BJT Small-Signal Analysis

Common-Emitter Configuration:

The voltage divider circuit of Fig. 13-1 includes an emitter resistor (R_E) that may or may not be bypassed by an emitter capacitor (C_E) in the ac domain.

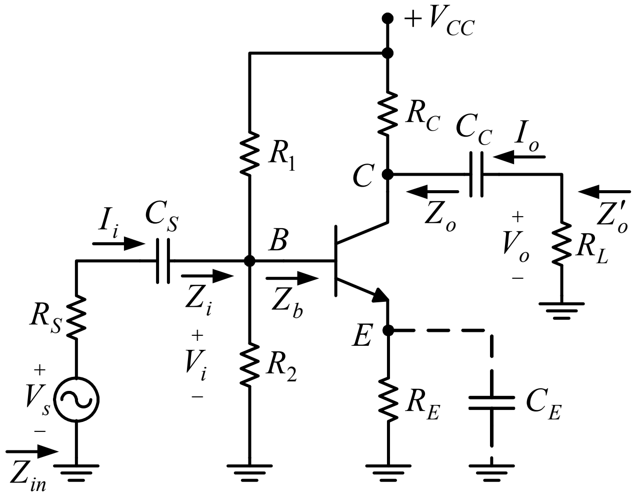

Fig. 13-1

Bypassed (absence of R_E):

For the ac equivalent circuit of Fig. 13-2,

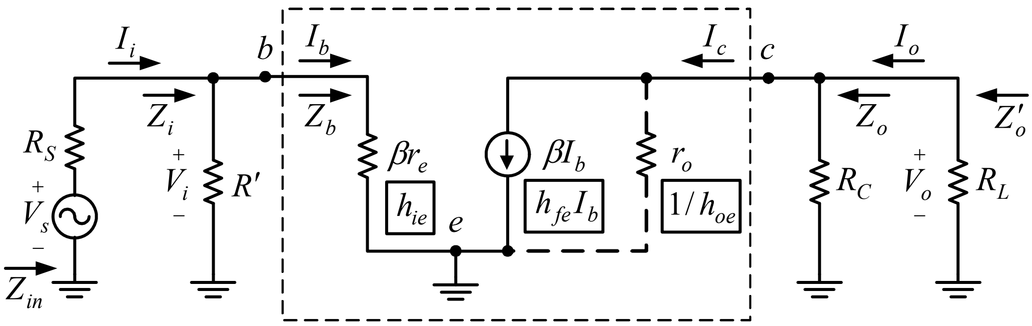

Fig. 13-2

Using r_e equivalent model:

Input impedance:

$$R' = R_1 \| R_2$$

$$Z_b = \beta r_e$$

$$Z_i = R' \| Z_b = R' \| \beta r_e$$

$$Z_{in} = R_S + Z_i = R_S + (R' \| \beta r_e)$$

Output impedance:

Approximate (neglecting r_o);

$$Z_o = R_C$$

$$Z'_o = R_L \| Z_o = R_L \| R_C$$

Exact (including r_o);

$$Z_o = R_C \| r_o$$

$$Z'_o = R_L \| R_C \| r_o$$

Voltage gain:

Approximate (neglecting r_o);

$$V_o = -I_c Z'_o = -\beta I_b (R_L \| R_C)$$

$$I_b = \frac{V_i}{Z_b} = \frac{V_i}{\beta r_e}$$

$$A_v = \frac{V_o}{V_i} = -\frac{R_L \| R_C}{r_e}$$

$$A_{v_s} = \frac{V_o}{V_s} = \frac{V_o}{V_i} \cdot \frac{V_i}{V_s} = A_v \cdot \frac{Z_i}{Z_i + R_S}$$

Exact (including r_o);

$$A_v = -\frac{R_L \| R_C \| r_o}{r_e}$$

Current gain:

Approximate (neglecting r_o);

$$A_i = \frac{I_o}{I_i} = \frac{I_o}{I_c} \cdot \frac{I_c}{I_b} \cdot \frac{I_b}{I_i}$$

$$= \frac{R_C}{R_C + R_L} \cdot \beta \cdot \frac{R'}{R' + Z_b}$$

$$= \frac{\beta R_C R'}{(R_C + R_L)(R' + \beta r_e)}$$

$$A_{i_s} = \frac{I_o}{I_s} = \frac{I_o}{I_i} \cdot \frac{I_i}{I_s} = A_i \cdot \frac{R_S}{R_S + Z_i}$$

Exact (including r_o);

$$A_i = \frac{\beta r_o R_C R'}{[r_o + (R_C \| R_L)](R_C + R_L)(R' + \beta r_e)}$$

As an option:

$$A_v = \frac{V_o}{V_i} = \frac{-I_o R_L}{I_i Z_i} = -\frac{I_o}{I_i} \cdot \frac{R_L}{Z_i} = -A_i \cdot \frac{R_L}{Z_i}$$

$$A_i = \frac{I_o}{I_i} = \frac{-V_o / R_L}{V_i / Z_i} = -\frac{V_o}{V_i} \cdot \frac{Z_i}{R_L} = -A_v \cdot \frac{Z_i}{R_L}$$

Phase relationship:

The negative sign in the resulting equation for A_v reveals that a 180° phase shift occurs between the input and output voltage signals.

Using *hybrid* equivalent model:

Approximate (neglecting h_{oe});

$$Z_b = h_{ie}$$

$$A_v = -\frac{h_{fe}(R_L \| R_C)}{h_{ie}}$$

$$A_i = \frac{h_{fe} R_C R'}{(R_C + R_L)(R' + h_{ie})}$$

Exact (including h_{oe});

$$A_v = -\frac{h_{fe}(R_L \| R_C \| 1/h_{oe})}{h_{ie}}$$

$$A_i = \frac{h_{fe} R_C R'/h_{oe}}{[1/h_{oe} + (R_C \| R_L)](R_C + R_L)(R' + h_{ie})}$$

Unbypassed (include of R_E):

For the approximate ac equivalent circuit ($r_o = 1/h_{oe} \approx \infty\Omega$) of Fig. 13-3,

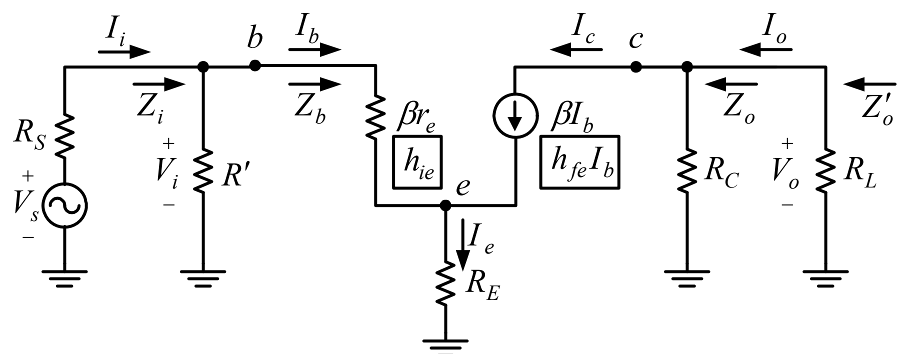

Fig. 13-3

Using r_e equivalent model:

Input impedance:

$$V_i = I_b \beta r_e + I_e R_E = I_b[\beta r_e + (\beta + 1)R_E]$$

$$Z_b = \frac{V_i}{I_b} = \beta r_e + (\beta + 1)R_E \approx \beta(r_e + R_E) \approx \beta R_E$$

$$Z_i = R' \| Z_b = R' \| [\beta r_e + (\beta + 1)R_E] \approx R' \| \beta(r_e + R_E) \approx R' \| \beta R_E$$

Output impedance:

$$Z_o = R_C$$

$$Z'_o = R_L \| Z_o = R_L \| R_C$$

Voltage gain:

$$V_o = -I_c Z'_o = -\beta I_b (R_L \| R_C)$$

$$I_b = \frac{V_i}{Z_b} = \frac{V_i}{\beta(r_e + R_E)}$$

$$A_v = \frac{V_o}{V_i} = -\frac{R_L \| R_C}{r_e + R_E} \approx -\frac{R_L \| R_C}{R_E}$$

Current gain:

$$A_i = \frac{I_o}{I_i} = \frac{I_o}{I_c} \cdot \frac{I_c}{I_b} \cdot \frac{I_b}{I_i}$$
$$= \frac{R_C}{R_C + R_L} \cdot \beta \cdot \frac{R'}{R' + Z_b}$$
$$= \frac{\beta R_C R'}{(R_C + R_L)[R' + \beta(r_e + R_E)]} \approx \frac{\beta R_C R'}{(R_C + R_L)(R' + \beta R_E)}$$

Phase relationship:

V_o and V_i are out-of-phase by 180°.

Using *hybrid* equivalent model:

$$Z_b = h_{ie} + (h_{fe} + 1)R_E \approx h_{ie} + h_{fe}R_E \approx h_{fe}R_E$$
$$A_v = -\frac{h_{fe}(R_L \| R_C)}{h_{ie} + h_{fe}R_E} \approx -\frac{R_L \| R_C}{R_E}$$
$$A_i = \frac{h_{fe}R_C R'}{(R_C + R_L)(R' + h_{ie} + h_{fe}R_E)} \approx \frac{h_{fe}R_C R'}{(R_C + R_L)(R' + h_{fe}R_E)}$$

Common-Base Configuration:

The common-base configuration of Fig. 13-4 is characterized as having a relatively low input and a high output impedance and a current gain less than 1. The voltage gain, however, can be quite large.

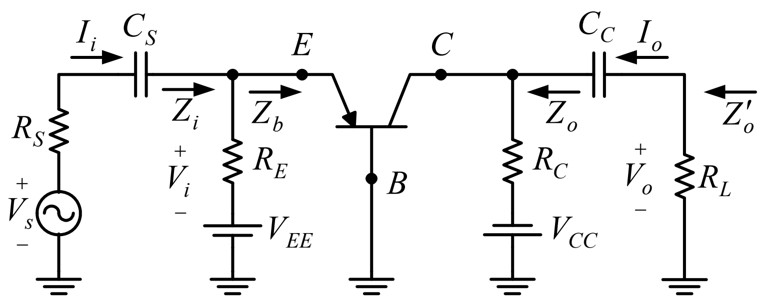

Fig. 13-4

Using r_e equivalent model:

For the approximate ac equivalent circuit ($r_o \approx \infty\Omega$) of Fig. 13-5,

Input impedance:

$Z_b = r_e$

$Z_i = R_E \| r_e$ [*low*]

Output impedance:

$Z_o = R_C$ [*high*]

$Z'_o = R_L \| R_C$

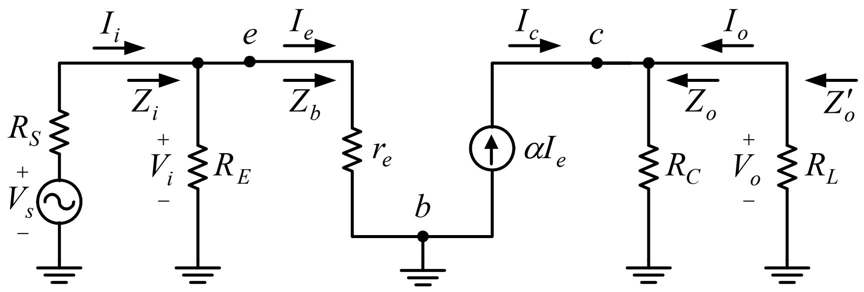

Fig. 13-5

Voltage gain:

$$V_o = I_c Z'_o = \alpha I_e (R_L \| R_C)$$

$$I_e = V_i / r_e$$

$$A_v = \frac{\alpha(R_L \| R_C)}{r_e} \cong \frac{R_L \| R_C}{r_e} \qquad [high]$$

Current gain:

$$A_i = \frac{I_o}{I_i} = \frac{I_o}{I_c} \cdot \frac{I_c}{I_e} \cdot \frac{I_e}{I_i} = -\frac{R_C}{R_C + R_L} \cdot \alpha \cdot \frac{R_E}{R_E + r_e}$$

$$= -\frac{\alpha R_C R_E}{(R_C + R_L)(R_E + r_e)} \qquad [less\ than\ 1]$$

Phase relationship:

V_o and V_i are in-phase.

Using *hybrid* equivalent model:

For the approximate ac equivalent circuit ($1/h_{ob} \approx \infty\Omega$) of Fig. 13-6,

$$Z_b = h_{ib}$$

$$Z_i = R_E \| h_{ib}$$

$$A_v = -\frac{h_{fb}(R_L \| R_C)}{h_{ib}}$$

$$A_i = \frac{h_{fb} R_C R_E}{(R_C + R_L)(R_E + h_{ib})}$$

[h_{fb}: *-ve quantity*]

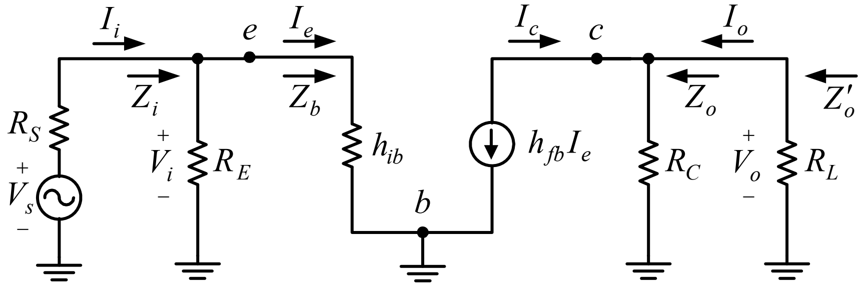

Fig. 13-6

Common-Collector (Emitter-Follower] Configuration:

When the output is taken from the emitter terminal of the transistor, an amplifier circuit is referred to as emitter-follower as shown in Fig. 13-7. The emitter-follower configuration is frequently used for impedance-matching purposes. It presents a high impedance at the input and a low impedance at the output. Also, the output voltage is always slightly less than the input signal with an in-phase relationship between them.

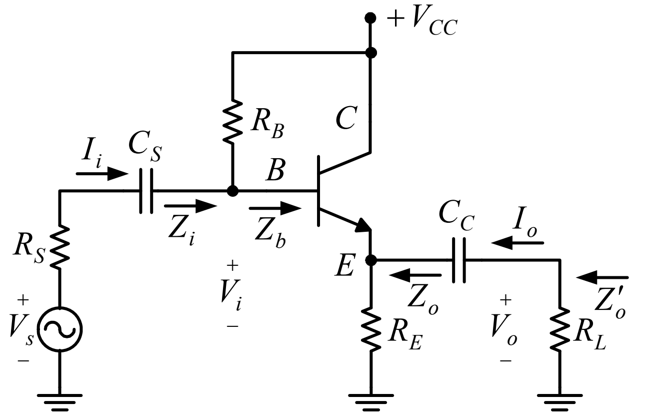

Fig. 13-7

Using r_e equivalent model:

For the ac equivalent circuit of Fig. 13-8,

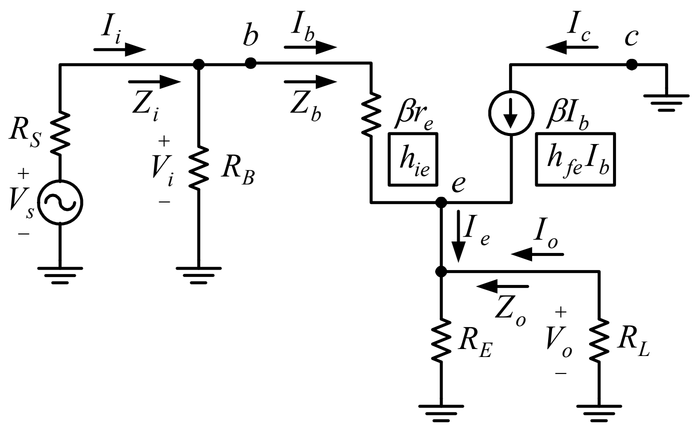

Fig. 13-8

Input impedance:

$$R' = R_L \| R_E$$

$$V_i = I_b \beta r_e + I_e R' = I_b[\beta r_e + (\beta + 1)R']$$

$$Z_b = V_i / I_b = \beta r_e + (\beta + 1)R'$$

$$\approx \beta(r_e + R') \approx \beta R' \qquad [high]$$

$$Z_i = R_B \| Z_b$$

Output impedance:

$V_s - I_i R_S - I_b \beta r_e - I_e R' = 0$ [KVL]

For the circuit of Fig. 13-9a,

$R_{Th} = R_S \| R_B$, and $E_{Th} = \dfrac{V_s R_B}{R_S + R_B}$

where $R_B >> R_S$ =>

$R_{Th} \approx R_S$, $E_{Th} \approx V_s$, and $I_i \approx I_b$

$V_s - I_b R_S - I_b \beta r_e - I_b(\beta+1)R' = 0$

$$I_b = \frac{V_s}{R_S + \beta r_e + (\beta+1)R'}$$

$$I_e = (\beta+1)I_b = \frac{(\beta+1)V_s}{R_S + \beta r_e + (\beta+1)R'}$$

$$\approx \frac{V_s}{R_S/\beta + r_e + R'}$$

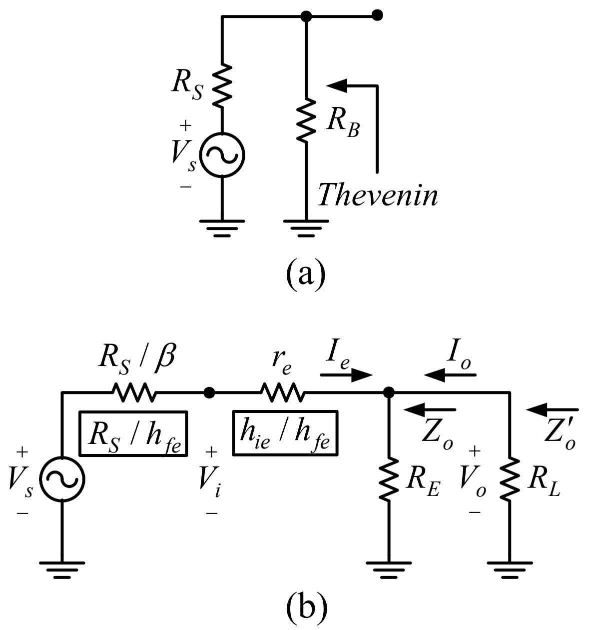

Fig. 13-9

Drawing the circuit to "fit" the above last equation will result in the configuration of Fig. 13-9b. Thus

$Z_o = R_E \| (R_S/\beta + r_e)$ [*low*]

$Z'_o = R_L \| Z_o$

Voltage gain:

$$A_v = \frac{V_o}{V_i} = \frac{I_e R'}{I_e(R' + r_e)} = \frac{R'}{R' + r_e}$$ [*less than 1*]

$$A_{v_s} = \frac{V_o}{V_s} = \frac{R'}{R' + R_s/\beta + r_e}$$

Current gain:

$$A_i = \frac{I_o}{I_i} = \frac{I_o}{I_e} \cdot \frac{I_e}{I_b} \cdot \frac{I_b}{I_i} = -\frac{R_E}{R_E + R_L} \cdot (\beta+1) \cdot \frac{R_B}{R_B + Z_b}$$

$$\approx -\frac{\beta R_E R_B}{(R_E + R_L)(R_B + \beta R')}$$ [*high*]

Phase relationship:

V_o and V_i are in-phase.

Using *hybrid* equivalent model:

$$Z_b = h_{ie} + (h_{fe} + 1)R' \approx h_{fe}R'$$

$$Z_o = R_E \| (R_S + h_{ie}) / h_{fe}$$

$$A_v = \frac{R'}{R' + h_{ie} / h_{fe}}$$

$$A_{v_s} = \frac{R'}{R' + (R_s + h_{ie}) / h_{fe}}$$

$$A_i = -\frac{h_{fe} R_E R_B}{(R_E + R_L)(R_B + h_{fe}R')}$$

Example 13-1:

For the BJT amplifier circuit of Fig. 13-10 with the following parameters: $V_{BE} = 0.7$ V, $\beta = h_{fe} \approx 250$, and $r_o = 1/h_{oe} \approx \infty\ \Omega$, determine:

(a) r_e, and dc output voltage (V_C).
(b) h_{ie}, Z_b, Z_i, Z_o, and Z'_o.
(c) $A_v = V_o/V_i$, and $A_i = I_o/I_i$.
(d) $A_{v_s} = V_o / V_s$, and ac output voltage (V_o).

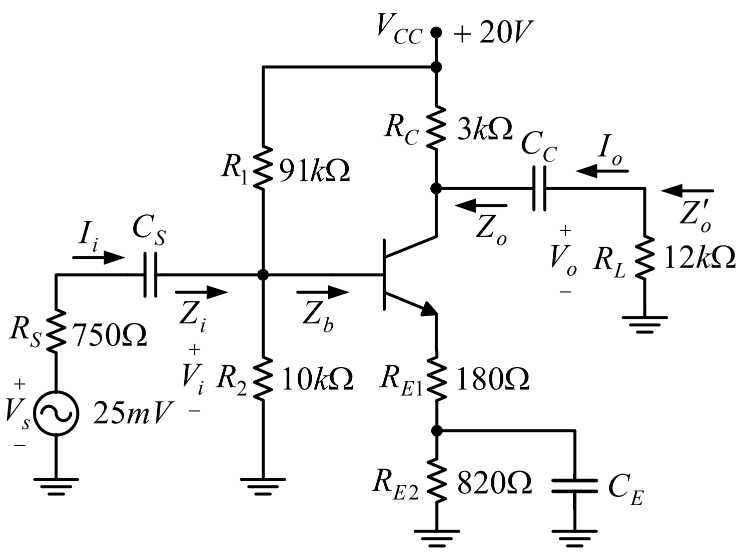

Fig. 13-10

Solution:

Testing: $\beta R_E \geq 10R_2$, $R_E = R_{E1} + R_{E2} = 0.18k + 0.82k = 1k\Omega$,
$250(1k) \geq 10(10k)$, $250k > 100k$ Satisfied,

$$V_B = \frac{V_{CC} \cdot R_2}{R_1 + R_2} = \frac{20(10k)}{10k + 91k} = 1.98V, \quad I_E = \frac{V_B - V_{BE}}{R_E} = \frac{1.98 - 0.7}{1k} = 1.28mA,$$

$$r_e = \frac{26mV}{I_E} = \frac{26m}{1.28m} = 20.3\Omega, \quad I_C \approx I_E = 1.28mA, \text{ and}$$

$$V_C = V_{CC} - I_C R_C = 20 - 1.28m(3k) = 16.16V.$$

$$h_{ie} = \beta r_e = 250(20.3) = 5.075k\Omega,$$

$$Z_b = h_{ie} + (h_{fe} + 1)R_{E1} = 5.075k + 251(0.18k) = 50.26k\Omega,$$

$$R' = R_1 \| R_2 = 91k \| 10k = 9.01k\Omega, \quad Z_i = R' \| Z_b = 9.01k \| 50.26k = 7.64k\Omega,$$

$$Z_o = R_C = 3k\Omega, \text{ and } Z'_o = R_L \| R_C = 12k \| 3k = 2.4k\Omega.$$

$$A_v = -\frac{h_{fe} Z'_o}{Z_b} = -\frac{250(2.4k)}{50.26k} = -11.94, \text{ and } A_i = -A_v \frac{Z_i}{R_L} = \frac{11.94(7.64k)}{12k} = 7.6.$$

$$A_{v_s} = A_v \frac{V_i}{V_s} = A_v \frac{Z_i}{Z_i + R_S} = \frac{-11.94(7.64k)}{7.64k + 0.75k} = -10.87, \text{ and}$$

$$V_o = A_{v_s} \cdot V_s = -10.87(25m) = -271.75mV.$$

Example 13-2:

Design the BJT amplifier circuit shown in Fig. 13-11 to have a voltage gain magnitude of 4, Z_i = 3.37 kΩ, Z_o = 3 kΩ, and Z'_o = 2kΩ. Assume that the transistor is silicon with $\beta = 100$, h_{ie} = 1 kΩ, $r_o = 1/h_{oe} \approx \infty$ Ω, and $\beta R_E > 10R_2$.

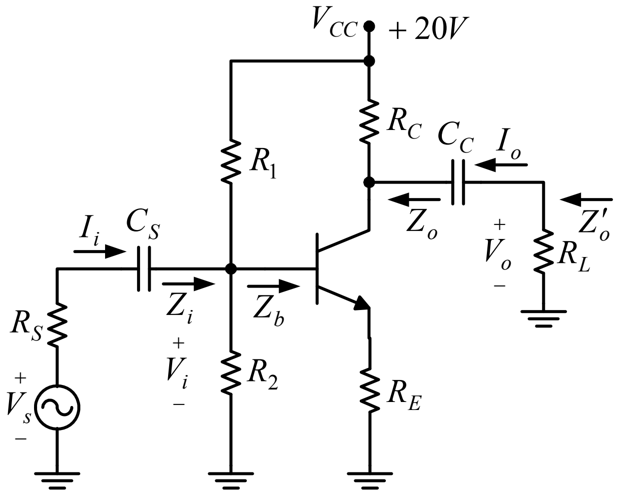

Fig. 13-11

Solution:

$R_C = Z_o = 3k\Omega$, $Z'_o = R_L \| R_C \Rightarrow 2k = R_L \| 3k \Rightarrow R_L = 6k\Omega$.

$A_v \approx \dfrac{Z'_o}{R_E} \Rightarrow 4 \approx \dfrac{2}{R_E} \Rightarrow R_E \approx 0.5k\Omega$.

$h_{ie} = \beta r_e \Rightarrow 1k = 100 r_e \Rightarrow r_e = 10\Omega$, $r_e = \dfrac{26mV}{I_E} \Rightarrow 10 = \dfrac{26m}{I_E} \Rightarrow I_E = 2.6mA$.

$V_E = I_E R_E = 2.6m(0.5k) = 1.3V$, $V_B = V_E + V_{BE} = 1.3 + 0.7 = 2V$.

$\because \beta R_E > 10R_2 \Rightarrow \dfrac{V_B}{V_{CC}} = \dfrac{R_2}{R_1 + R_2} \Rightarrow \dfrac{2}{20} = \dfrac{R_2}{R_1 + R_2} \Rightarrow \dfrac{R_2}{R_1 + R_2} = 0.1$ ----- [a]

$Z_b = h_{ie} + (\beta + 1)R_E = 1k + 101(0.5k) = 51.5k\Omega$.

$Z_i = R' \| Z_b \Rightarrow 3.37k = R' \| 51.5k \Rightarrow R' = 3.6k\Omega$.

$R' = R_1 \| R_2 = \dfrac{R_1 R_2}{R_1 + R_2} \Rightarrow \dfrac{3.6k}{R_1} = \dfrac{R_2}{R_1 + R_2}$ ----- [b]

From Eqs. [a] and [b]: $\dfrac{3.6k}{R_1} = 0.1 \Rightarrow R_1 = 36k\Omega$.

From Eq. [a]: $\dfrac{R_2}{36k + R_2} = 0.1 \Rightarrow R_2 = 4k\Omega$.

Example 13-3:

Complete the design of the BJT amplifier circuit shown in Fig. 13-12 for a voltage gain of 125, Z_o = 2.4 kΩ, Z'_o = 2 kΩ. Assume that $\alpha = 0.985$, $|V_{BE}|$ = 0.7 V, and $r_o = 1/h_{ob} \approx \infty$ Ω. Calculate A_{v_s}, and V_o.

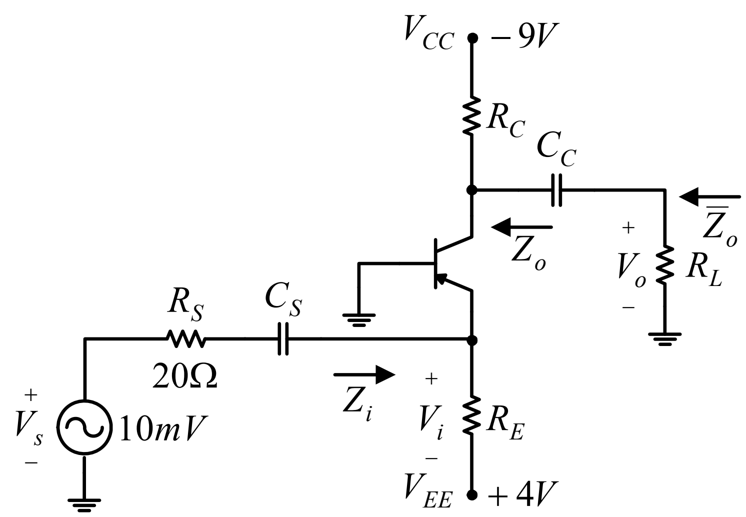

Fig. 13-12

Solution:

$R_C = Z_o = 2.4k\Omega$.

$Z'_o = R_L \| R_C \Rightarrow 2k = R_L \| 2.4k \Rightarrow R_L = 12k\Omega$.

$A_v = \dfrac{\alpha Z'_o}{r_e} \Rightarrow 125 = \dfrac{0.985(2k)}{r_e} \Rightarrow r_e = 15.76\Omega$.

$r_e = \dfrac{26mV}{I_E} \Rightarrow 15.76 = \dfrac{26m}{I_E} \Rightarrow I_E = 1.65mA$.

$R_E = \dfrac{V_{EE} - V_{BE}}{I_E} = \dfrac{4 - 0.7}{1.65m} = 2k\Omega$.

$Z_i = r_e \| R_E = 15.76 \| 2k = 15.64\Omega$.

$A_{v_s} = A_v \dfrac{Z_i}{Z_i + R_s} = \dfrac{125(15.64)}{15.64 + 20} = 55$.

$V_o = A_{v_s} \cdot V_s = 55(10m) = 550mV$.

Exercises:

1. For each one of the circuits shown in Fig. 13-13, write a mathematical expression to determine each of the following parameters by using *hybrid* or r_e equivalent model.
   (a) Z_b and Z_i. (b) Z_o and Z'_o. (c) A_i and A_v.

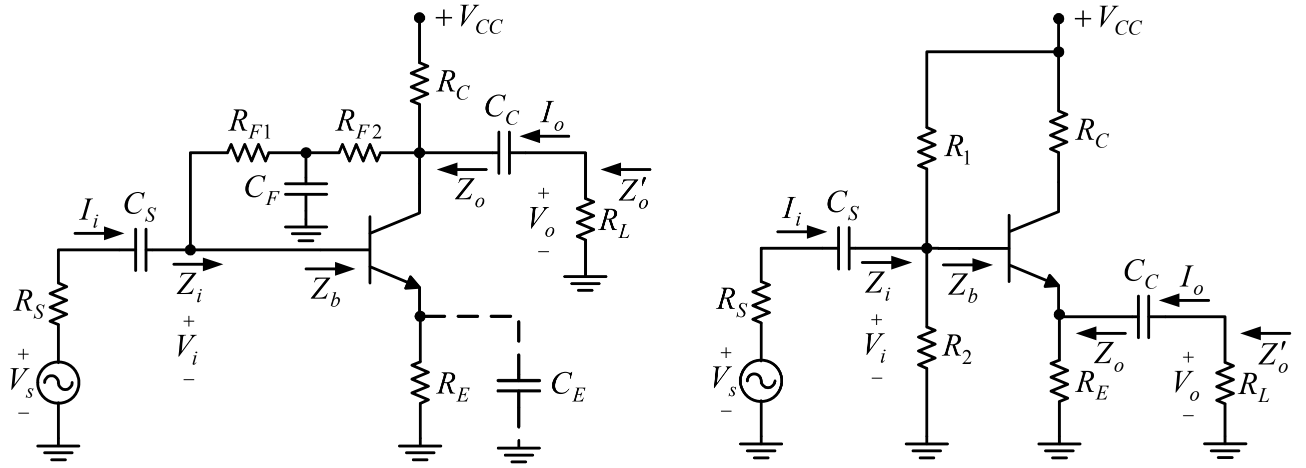

Fig. 13-13

2. For the common-base amplifier of Fig. 13-14, determine the following parameters using the complete hybrid equivalent model and compare the results to these obtained using the approximate model.
   (a) Z_b and Z_i. (b) Z_o and Z_o'. (c) A_i and A_{v_s}. (d) A_i and A_{i_s}.

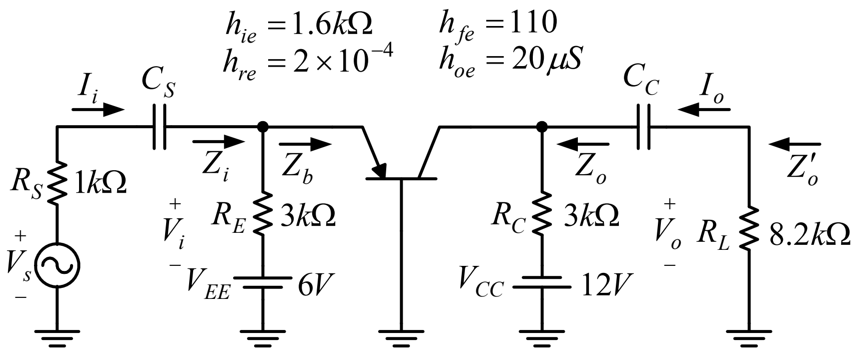

Fig. 13-14

3. Complete the design of the BJT amplifier circuit shown in Fig. 13-15 for a voltage gain magnitude of 205, Z_i =1.5k Ω, and Z_o'= 3.2 kΩ. Assume that $\beta = 100$, V_{BE} = 0.7 V, R_{F1}/R_{F2} =1.95, and $r_o = 1/h_{oe} \approx \infty$ Ω. Sketch V_o to the same time scale as V_s.

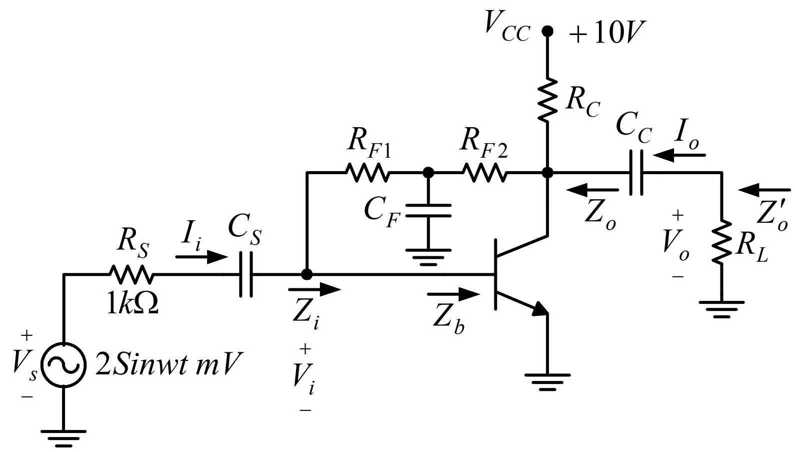

Fig. 13-15