Estimation of Random Errors

These errors are due to unknown causes and occur even when all systematic errors have been accounted for. In well designed experiments few random errors usually occur, but they become important in high accuracy work. The only way to offset these errors is by increasing the number of readings and using statistical means to obtain the best approximation of the true value of the quantity under measurement.

Statistical Analysis of Data

To make statistical methods useful, the systematic errors should be small compared with random errors because statistical treatment can not improve the accuracy of measurement.

1- ***Arithmetic Mean(X`):***

It's the value lie in the medial number of measured variable and represents the most accurate measured value for the true value. Arithmetic mean is given by:

$$\overline{X} = \frac{\sum F_i \cdot X_i}{\sum F_i}$$

, where X_i is the reading values taken, and F_i is the number that each reading is occur in the measurements, or the frequency number of each reading.

2- ***Deviation From The Mean(d_i):***

Deviation is the departure of a given reading from the mean value. It's given by:

$$d_i = X_i - \overline{X}$$

The deviation from the mean may have a positive or a negative value and the algebraic sum of all the deviation must be zero in symmetrical curve.

3- ***Average Deviation(D):***

The average deviation is the sum of the ***absolute*** values of deviations divided by the number of readings

$$D = \frac{\sum |F_i \cdot d_i|}{\sum F_i}$$

where $\sum F_i = n$, and n= number of all readings

4- ***Standard Deviation(σ):***

It's the root mean square deviation, and the standard deviation represents the variation of the reading from the mean value. For a finite number of reading

$$\sigma = \sqrt{\frac{\sum F_i .(d_i)^2}{n-1}}$$

5- ***Variance(v):***

It's defined as mean square standard deviation

$$v = \sigma^2$$

6- ***Probable Error (r):***

It's the maximum chance (50%) that any given measurement will have a random error no greater than ±r

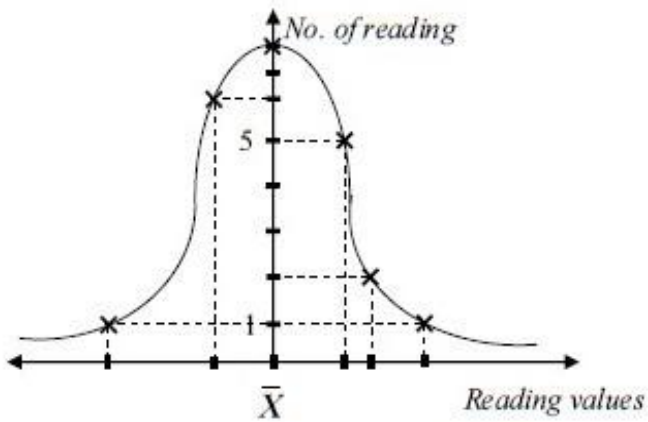
$$r = \pm 0.6745\sigma$$

E-mail:zainab.kadum.jaber@uomus.edu.iq

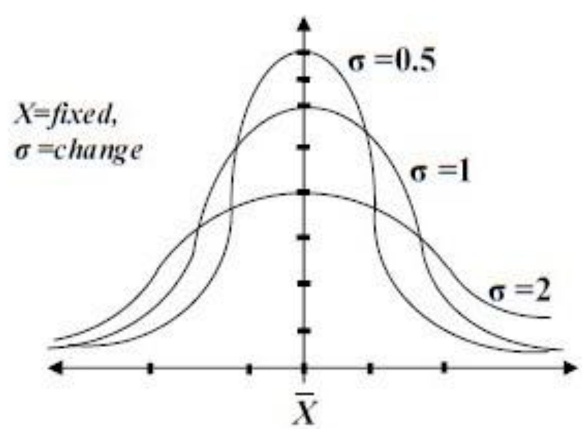
7- *Gaussian Distribution Curve:*

It's the normal distribution curve for random errors where $\overline{X}$ in the centre of this curve. The random errors may be positive or negative with respect to $\overline{X}$ thus lie at the two side of the curve, small errors are more probable than large errors. Gaussian curve is drawn by the following equation:

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\overline{x})^2/2\sigma^2} \qquad -\infty < X < +\infty \qquad -\infty < \sigma < +\infty$$



There are two factors effecting Gaussian curve shape $\overline{X}$ and σ as shown:



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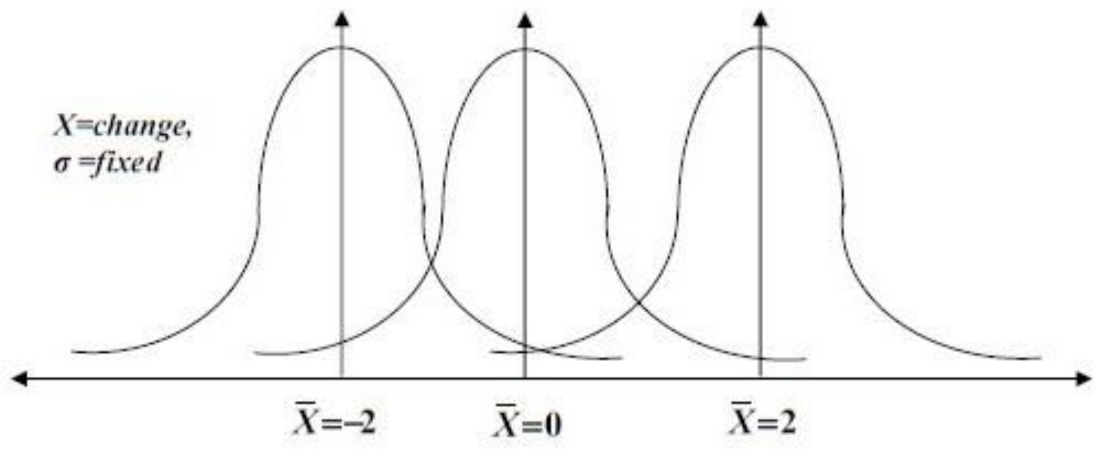
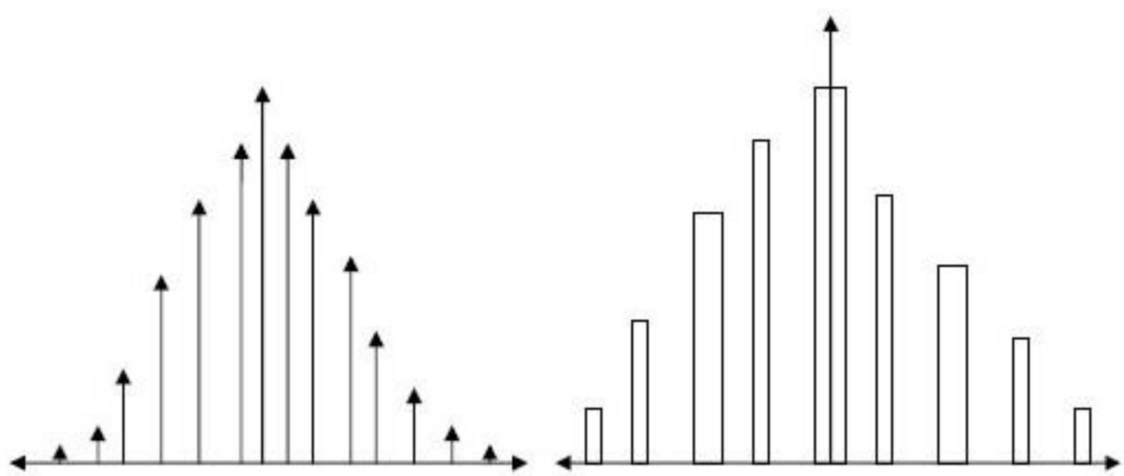


8- ***Histogram:***

Graphically represent the number of observed reading against the observed value is called the histogram and the connection between the distributions of observation is called Gaussian curve



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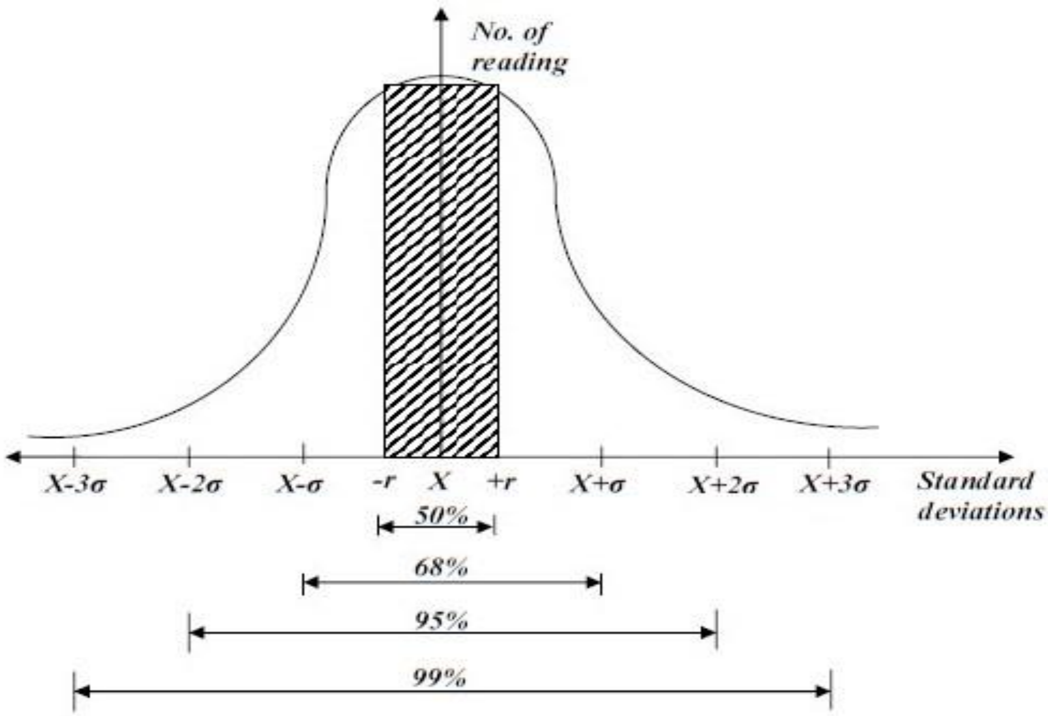


Example:

The following readings were recorded for voltage measurement:

10.1, 9.7, 10.2, 9.6, 9.7, 10.1, 9.6, 9.7, 10.1

Calculate:

1. Arithmetic mean (X)
2. Deviation from the mean (d)
3. Average deviation (D)
4. standard deviation (σ)
5. Variance (V)
6. probable error (±r)

Sol.:

Rearrangement the reading in two columns with its frequency or (number of reading), thus

Reading values	*No. of reading*	*di(xi-x)*
10.1	3	0.3
9.7	3	-0.1
9.6	2	-0.2
10.2	1	0.4

1- $\overline{X} = \dfrac{\sum F_i \cdot X_i}{\sum F_i} = \dfrac{3(10.1) + 3(9.7) + 2(9.6) + (10.1)}{9} = 9.8 volt$

2- $d_i = X_i - \overline{X}$

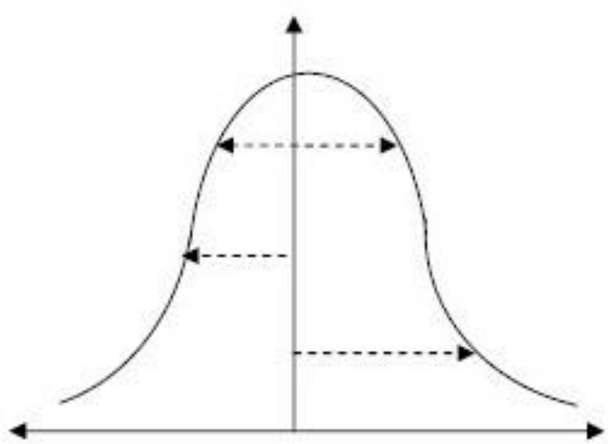
$$d_1 = 10.1 - 9.8 = 0.3 volt$$
$$d_4 = 9.7 - 9.8 = -0.1 volt$$
$$d_7 = 9.6 - 9.8 = -0.2 volt$$
$$d_9 = 10.2 - 9.8 = 0.4 volt$$

3- $D = \dfrac{\sum \left| F_i \cdot d_i \right|}{\sum F_i} = \dfrac{3(0.3) + 3(0.1) + 2(0.2) + (0.4)}{9} = 0.22 volt$

4- $\sigma = \sqrt{\dfrac{\sum F_i .(d_i)^2}{n-1}} = \sqrt{\dfrac{3(0.09) + 3(0.01) + 2(0.04) + (0.16)}{8}} = 0.26 volt$

5- $\nu = \sigma^2 = (0.26)^2 = 0.067 volt^2$

6- $r = \pm 0.6745\sigma = \pm 0.6745(0.26) = \pm 0.175 volt$



E-mail:zainab.kadum.jaber@uomus.edu.iq