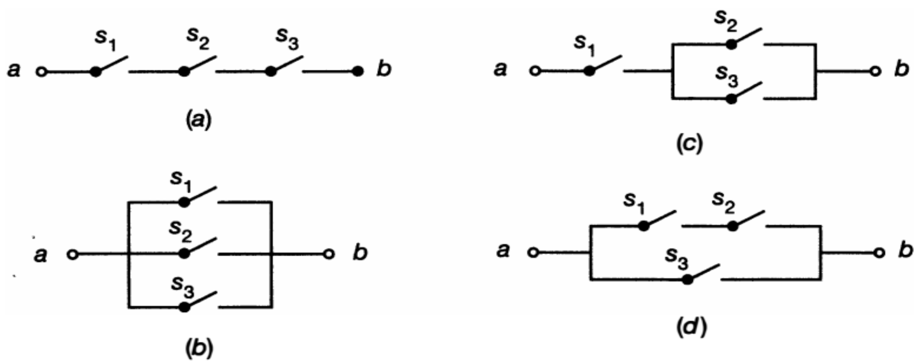
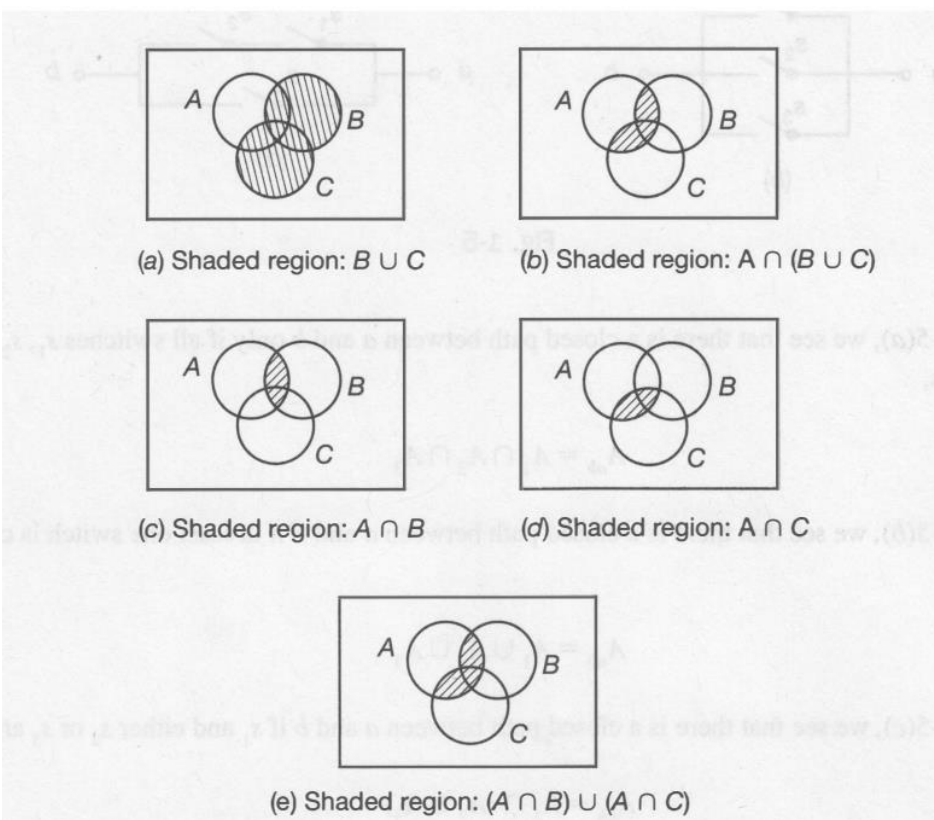
Solved Problems Set#1

Q1. Consider the switching networks shown in Figure below. Let A1, A2, and A3 denote the events that the switches s1, s2, and s3 are closed, respectively. Let A**ab** denote the event that there is a closed path between terminals a and b. Express Aab in terms of A1, A2, and A3 for each of the networks shown



(a)	$A_{ab} = A_1 \cap A_2 \cap A_3$	(c)	$A_{ab} = A_1 \cap (A_2 \cup A_3)$
(b)	$A_{ab} = A_1 \cup A_2 \cup A_3$	(d)	$A_{ab} = (A_1 \cap A_2) \cup A_3$

Q2. Identify the shaded set:



Q3. Prove :

$(a)\quad P(A \mid B) \geq 0$

$(b)\quad P(S \mid B) = 1$

$(c)\quad P(A_1 \cup A_2 \mid B) = P(A_1 \mid B) + P(A_2 \mid B)$ if $A_1 \cap A_2 = \varnothing$

Solution:

<table>
<tr>
<td>(a)</td>
<td>$$P(A|B)=\frac{P(A\cap B)}{P(B)} \qquad P(B)>0$$
l, $P(A \cap B) \geq 0$. Thus,
$$P(A \mid B) \geq 0$$</td>
<td rowspan="2">(c)</td>
<td rowspan="2">$$P(A_1 \cup A_2|B)=\frac{P[(A_1 \cup A_2)\cap B]}{P(B)}$$
$$(A_1 \cup A_2) \cap B = (A_1 \cap B) \cup (A_2 \cap B)$$
$$(A_1 \cap B) \cap (A_2 \cap B) = \varnothing$$
$$P(A_1 \cup A_2|B)=\frac{P(A_1 \cap B)+P(A_2 \cap B)}{P(B)}=\frac{P(A_1 \cap B)}{P(B)}+\frac{P(A_2 \cap B)}{P(B)}$$
$$=P(A_1|B)+P(A_2|B) \qquad \text{if } A_1 \cap A_2 = \varnothing$$</td>
</tr>
<tr>
<td>(b)</td>
<td>$S \cap B = B$. Then
$$P(S|B)=\frac{P(S\cap B)}{P(B)}=\frac{P(B)}{P(B)}=1$$</td>
</tr>
</table>

Q3. Prove

$$P(\bar{A}|B)=1-P(A|B)$$

Solution:

$$A \cup \bar{A} = S, \qquad A \cap \bar{A} = \varnothing$$

$$P(A \cup \bar{A}|B)=P(S|B)=1=P(A|B)+P(\bar{A}|B)$$

$$P(\bar{A}|B)=1-P(A|B)$$

Q4. You toss a fair coin three times:

a. What is the probability of three heads, HHH?

b. What is the probability that you observe exactly one head?

c. Given that you have observed at least one head?

Solution:

a. Since each toss is independent of other then

$P(HHH)=P(H).P(H).P(H)=(0.5)^3=0.125$

Or by using binomial distribution or repeated trials: n=3, k=0,1,2,3 , since we have 3 Heads then k=3 with $P_s = P(H)=0.5$, $P_F=1-P_s=0.5$

$$P(HHH) = P_3 = C_k^n P_s^k P_f^{n-k} = C_3^3 (0.5)^3 (0.5)^0 = 0.125$$

b. Using binomial distribution or repeated trials: n=3, k=0,1,2,3 , since we have one Head then k=1 with $P_s = P(H)=0.5$, $P_F=1-P_s=0.5$

$$P(H) = P_1 = C_k^n P_s^k P_f^{n-k} = C_1^3 (0.5)^1 (0.5)^2 = \frac{3!}{1!.2!} (0.5)^3 = 0.375$$

c. Using binomial distribution or repeated trials: n=3, k=0,1,2,3 , since we have at least one Head then k=1,2,3 with $P_s = P(H)=0.5$, $P_F=1-P_s=0.5$

$$P(at\ least\ 1\ Head\ ) = P_{1or2or\ 3} = \sum_{k=1}^{3} C_k^n P_s^k P_f^{n-k} = \sum_{k=1}^{3} C_k^3 P_s^k P_f^{3-k}$$

$$= C_1^3 (0.5)^1 (0.5)^2 + C_2^3 (0.5)^2 (0.5)^1 + C_3^3 (0.5)^3 (0.5)^0 = 0.875$$