ahmed.adnan@mustaqbal-college.edu.iq

Lecture Three:

Signals

1. Introduction

The concept and theory of signals and systems are needed in many engineering and scientific disciplines as well. An introduction to mathematical description and representation of signals and systems and their classifications will be illustrated. We also define several important basic signals essential to our studies.

2. Signals and Classification Of Signals

A signal is a function representing a physical quantity or variable, and typically it contains information about the behavior or nature of the phenomenon. Mathematically, a signal is represented as a function of an independent variable t. usually t represents time. Thus, a signal is denoted by x(t).

A. Continuous-Time and Discrete-Time Signals:

A signal x(t) is a continuous-time signal if t is a continuous variable. If t is a discrete variable, that is, x(t) is defined at discrete times, then x(t) is a discrete-time signal. Since a discrete-time signal is defined at discrete times, a discrete-time signal is often identified as a sequence of numbers, denoted by $\{x_n\}$ o r x[n], where n = integer. Illustrations of a continuous-time signal x(t) and of a discrete-time signal x[n] are shown in Fig. 1.

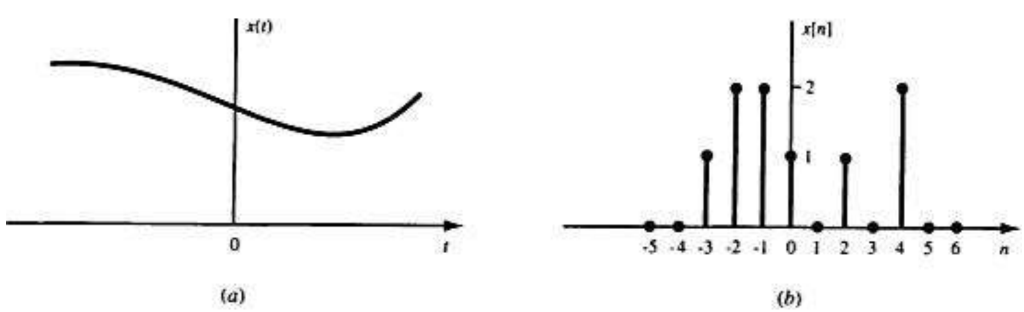


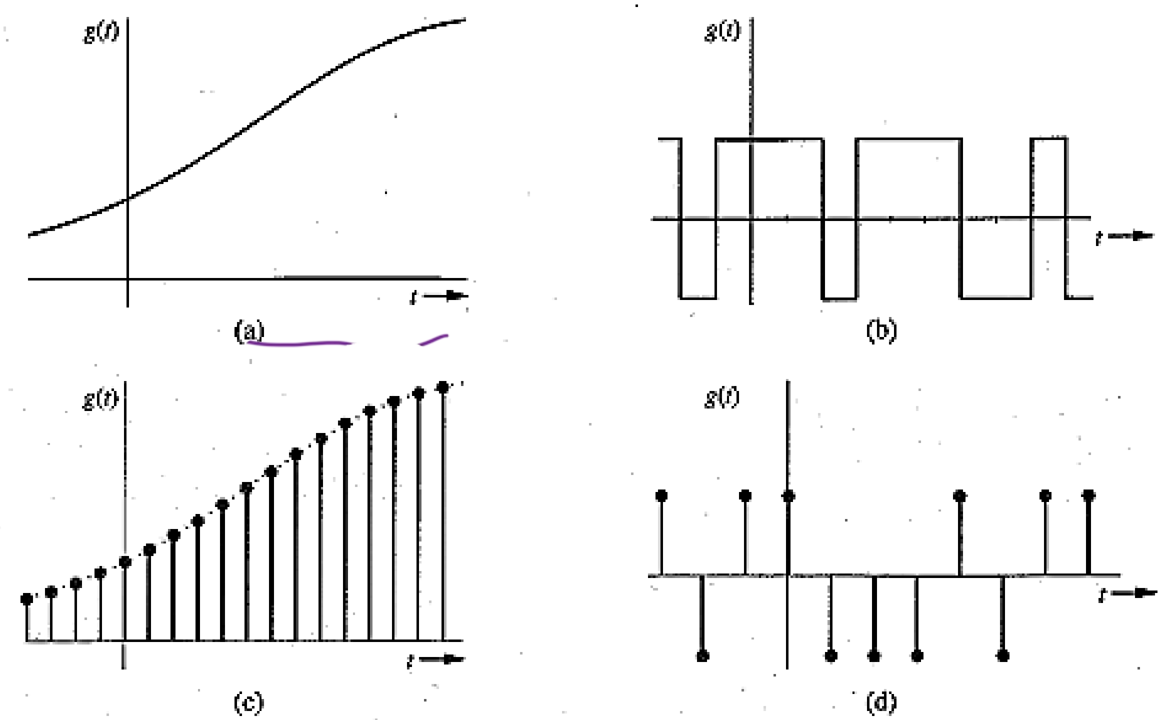
Fig. 1: Graphical representation of (a) continuous-time and (b) discrete-time

signals.

ahmed.adnan@mustaqbal-college.edu.iq

B. Analog and Digital Signals:

The concept of continuous time is often confused with that of analog. The two are not the same. The same is true of the concepts of discrete time and digital. A signal whose amplitude can take on any value in a continuous range is an **analog signal**. This means that analog signal amplitude can take on an infinite number of values. A **digital signal**, on the other hand, is one whose amplitude can take on only a finite number of values.



Example of signals is shown above. (a) is an analog signal and its at the same time continuous time. (b) Digital signal and its continuous. (c) Analog signal but its discrete. (d) Digital and discrete time.

C. Real and Complex Signals:

A signal x(*t*) is a real signal if its value is a real number, and a signal x(*t*) is a complex signal if its value is a complex number. A general complex signal x(t) is a function of the form

$$x(t) = x_1(t) + jx_2(t)$$

ahmed.adnan@mustaqbal-college.edu.iq

Where $x_1(t)$ *and* $x_2(t)$ are real signals and $= \sqrt{-1}$.

D. Deterministic and Random Signals:

Deterministic signals are those signals whose values are completely specified for any given time. *Random* signals are those signals that take random values at any given time and must be characterized statistically.

E. Energy and Power Signals

The normalized energy content E of a signal x(t) is defined as

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt \qquad (1)$$

The normalized average power P of a signal x(t) is defined as

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-\infty}^{\infty} |x(t)|^2 dt \qquad (2)$$

- x(t) is said to be an *energy* signal if and only if $0 < E < \infty$ and so P=0.
- x(t) is said to be a *power* signal if and only if $0 < P < \infty$, thus implying that $E=\infty$.

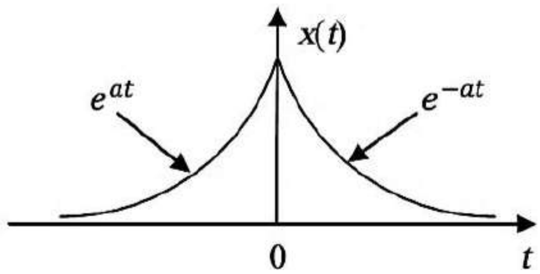
Example 1:

Determine whether the signal $x(t) = e^{-a|t|}$ is power or energy signals or neither

Sol: $$x(t) = e^{-a|t|} = \begin{cases} e^{-at} & t > 0 \\ e^{at} & t < 0 \end{cases}$$

$$E = \int_{-\infty}^{\infty} [x(t)]^2 dt = \int_{-\infty}^{\infty} e^{-2a|t|}\, dt$$

$$= 2\int_{0}^{\infty} e^{-2at} dt = \frac{1}{a} < \infty \text{ is an energy signal.}$$



F. Periodic and Non-periodic Signals

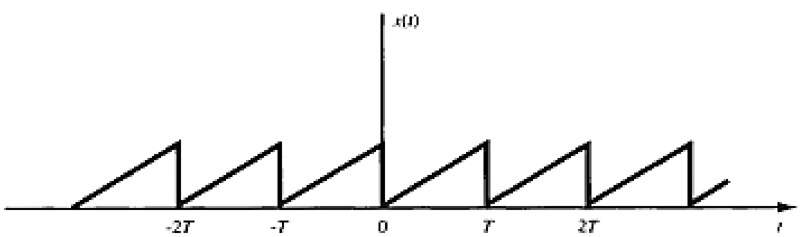
A signal x(t) is periodic if there is a positive number T_o such that

$$x(t + nT_o) = x(t) \qquad \text{all t} \tag{3}$$

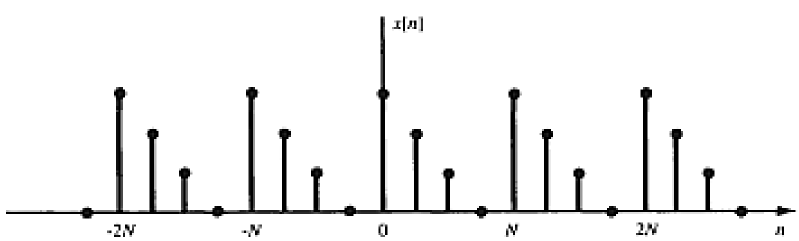
The smallest positive number T_o is called the period, and the reciprocal of the period is called the fundamental frequency f_o

$$f_o = \frac{1}{T_o} \quad hertz\ (Hz) \tag{4}$$

Any signal for which there is no value of T_o satisfying Eq.(3) is said to be non-periodic or aperiodic, A periodic signal is a power signal if its energy content per period is finite, and then the average power of the signal need only be calculated over a period.



(a)



(b)

ahmed.adnan@mustaqbal-college.edu.iq

Example 2:

Let $x_1(t)$ and $x_2(t)$ be periodic signals with periods T_1 and T_2, respectively: Under what conditions is the sum.:

$$x(t) = x_1(t) + x_2(t)$$

periodic, and what is the period of x(t) .if it is periodic?

Sol:

From Eq. (3)

$$x_1(t) = x_1(t + mT_1) \quad \text{m is an integer}$$

$$x_2(t) = x_2(t + nT_2) \quad \text{n is an integer}$$

If, therefore, T_1 and $T_2$are such that

$$mT_1 = \text{n}\, T_2 = T$$

then, $\quad x(t + T) = x_1(t + T) + x_2(t + T) = x_1(t) + x_2(t) = x(t)$

that is, x(t) is periodic. Thus, the condition for x(t) to be periodic is

$$\frac{T_1}{T_2} = \frac{n}{m} = rational\ number$$

The smallest common period is the least common multiple of T_1 and T_2. If the ratio is T_1/T_2 an irrational number, then the signals $x_1(t)$ and $x_2(t)$) do not have a common period and x(t(cannot be periodic.

To check $x(t) = \cos\left(\frac{1}{3}t\right) + \sin\left(\frac{1}{4}t\right)$ for periodicity, $\cos\left(\frac{1}{3}t\right)$ is periodic with period $T_1 = 6\pi$, and $\sin\left(\frac{1}{4}t\right)$ is periodic with period $T_2 = 8\pi$. Since $\frac{T1}{T_2} = \frac{6\pi}{8\pi} = \frac{3}{4}$ is a rational number, (t) is periodic with period T = 4T_1 =3T_2 = 24π.

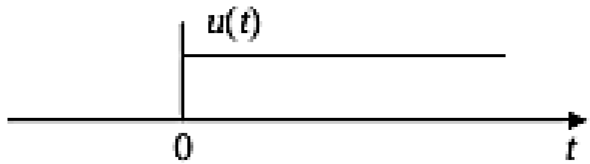
G. Singularity Functions

An important subclass of non-periodic signals in communication theory is the singularity functions (or, as they are sometimes called, the generalized functions).

G.1 Unit Step Function

The unit step function u(t) is defined as

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases} \tag{5}$$



Note that it is discontinuous at t = 0 and that the value at t = 0 is undefined.

G.2 Unit Impulse Function

The unit impulse function, also known as the Dirac delta function, δ(t) is not an ordinary function and is defined in terms of the following process:

$$\int_{-\infty}^{\infty} \phi(t)\delta(t)dt = \phi(0) \tag{6}$$

where ϕ (t) is any test function continuous at t = 0.

Some additional properties of δ(t) are

$$\int_{-\infty}^{\infty} \phi(t)\delta(t - t_0)dt = \phi(t_0) \tag{7}$$

$$\delta(at) = \frac{1}{|a|}\delta(t) \tag{8}$$

$$\delta(-t) = \delta(t) \tag{9}$$

$$x(t)\delta(t) = x(0)\delta(t) \qquad (10)$$

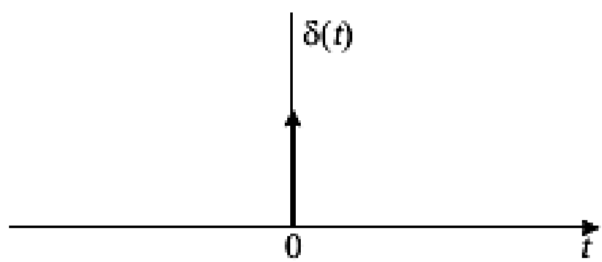
$$x(t)\delta(t - t_0) = x(t_0)\delta(t - t_0) \qquad (11)$$

An alternate definition of δ(t) is provided by the following two conditions

$$\int_{t_1}^{t_2} \delta(t - t_0)dt = 1 \qquad t_1 < t_0 < t_2 \qquad (12)$$

$$\delta(t - t_0) = 0 \qquad t \text{ G } t_0 \qquad (13)$$

Conditions in Eq.(12) and (13) correspond to the intuitive notion of a unit impulse as the limit of a suitably chosen conventional function having unity area in an infinitesimally small width. For convenience, δ(t) is shown schematically below.



If g(t) is a generalized function, its derivative g'(t) is defined by the following relation:

$$\int_{-\infty}^{\infty} g'(t)\phi(t)dt = -\int_{-\infty}^{\infty} g(t)\phi'(t)dt \qquad (14)$$

By using Eq. (14), the derivative of u(t) can be shown to be δ(t); that is,

$$\delta(t) = u'(t) = \frac{du(t)}{dt} \qquad (15)$$

Example 3:

Evaluate the integral, $\int_{-\infty}^{\infty}(t^2 + cos\pi t)\delta(t - 1)dt$

Sol:- $\int_{-\infty}^{\infty}(t^2 + cos\pi t)\delta(t - 1)dt = t^2 + cos\pi t|_{t=1} = 1 + cos\pi = 1 - 1 = 0$