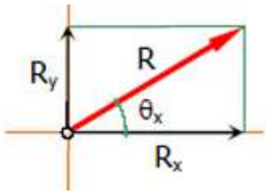
2.3 Resultant of Coplanar General Force System

The resultant of non-concurrent and non-parallel force system is defined according to magnitude, direction, and position. The magnitude of the resultant can be found as follows:

$$R_x = \sum F_x \quad \rightarrow^{+}$$

$$R_y = \sum F_y \quad \uparrow^{+}$$



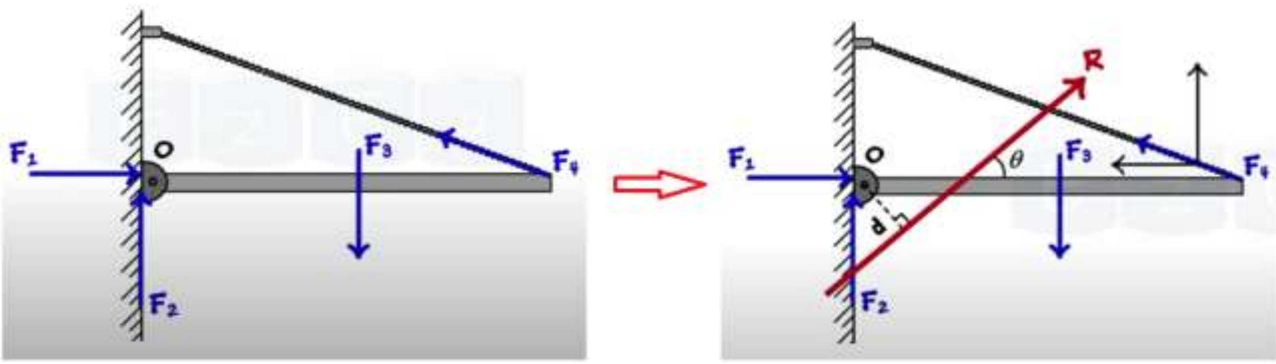
$$R = \sqrt{{R_x}^2 + {R_y}^2}$$

The direction of the resultant from the horizontal is defined by

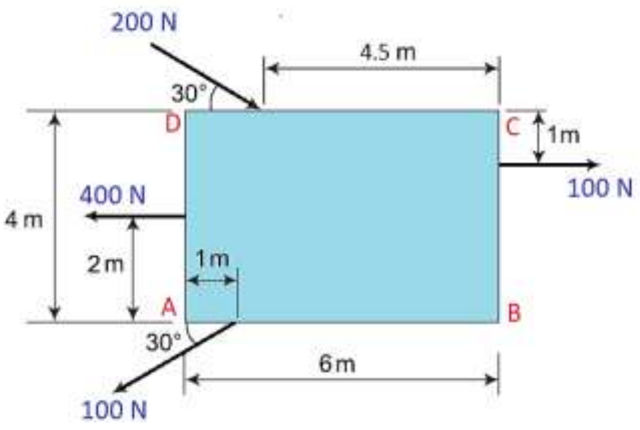
$$\theta_x = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

The position of the resultant can be determined according to the principle of moments.

$$\circlearrowright^{+} \; R \cdot d = \sum F_i \cdot d_i$$



Example No. 1: A rectangular block ABCD is subjected to four forces as shown in figure. Determine (a) magnitude and direction of the resultant force and (b) position of the resultant force from point A.



Solution:

a. The magnitude and direction of the resultant force

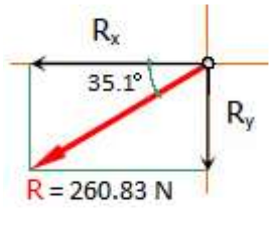
$$\xrightarrow{+} R_x = \sum F_x = 100 + 200\cos 30 - 400 - 100\cos 30$$

$$R_x = -213.39\ N = 213.39\ N \leftarrow$$

$$\uparrow^{+} R_y = \sum F_y = -200\sin 30 - 100\sin 30$$

$$R_y = -150\ N = 150\ N \ \downarrow$$

$$R = \sqrt{{R_x}^2 + {R_y}^2} = \sqrt{(213.39\,)^2 + (150)^2} = 260.83\ N$$



$$\theta_x = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{150}{213.39}\right) = 35.10^\circ$$

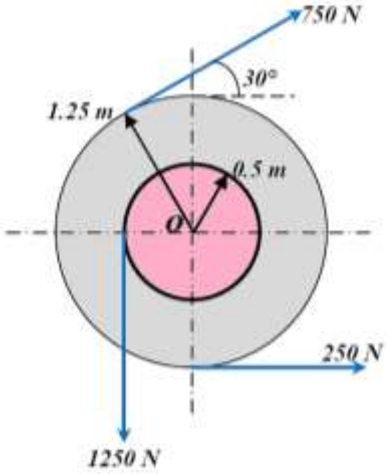
b. The position of the resultant force from point A

$$\circlearrowright^{+} R \cdot d = \sum F_i \cdot d_i \quad (about\ point\ A)$$

$$260.83 \times d = 100 \times 3 + 200\cos 30 \times 4 + 200\sin 30 \times 1.5 - 400 \times 2 + 100\sin 30 \times 1$$

$d = 1.5\ m$ from point A

Example No. 2: Determine the resultant of the forces acting on the step pulley shown in Figure.



Solution:

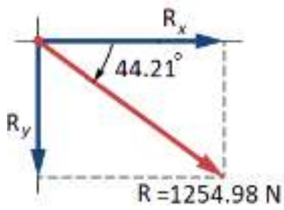
$$\xrightarrow{+} R_x = \sum F_x = 750\cos 30 + 250$$

$$R_x = 899.52\ N \rightarrow$$

$$\uparrow^{+} R_y = \sum F_y = 750\sin 30 - 1250$$

$$R_y = -875\ N = 875\ N\ \downarrow$$

$$R = \sqrt{{R_x}^2 + {R_y}^2} = \sqrt{(899.52\,)^2 + (875)^2}$$



$$R = 1254.89\ N$$

$$\theta_x = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{875}{899.52}\right) = 44.21^{\circ}$$

$$\circlearrowright^{+} R \cdot d = \sum F_i \cdot d_i \quad (about\ point\ o)$$

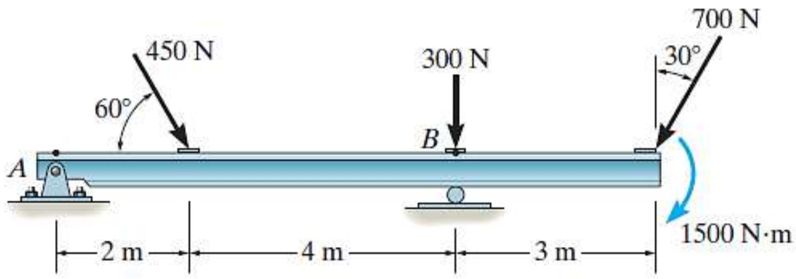
$$1254.89\ \times d = -250 \times 1.25 - 1250 \times 0.5 + 750 \times 1.25$$

$d = 0$ i.e. the resultant passes through point O

Dr. Mayadah W.Falah

Problems:

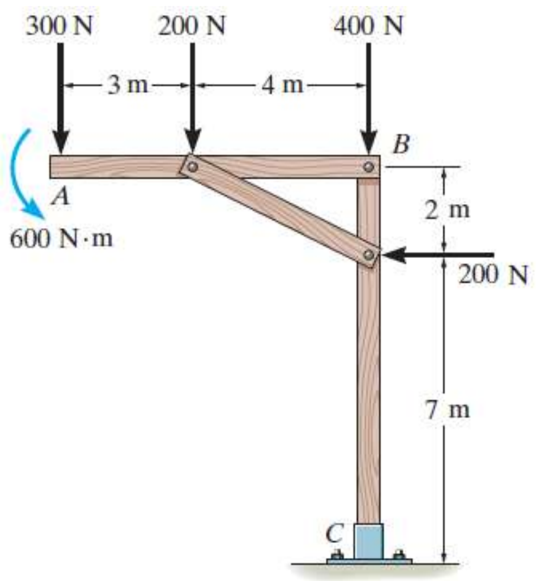
1. Determine the magnitude and direction of the resultant force, then determine position of the resultant force from (a) point A, (b) point B.



Answer: $(\boldsymbol{a})\ R = 1302\ N,\ \ \theta_x = 84.5^{\circ}$ ↙ , $d = 7.32\ m$

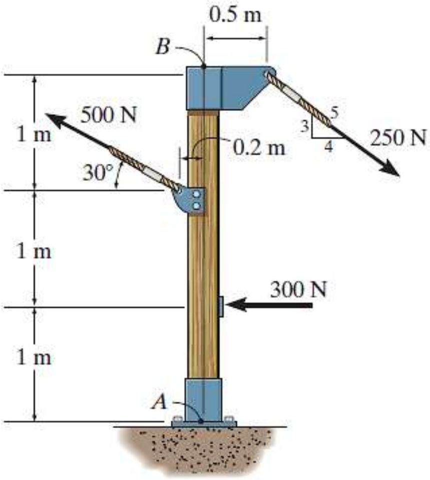
$(\boldsymbol{b})\ R = 1302\ N,\ \ \theta_x = 84.5^{\circ}$ ↙ , $d = 1.35\ m$

2. Replace the loading on the frame by a single resultant force where its line of action intersects member AB, measured from A.



Answer: $R = 922\ N,\ \ \theta_x = 77.5^{\circ}$ ↙ , $d = 3.56\ m$

3. Replace the force system acting on the post by a resultant force where its line of action intersects the post AB measured from point B.



Answer: $R = 542\ N,\ \ \theta_x = 10.6^{\circ}$, $d = 2.14\ m$