COLLEGE OF ENGINEERING AND TECHNOLOGIES
ALMUSTAQBAL UNIVERSITY

Digital Signal Processing (DSP)

CTE 306

Lecture 9

- Convolution I -

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Convolution

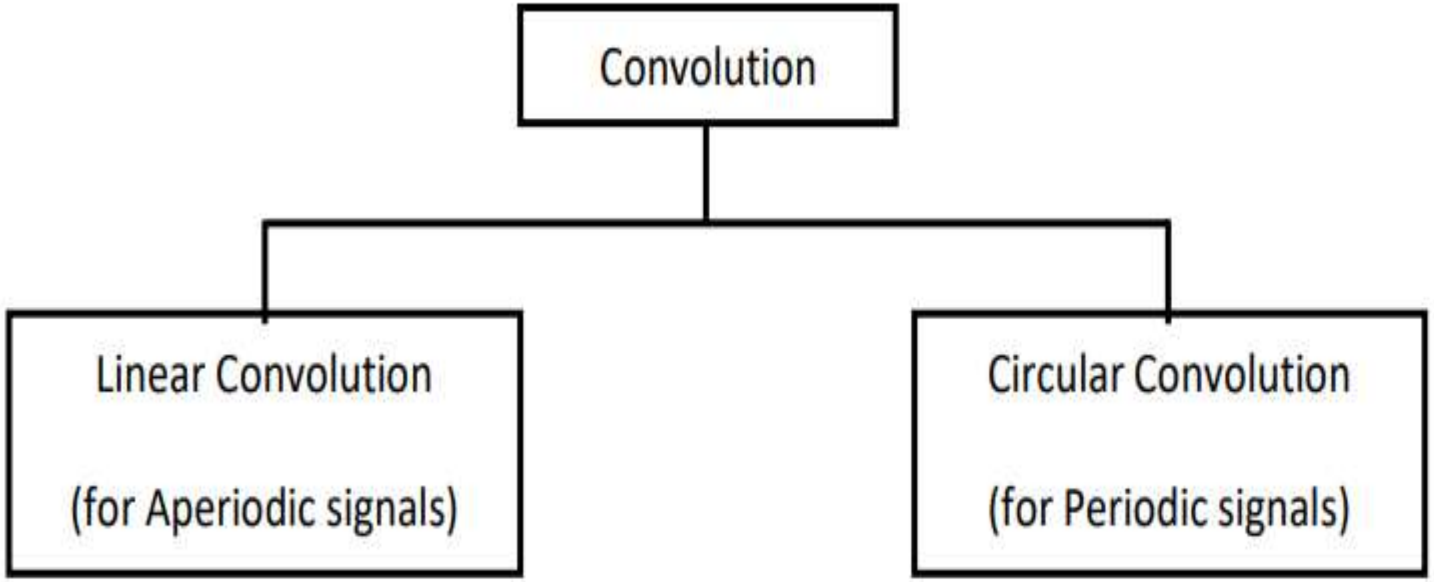
- Convolution is a mathematical operation on two functions producing a third function that is affected at any time by all previous input values.
- Convolution is the most important and fundamental concept in signal processing and analysis.
- By using convolution, we can construct the output of any LTI system for any arbitrary input signal, if we know the impulse response of that system.

Convolution

- The impulse response goes by a different name in some applications.
- If the system being considered is a filter, the impulse response is called the filter kernel, the convolution kernel, or simply, the kernel.
- In image processing, the impulse response is called the point spread function.
- The convolution is performed by sliding the kernel along the input signal

Classification of convolution

- Convolution can be classified into two categories according to the signals that will be convolved as shown in the figure below.



Methods

Methods of calculation of the convolutions

- There are several different approaches that may be used, and the one that is the easiest will depend upon the form and type of sequences that are to be convolved.

I. Graphical Method.

II. Tabular Method.

III. Matlab Method.

Graphical Method

The steps involved in using the graphical approach are as follows:

1) Plot both sequences, $x(k)$ and $h(k)$, as functions of k.

2) Choose one of the sequences, say $h(k)$, and time-reverse it to form the sequence $h(-k)$.

3) Shift the time-reversed sequence by n.

[if $n > 0$, this corresponds to a shift to the right (delay), whereas if $n < 0$, this corresponds to a shift to the left (advance)].

4) Multiply the two sequences $x(k)$ and $h(n-k)$ and sum the product for all values of k.

The resulting value will be equal to $y(k)$. This process is repeated for all possible shifts, n.

- The convolution of two discrete-time signals $x[n]$ and $h[n]$ to produce a new signal $y[n]$ is denoted by:

$$y[n] = x[n] * h[n]$$

and defined by

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

- This equation referred to as the *convolution sum*

Convolution of discrete-time signals

i. Commutative property:

$$y[n] = h[n] * x[n] = x[n] * h[n]$$

ii. Associative property

$$(x[n] * h_1[n]) * h_2[n] = x[n] * (h_1[n] * h_2[n])$$

iii. Distributive property

$$x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$$

Convolution of discrete-time signals

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$
$$= \ldots + h(-1)x(n+1) + h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + \ldots$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$
$$= \ldots + x(-1)h(n+1) + x(0)h(n) + x(1)h(n-1) + x(2)h(n-2) + \ldots$$

Using a conventional notation, we express the digital convolution as

$$y(n) = h(n)*x(n).$$

Note that for a causal system, which implies its impulse response

$$h(n) = 0 \text{ for } n < 0,$$

Convolution of discrete-time signals

The lower limit of the convolution sum begins at 0 instead of 1, that is

$$y(n) = \sum_{k=0}^{\infty} h(k)x(n-k) = \sum_{k=0}^{\infty} x(k)h(n-k).$$

We will focus on evaluating the convolution sum based on Equation (4).
Let us examine first a few outputs from Equation (4):

$$y(0) = \sum_{k=-\infty}^{\infty} x(k)h(-k) = \ldots + x(-1)h(1) + x(0)h(0) + x(1)h(-1) + x(2)h(-2) + \ldots$$

$$y(1) = \sum_{k=-\infty}^{\infty} x(k)h(1-k) = \ldots + x(-1)h(2) + x(0)h(1) + x(1)h(0) + x(2)h(-1) + \ldots$$

$$y(2) = \sum_{k=-\infty}^{\infty} x(k)h(2-k) = \ldots + x(-1)h(3) + x(0)h(2) + x(1)h(1) + x(2)h(0) + \ldots$$

$$\ldots$$

Tabular Method (Slide Rule Method)

The slide rule method is particularly convenient when both $x(k)$ and $h(n)$ are finite in length and short in duration.

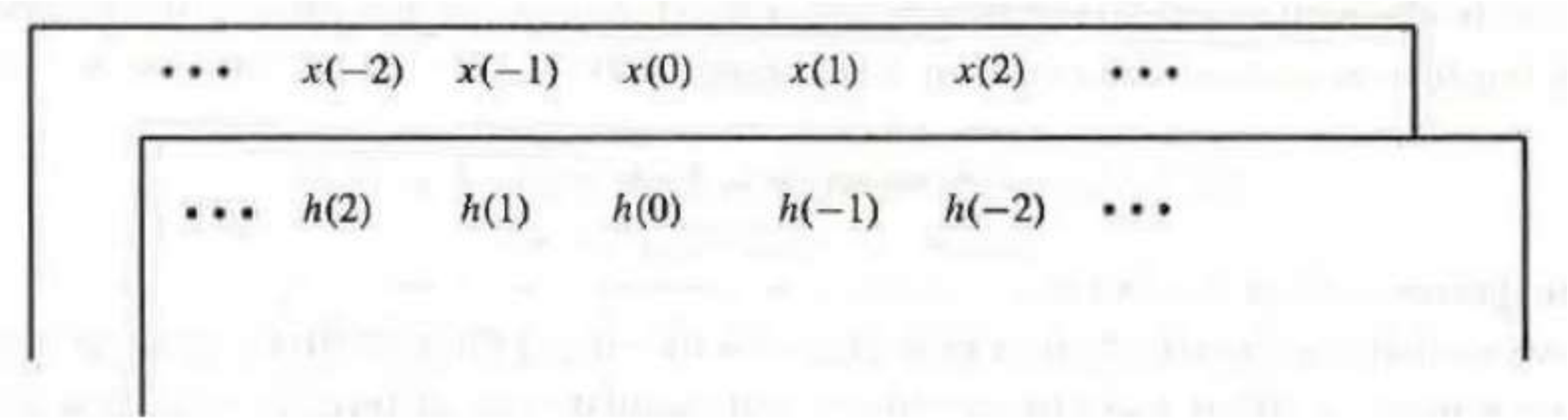
The steps involved in the slide rule method are as follows:

1) Write the values of $x(k)$ along the top of a piece of paper, and the values of $h(-k)$ along the top of another piece of paper as illustrated in figure.

2) 2) Line up the two sequences values $x(0)$ and $h(0)$, multiply each pair of numbers, and add the products to form the values of $y(0)$.

Tabular Method (Slide Rule Method)

3) Slide the paper with the time-reversed sequence $h(-k)$ to the right by one, multiply each pair of numbers, sum the products to find the value $y(1)$, and repeat for all shifts to the right by $n > 0$. Do the same, shifting the time-reversed sequence to the left, to find the values of $y(n)$ for $n < 0$

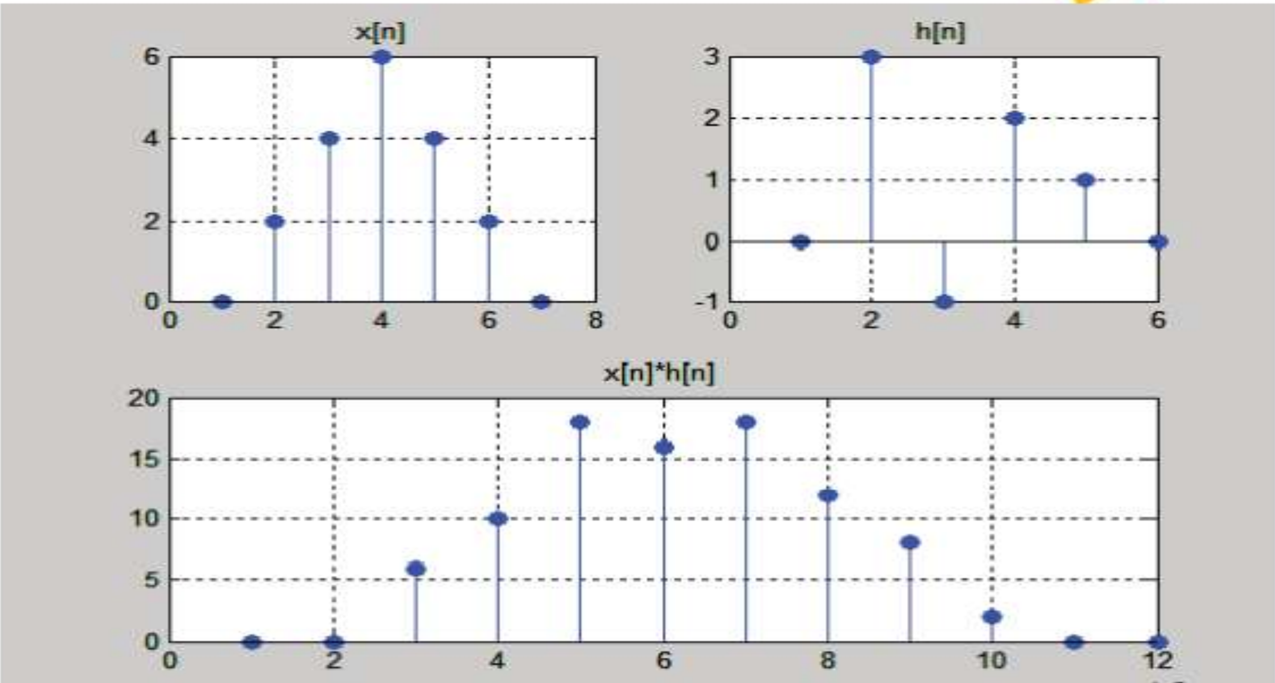


Matlab Method

Solution : Ex.1 (Method 3 – using MATLAB)

The convolution of two discrete-time signals can be carried out with the **MATLAB** M-file *conv*.

```
x=[0 2 4 6 4 2 0];
h=[0 3 -1 2 1 0];
y = conv(x,h);
```



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