TUTORIAL 4

Q.14: The joint probability of channel is $p(x,y) = \begin{pmatrix} 0.2 & 0.1 \\ 0.25 & 0.1 \\ 0.25 & 0.1 \end{pmatrix}$; Find:

1) The receiver entropy H(y)

2) Joint entropy H(x,y)

3) Noise entropy H(y/x)

4) Loss entropy H(x/y)

Solution:

P(x)= [0.3, 0.35, 0.35], P(y)=[0.7, 0.3]

$$1)\ H(y) = -\sum_{j=1}^{m} P(yj) log_2 P(yj) = -\left[\frac{0.7\ ln0.7 + 0.3 ln0.3}{ln2}\right] = 0.881\ bit/symbol$$

$$2)\ H(x,y) = -\sum_{j=1}^{m}\sum_{i=1}^{n} P(xi,yj) log_2 P(xi,yj)$$

$$= -\left[\frac{0.2 ln0.2 + 3 * 0.1 ln0.1 + 2 * 0.25 ln0.25}{ln2}\right] = \quad 2.460\ bit/symbol$$

$$3)\ H(y/x) = H(x,y) - H(x)$$

$$H(x) = -\sum_{i=1}^{n} P(xi) log_2 P(xi) = -\left[\frac{0.3 ln0.3 + 2 * 0.35 ln0.35}{ln2}\right]$$

$$= 1.581\ bit/symbol$$

$$\therefore H(y/x) = 2.640 - 1.581 = 0.879\ bit/symbol$$

$$4)\ H(x/y) = H(x,y) - H(y) = 2.640 - 0.881 = 1.579\ bit/symbol$$

Q.15: The joint probability of channel is defined as: $P(x,y) = \begin{pmatrix} 0.2 & 0.1 \\ 0.25 & 0.2 \\ 0.25 & 0 \end{pmatrix}$, Find:

1) The receiver entropy H(y) and source entropy H(x)

2) Joint entropy H(x,y)

3) Noise entropy H(y/x)

4) transinformation I(x,y)

Solution:

P(x)=[0.3, 0.45, 0.25], P(y)= [0.7, 0.3]

1) $H(y) = -\sum_{j=1}^{m} P(yj) log_2 P(yj) = -\left[\frac{0.7ln0.7 + 0.3ln0.3}{ln2}\right] = 0.881\ bit/symbol$

$$H(x) = -\sum_{i=1}^{n} P(xi) log_2 P(xi) = -\left[\frac{0.3ln0.3 + 0.25ln0.25 + 0.45ln0.45}{ln2}\right]$$

$$= 1.535\ bit/symbol$$

2) $H(x,y) = -\sum_{j=1}^{m}\sum_{i=1}^{n} P(xi,yj) log_2 P(xi,yj)$

$$= -\left[\frac{2 * 0.2ln0.2 + 0.1ln0.1 + 2 * 0.25ln0.25}{ln2}\right] = \quad 2.260\ bit/symbol$$

3) $H(y/x) = H(x,y) - H(x)$

$$\therefore H(y/x) = 2.260 - 1.535 = 0.725\ bit/symbol$$

4) $I(x,y) = H(y) - H(y/x) = 0.881 - 0.725 = 0.156\ bit/symbol$

Q.16 :A system having the following joint probability matrix:

$$\text{determine: } P(X,Y) = \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix}\begin{bmatrix} 0 & 0.125 \\ 0.0625 & 0.0625 \\ 0.5 & 0.25 \end{bmatrix}$$

a- Reciver entropy H(y).
b- Source entropy H(x).
c- Joint entropy H(x,y).
d- Conditional entropy H(y/x).
e- Transinformation I(x, y).

Solution:

P(x)=[0.125, 0.125, 0.75], P(y)= [0.5625, 0.4375]

$$a)\ H(y) = -\sum_{j=1}^{m} P(yj)log_2P(yj) = -\left[\frac{0.5625ln0.5625 + 0.4375ln0.4375}{ln2}\right]$$

$$= 0.988\ bit/symbol$$

$$b)\ H(x) = -\sum_{i=1}^{n} P(xi)log_2P(xi) = -\left[\frac{2 * 0.125ln0.125 + 0.75ln0.75}{ln2}\right]$$

$$= 1.061\ bit/symbol$$

$$c)\ H(x,y) = -\sum_{j=1}^{m}\sum_{i=1}^{n} P(xi,yj)log_2P(xi,yj)$$

$$= -\left[\frac{0.5ln0.5 + 0.25ln0.25 + 2 * 0.0625ln0.0625 + 0.125ln0.125}{ln2}\right]$$

$$= \quad 1.875\ bit/symbol$$

$$d)\ H(y/x) = H(x,y) - H(x)$$

$$\therefore H(y/x) = 1.875 - 1.061 = 0.814\ bit/symbol$$

$$e)\ I(x,y) = H(y) - H(y/x) = 0.988 - 0.814 = 0.174\ \ bit/symbol$$