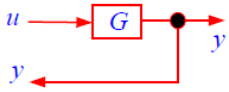
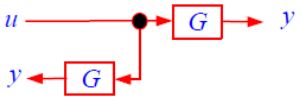
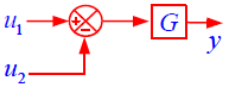
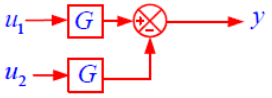
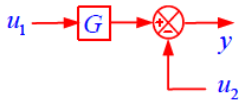
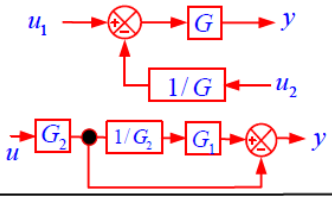
Figure 8: Branch Points

Block Diagram Reduction Rules

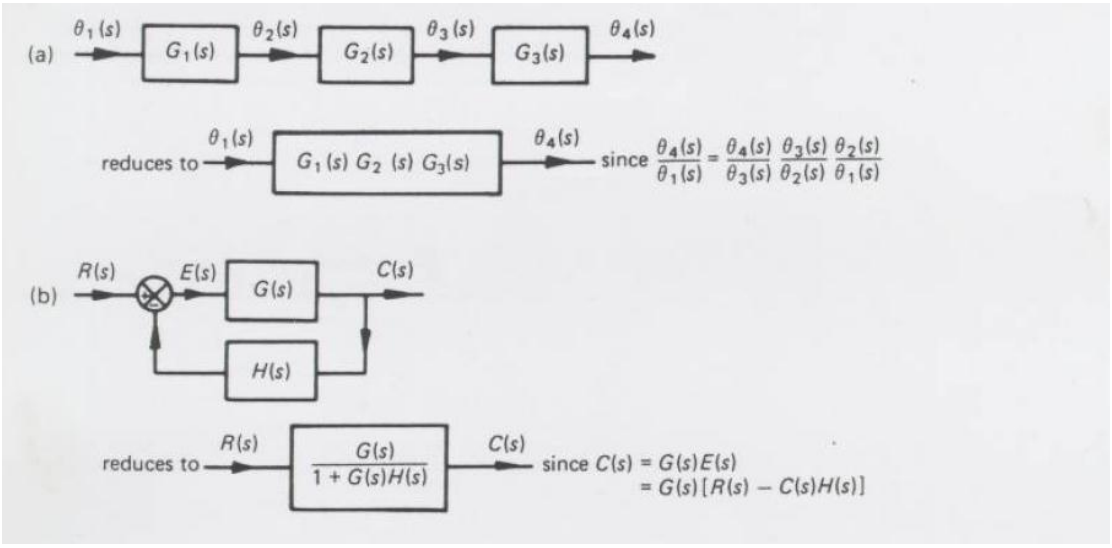
Table 1: Block Diagram Reduction Rules

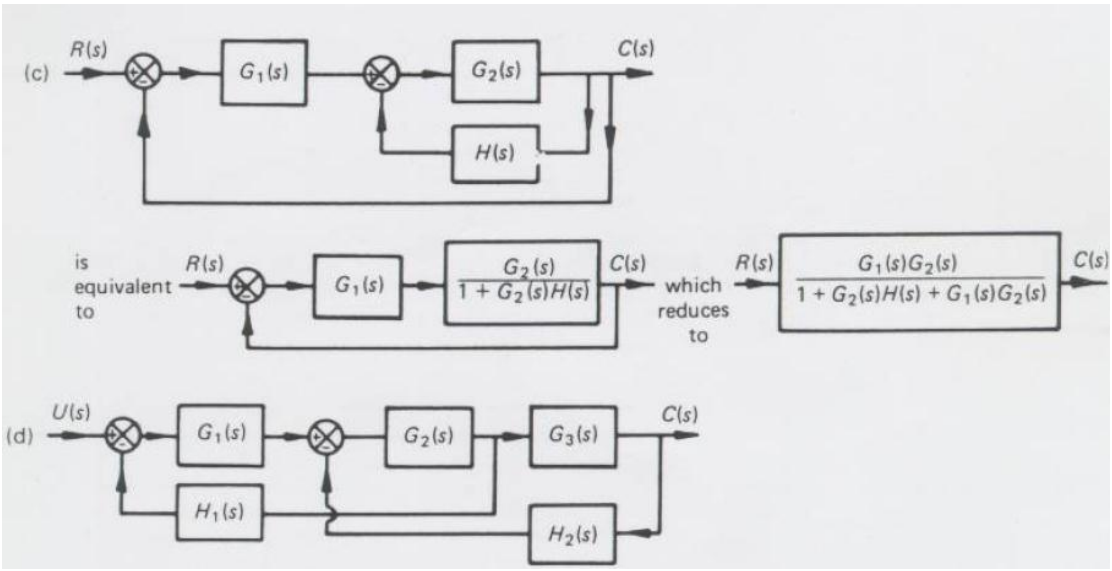
1.	**Combine all cascade blocks**
2.	**Combine all parallel blocks**
3.	**Eliminate all minor (interior) feedback loops**
4.	**Shift summing points to left**
5.	**Shift takeoff points to the right**
6.	**Repeat Steps 1 to 5 until the canonical form is obtained**

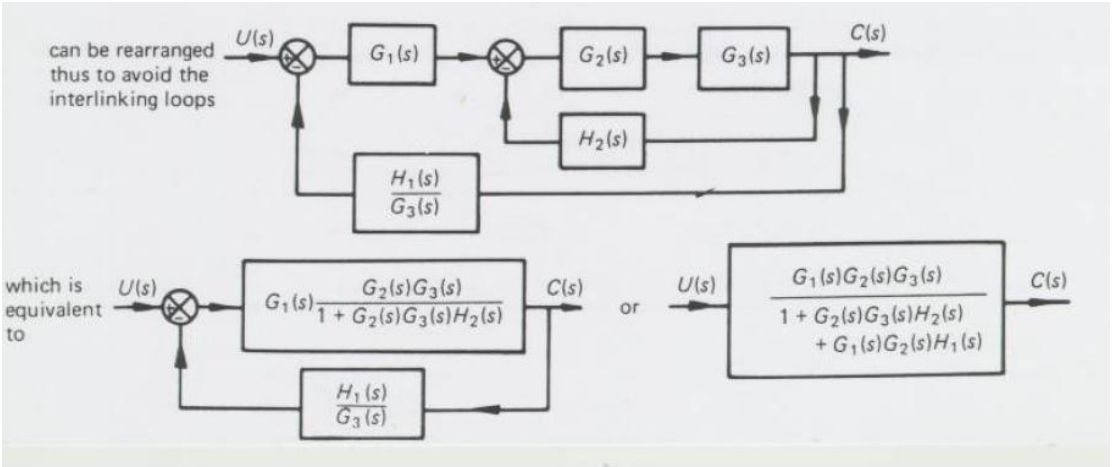
Table 2: Basic rules with block diagram transformation

	Manipulation	Original Block Diagram	Equivalent Block Diagram	Equation
1	Combining Blocks in Cascade			$Y=(G_1G_2)X$
2	Combining Blocks in Parallel; or Eliminating a Forward Loop			$Y=(G_1\pm G_2)X$
3	Moving a pickoff point behind a block			$y = G\,u$ $u = \frac{1}{G}y$
4	Moving a pickoff point ahead of a block			$y = Gu$
5	Moving a summing point behind a block			$e_2 = G(u_1 - u_2)$
6	Moving a summing point ahead of a block			$y = Gu_1 - u_2$ $y = (G_1 - G_2)u$

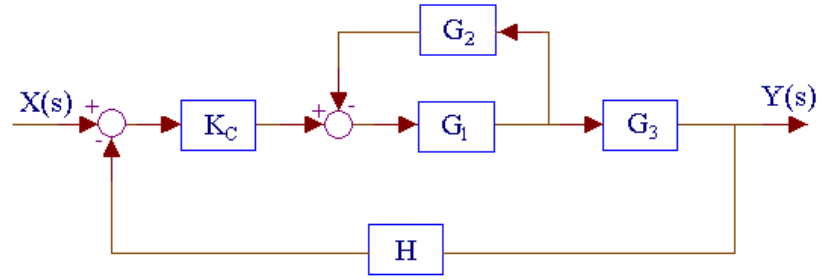
Example 1:

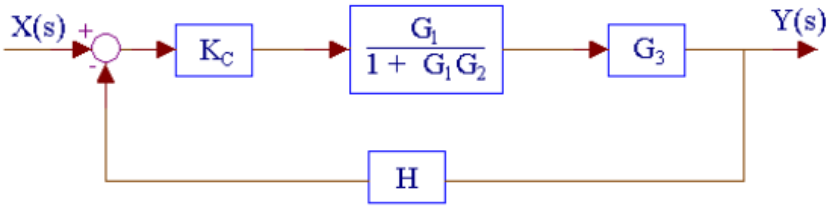

(a)
$\theta_1(s)$
$G_1(s)$
$\theta_2(s)$
$G_2(s)$
$\theta_3(s)$
$G_3(s)$
$\theta_4(s)$
reduces to
$G_1(s)\,G_2(s)\,G_3(s)$
since $\frac{\theta_4(s)}{\theta_1(s)} = \frac{\theta_4(s)}{\theta_3(s)}\frac{\theta_3(s)}{\theta_2(s)}\frac{\theta_2(s)}{\theta_1(s)}$
(b)
$R(s)$
$E(s)$
$G(s)$
$C(s)$
$H(s)$
reduces to
$\frac{G(s)}{1 + G(s)H(s)}$
since $C(s) = G(s)E(s)$
$= G(s)[R(s) - C(s)H(s)]$

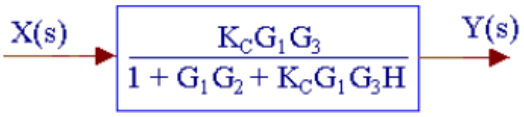

(c)
$R(s)$
$G_1(s)$
$G_2(s)$
$C(s)$
$H(s)$
is equivalent to
$\frac{G_2(s)}{1 + G_2(s)H(s)}$
which reduces to
$\frac{G_1(s)G_2(s)}{1 + G_2(s)H(s) + G_1(s)G_2(s)}$
(d)
$U(s)$
$G_3(s)$
$H_1(s)$
$H_2(s)$


can be rearranged thus to avoid the interlinking loops
$U(s)$
$G_1(s)$
$G_2(s)$
$G_3(s)$
$C(s)$
$H_2(s)$
$\frac{H_1(s)}{G_3(s)}$
which is equivalent to
$G_1(s)\frac{G_2(s)G_3(s)}{1 + G_2(s)G_3(s)H_2(s)}$
or
$\frac{G_1(s)G_2(s)G_3(s)}{1 + G_2(s)G_3(s)H_2(s) + G_1(s)G_2(s)H_1(s)}$

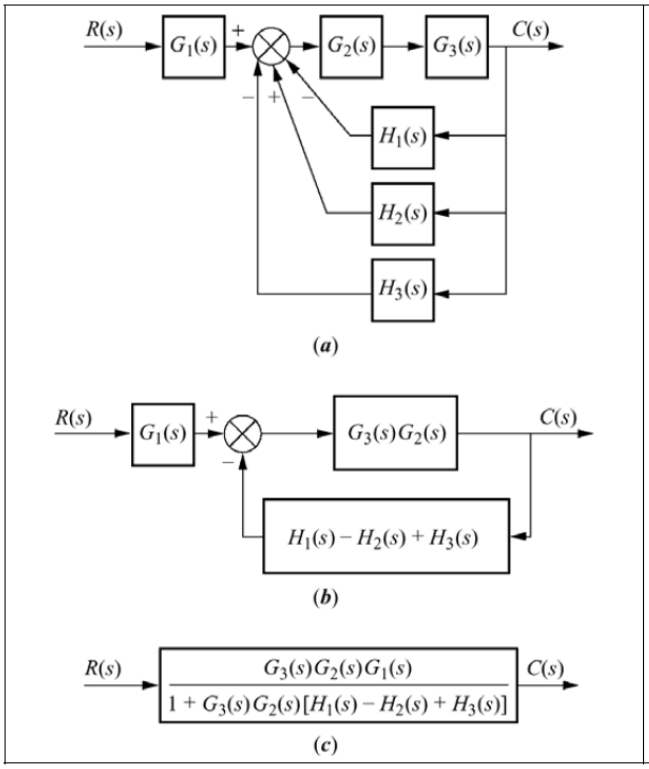
Example 2:







Example 3:



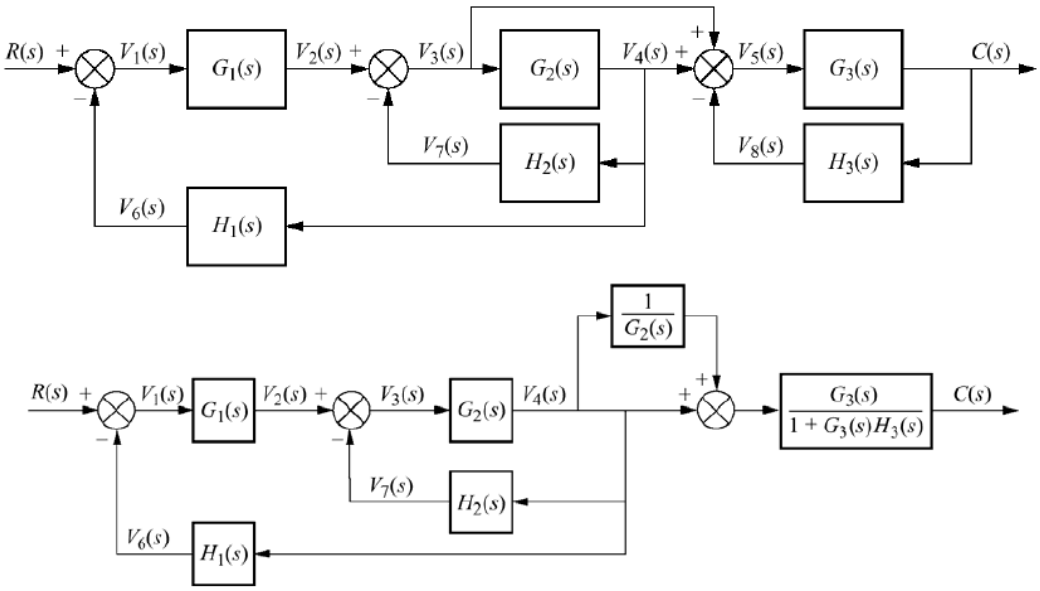
G_1 and G_2 are in series

H_1 and H_2 and H_3 are in parallel

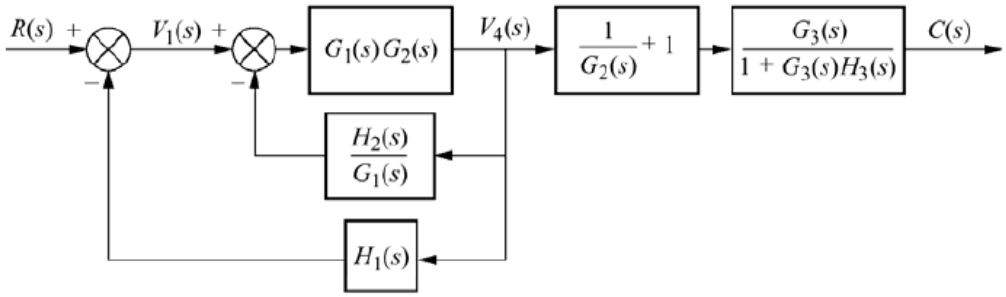
G_1 is in series with the feedback configuration.

$$\frac{C(s)}{R(s)} = G_1\left[\frac{G_3 G_2}{1 + G_3 G_2 (H_1 - H_2 + H_3)}\right]$$

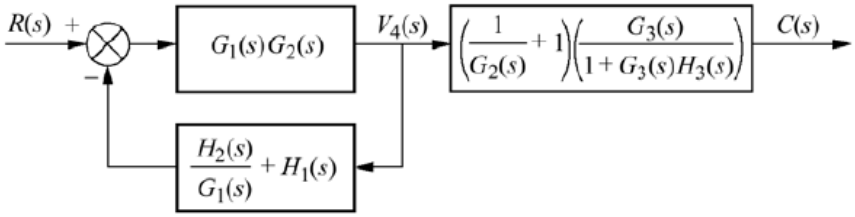
Example 4:



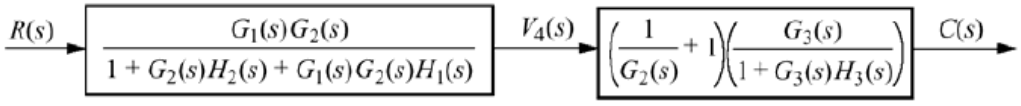
(*a*)



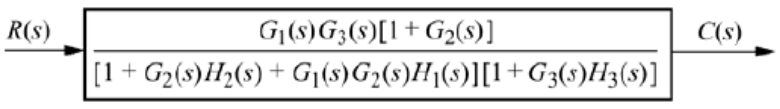
(*b*)



(*c*)

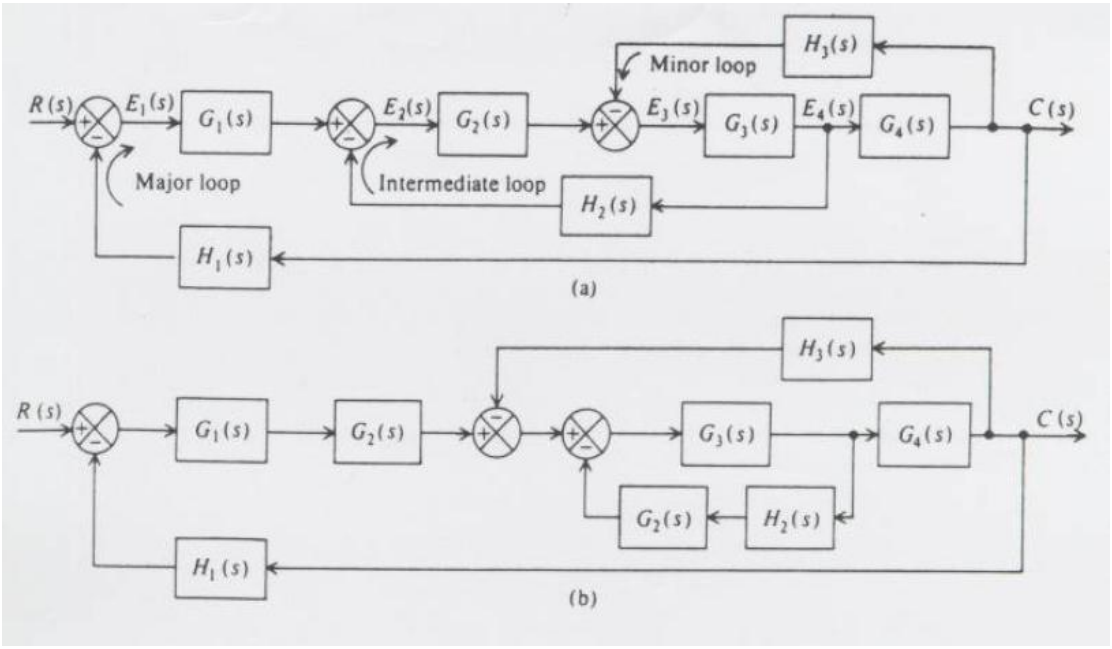


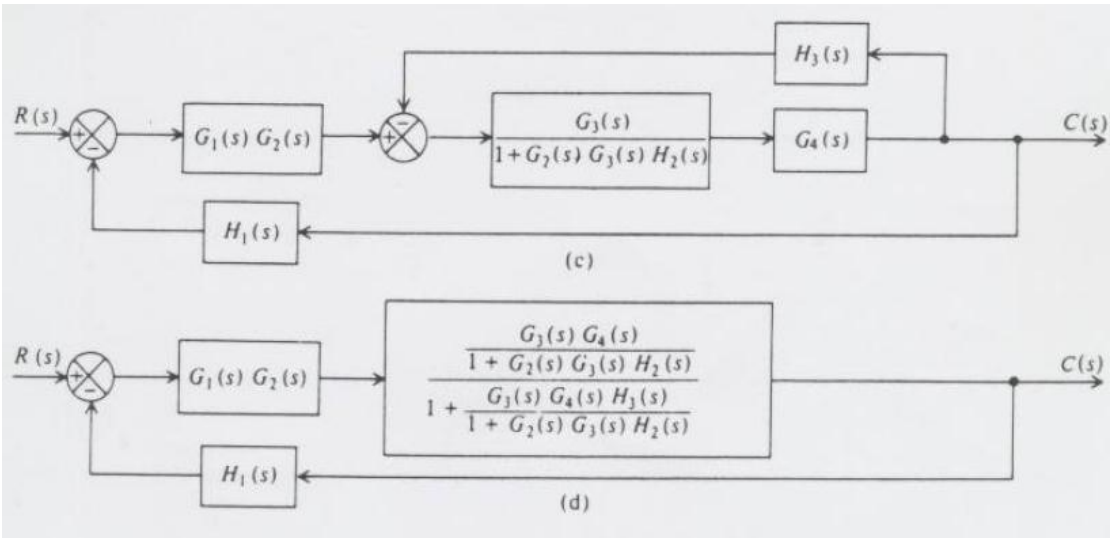
(*d*)

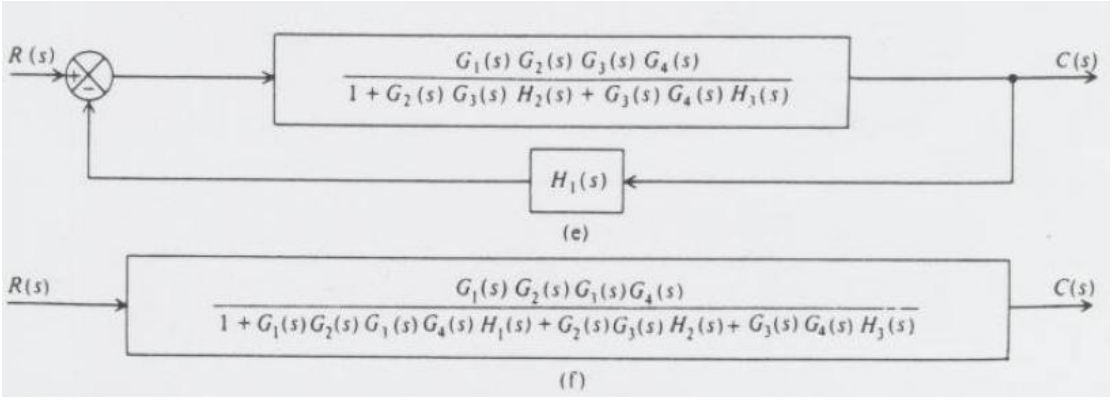


(*e*)

Example5:


R(s)
E1(s)
G1(s)
E2(s)
G2(s)
E3(s)
G3(s)
E4(s)
G4(s)
C(s)
H3(s)
Minor loop
Major loop
Intermediate loop
H2(s)
H1(s)
(a)
R(s)
G1(s)
G2(s)
G3(s)
G4(s)
C(s)
H3(s)
G2(s)
H2(s)
H1(s)
(b)


R(s)
G1(s) G2(s)
G3(s)
1+G2(s) G3(s) H2(s)
G4(s)
H3(s)
C(s)
H1(s)
(c)
R(s)
G1(s) G2(s)
G3(s) G4(s)
1 + G2(s) G3(s) H2(s)
1 + G3(s) G4(s) H3(s)
1 + G2(s) G3(s) H2(s)
C(s)
H1(s)
(d)


R(s)
G1(s) G2(s) G3(s) G4(s)
1 + G2(s) G3(s) H2(s) + G3(s) G4(s) H3(s)
C(s)
H1(s)
(e)
R(s)
G1(s) G2(s) G3(s) G4(s)
1 + G1(s) G2(s) G3(s) G4(s) H1(s) + G2(s) G3(s) H2(s) + G3(s) G4(s) H3(s)
C(s)
(f)