TUTORIAL 3

Q9: A source emits 3 characters (A, B, C) with probabilities of $(0.25,\ 0.35, 0.4)$ respectively, calculate:

1) The self-information for each character I(A), I(B), I(C)

2) The information rate R(x) if τ(A)= 2msec, τ(B) =3msec, τ(C)=5msec.

Solution:

1) $I(xi) = -log_2 P(xi)$

$I(A) = -log_2 P(0.25) = 2bit$

$I(B) = -log_2 P(0.35) = 1.514\ bit$

$I(C) = -log_2 P(C) = 1.321\ bit$

2) $\tau = \sum_{i=1}^{n} \tau_i P(xi) = 2 * 0.25 + 3 * 0.35 + 5 * 0.4 = 3.55\ msec$

$$H(x) = -\sum_{i=1}^{n} P(xi) log_2 P(xi) = -\left[\frac{0.4ln0.4 + 0.25ln0.25 + 0.35ln0.35}{ln2}\right]$$

$$= 1.558\ bit/symbol$$

$$R(x) = \frac{H(x)}{\tau} = \frac{1.558}{3.55 * 10^{-3}} = 0.439\ kbps$$

Q10: A source emits 12 characters with equal probability, one every 3msec, calculate:

1) Source entropy H(x)

2) The probability for each character

3) The information rate R(x)

Solution:

$$1)\ H(x) = log_2 n = log_2 12 = 3.584\ bit/symbol$$
$$2)\ P(xi) = \frac{1}{n} = \frac{1}{12}$$
$$3)\ R(x) = \frac{H(x)}{\tau} = \frac{3.584}{3*10^{-3}} = 1.194\ kbps$$

Q11: A source emits 8 characters with equal probability, one every 1msec, calculate:

1) The self-information for each character

2) Source entropy H(x)

3) The information rate R(x)

Solution:

$$1)\ P(xi) = \frac{1}{n} = \frac{1}{8} \quad ,\ I(xi) = -log_2 P(xi) = 3bit$$
$$2)\ H(x) = log_2 n = log_2 8 = 3\ bit/symbol$$
$$3)\ R(x) = \frac{H(x)}{\tau} = \frac{3}{1*10^{-3}} = 3\ kbps$$

Q12: Having the text (A A A B A C B C C B B A A A D B B B C D).Find

1. Find source Entropy H(x).
2. The self-information of A and B, (I(A), I(B))
3. If τ(A) = τ(B) = τ(C) = 0.1 μsec and τ(D) = 0.2 μsec. Calculate average Source Entropy Rate R(x).

Solution:

1.

$$H(x) = -\sum_{i=1}^{n} P(xi) log_2 P(xi)$$

$$P(A) = \frac{7}{20}, P(B) = \frac{7}{20}, P(C) = \frac{4}{20}, P(D) = \frac{2}{20}$$

$$H(x) = \frac{\left(\frac{7}{20} \times \ln\left(\frac{7}{20}\right) \times 2 + \frac{1}{5} \times \ln\left(\frac{1}{5}\right) + \frac{1}{10} \times \ln\left(\frac{1}{10}\right)\right)}{\ln(2)} = 1.85677 \frac{bits}{symbol}$$

2.

$$I(A) = I(B) = -\log_2 P(A) = -\left(\frac{\ln\left(\frac{7}{20}\right)}{\ln(2)}\right) = 1.514\ bits$$

3.

$$R(X) = \frac{H(X)}{\bar{\tau}}$$

$$\bar{\tau} = \sum_i \tau_i * P(x_i) = \left(\frac{7}{20} + \frac{7}{20} + \frac{1}{5}\right) * 0.1 \times 10^{-6} + \frac{1}{10} \times 0.2 \times 10^{-6}$$

$$= 1.199\ \mu sec$$

$$R(X) = \frac{1.85677}{1.199 \times 10^{-6}} = 1.54\ Mbps$$

Q.13: An information source has the following table

Symbol: xi	X1	X2	X3	X4	X5	X6	X7
P(xi)	0.15	0.1	0.03	0.4	0.02	0.1	0.2
τ(xi) m second	1	4	2	3	1	2	2

Find :

i-self information of x1, x4 (I(x1), I(x4)

ii-source entropy H(x)

iii-average information rate R(x)

Solution:

i-

$$I(xi) = -log_2 P(xi)$$

$$,I(x1) = -log_2 P(0.15) = 2.736\ bit$$

$$I(x4) = -log_2 P(0.4) = 1.321\ bit$$

ii-$H(x) = -\sum_{i=1}^{n} P(xi) log_2 P(xi)$

$$= -\left[\frac{0.15ln0.15 + 2 * 0.1ln0.1 + 0.03ln0.03 + 0.4ln0.4 + 0.02ln0.02 + 0.2ln0.2}{ln2}\right]$$

$$= 2.332\ bit/symbol$$

iii-

$$\tau = \sum_{i=1}^{n} \tau_i P(xi)$$

$$= 0.15 * 1 + 0.1 * 4 + 0.03 * 2 + 0.4 * 3 + 0.02 * 1 + 0.1 * 2 + 0.2 * 2$$

$$= 2.43\ msec$$

$$R(x) = \frac{H(x)}{\tau} = \frac{2.332}{2.43 * 10^{-3}} = 0.959\ kbps =$$