Frequency Response of BJT Amplifiers

Low-Frequency Response of BJT Amplifiers:

For the ***high-pass filter*** circuit of Fig. 14-1a, the output and the input voltages are related by the voltage-divider rule in the following manner:

$$V_o = \frac{RV_i}{R + X_C},$$

with the magnitude of V_o determined by

$$|V_o| = \frac{R \cdot |V_i|}{\sqrt{R^2 + X_C^2}}.$$

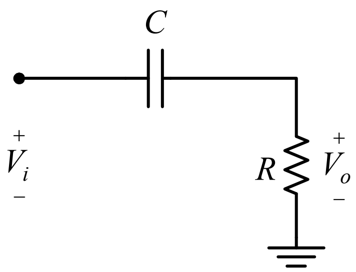
For special case where $X_C = R$,

$$|V_o| = \frac{1}{\sqrt{2}}|V_i|, \text{ and}$$

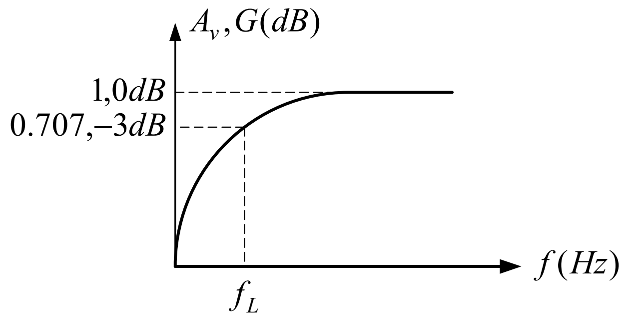
$$|A_v| = \frac{|V_o|}{|V_i|} = \frac{1}{\sqrt{2}} = 0.707\Big|_{X_C=R}.$$

In "***deciBel***" (dB):

$$G(dB) = 20\log_{10}|A_v| = 20\log_{10}\frac{1}{\sqrt{2}} = -3dB.$$



(a)



(b)

Fig. 14-1

At the frequency of witch $X_C = R$, the output will be 70.7 % of the input (a 3 dB drop in gain, see Fig. 14-1b) for the *RC* circuit. The frequency (f_L) at witch this occurs is determined from:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = R \quad =>$$

$$\boxed{f_L = \frac{1}{2\pi RC}}$$

f_L: the low-cutoff frequency.

The Capacitors C_S, C_C, and C_E will determine the lower-cutoff frequency (f_L) of the loaded voltage divider BJT bias configuration shown in Fig. 14-2, but the results can be applied to any BJT configuration.

For the amplifier circuit of Fig. 14-2:

The cutoff-frequency of C_S,

$$f_{L_S} = \frac{1}{2\pi(R_S + R_i)C_S}$$

where $R_i = R' \| h_{ie}$, and
$R' = R_1 \| R_2$.

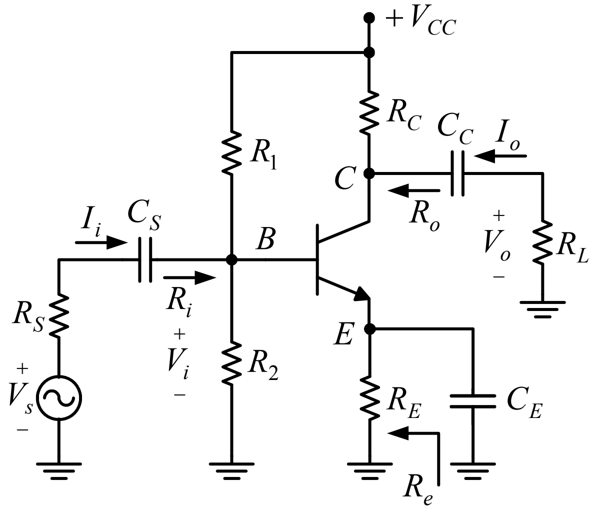


Fig. 14-2

The cutoff-frequency of C_C,

$$f_{L_C} = \frac{1}{2\pi(R_L + R_o)C_C}$$

where $R_o = R_C \| 1/h_{oe}$.

The cutoff-frequency of C_E,

$$f_{L_E} = \frac{1}{2\pi R_e C_E}$$

where $R_e = R_E \| \dfrac{R_S' + h_{ie}}{h_{fe} + 1}$, and
$R_S' = R_S \| R'$.

The lower-cutoff frequency,

$$f_L = Max.[f_{L_S}, f_{L_C}, f_{L_E}]$$

Miller's Theorem and Its Dual:

For the circuit of Fig. 14-3a,

$$I_i = \frac{V_i}{Z_i},\ I_1 = \frac{V_i}{R_i},\ \text{and}\ I_2 = \frac{V_i - V_o}{Z_F} = \frac{V_i - A_v V_i}{Z_F} = \frac{(1-A_v)V_i}{Z_F} = \frac{V_i}{Z_F/(1-A_v)}.$$

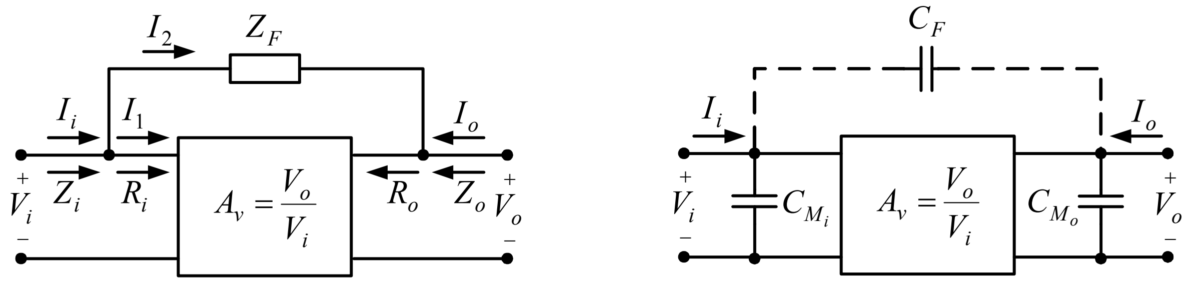
$$I_i = I_1 + I_2 \Rightarrow \frac{V_i}{Z_i} = \frac{V_i}{R_i} + \frac{Vi}{Z_F/(1-A_v)} \Rightarrow \frac{1}{Z_i} = \frac{1}{R_i} + \frac{1}{Z_F/(1-A_v)}.$$

when $Z_F = R_F \Rightarrow \frac{1}{Z_i} = \frac{1}{R_i} + \frac{1}{R_{M_i}}$, where $R_{M_i} = \frac{R_F}{1-A_v}$.

As shown in Fig. 14-3b,

when $Z_F = X_{C_F} \Rightarrow \frac{1}{Z_i} = \frac{1}{R_i} + \frac{1}{X_{C_{M_i}}}$, where $X_{C_{M_i}} = \frac{X_{C_F}}{1-A_v} = \frac{1}{\omega(1-A_v)C_F} \Rightarrow$

$$\boxed{C_{M_i} = (1-A_v)C_F}$$



(a) (b)

Fig. 14-3

In a similar way,

$$\frac{1}{Z_o} = \frac{1}{R_o} + \frac{1}{Z_F/(1-1/A_v)} \Rightarrow R_{M_o} = \frac{R_F}{1-1/A_v},\ \text{and}$$

$$X_{C_{M_o}} = \frac{X_{C_F}}{1-1/A_v} = \frac{1}{\omega(1-1/A_v)C_F} \Rightarrow$$

$$\boxed{C_{M_o} = (1-1/A_v)C_F}$$

The above shows us that:

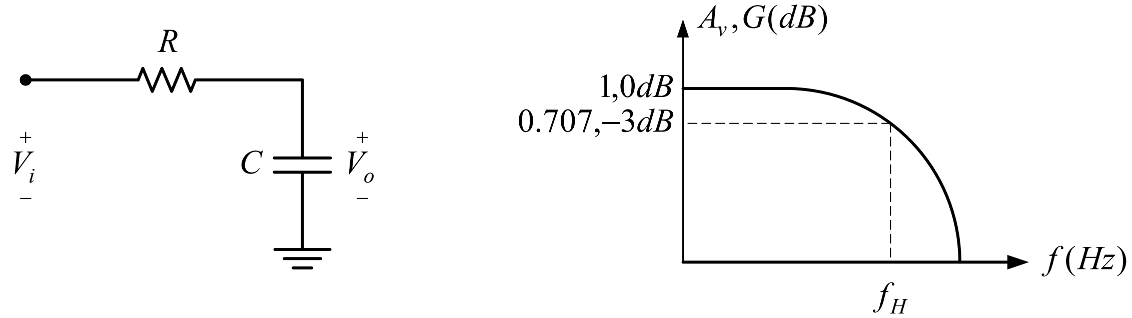
For any ***inverting*** amplifier (phase shift of 180^o between input and output resulting in a negative value for A_v), the input and output capacitance will be increased by a Miller effect capacitance sensitive to the gain of the amplifier and the interelectrode capacitance connected between the input and output terminals of the active device.

High-Frequency Response of BJT Amplifiers:

A frequency response of the ***low-pass filter*** circuit of Fig. 14-4a is given by Fig. 14-4b, where the high-cutoff frequency is determined from:

$$f_H = \frac{1}{2\pi RC}$$

f_H: the high-cutoff frequency.



(a) (b)

Fig. 14-4

At the high-frequency end, there are two factors that will define the -3 dB point: the circuit capacitance (parasitic and introduced) and the frequency dependence of h_{fe}.

Circuit (Capacitances) Parameters:

In high-frequency region the capacitive elements of the importance are the inter-electrode (between terminals) capacitances internal to the active device and the wiring capacitance between leads of the circuit. In Fig. 14-5, the various parasitic capacitances (C_{be}, C_{bc}, and C_{ce}) of the transistor have been included with the wiring capacitances (C_{W_i} and C_{W_o}) introduced during construction.

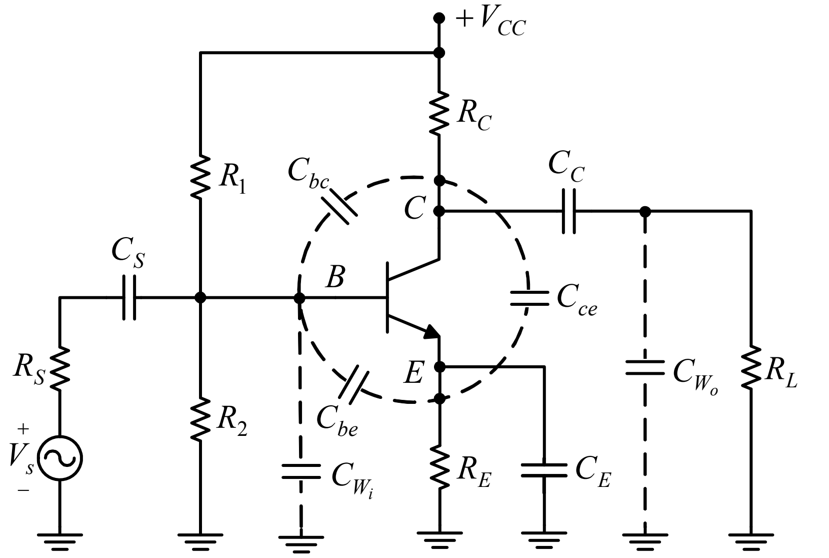


Fig. 14-5

The high-frequency equivalent model for the amplifier circuit of Fig. 14-5 appears in Fig. 14-6. Note the absence of the capacitors C_S, C_C, and C_E, which are all assumed to be in the short circuit state at these frequencies.

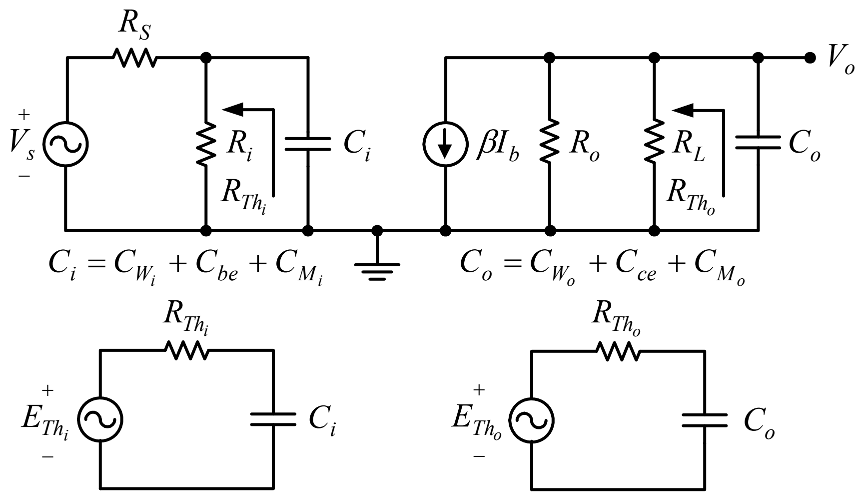


Fig. 14-6

For the circuit of Fig. 14-6:

The input high cutoff frequency,

$$\boxed{f_{H_i} = \frac{1}{2\pi R_{Th_i} C_i}}$$

where $R_{Th_i} = R_S \| R_i$, and $R_i = R_1 \| R_2 \| h_{ie}$.

$C_i = C_{W_i} + C_{be} + C_{M_i} = C_{W_i} + C_{be} + (1 - A_v)C_{bc}$.

The output high cutoff frequency,

$$\boxed{f_{H_o} = \frac{1}{2\pi R_{Th_o} C_o}}$$

where $R_{Th_o} = R_L \| R_o$, and $R_o = R_C \| 1/h_{oe}$

$C_o = C_{W_o} + C_{ce} + C_{M_o} = C_{W_o} + C_{ce} + (1 - 1/A_v)C_{bc}$.

The higher-cutoff frequency,

$$\boxed{f_H = Min.[f_{H_i}, f_{H_o}]}$$

h_{fe} (β) Variation:

The beta cutoff frequency (f_β) is another important transistor cutoff frequency. The f_β, the frequency where the β of the transistor drop to 0.707 of its low-frequency value, is given by

$$f_\beta \cong \frac{1}{2\pi\beta r_e(C_{be}+C_{bc})}$$

If the frequency of operation is increased above the f_β of the transistor, the β will continue to decrease. Eventually, we find a frequency where the $\beta = 1$; this frequency is called the f_T of the transistor. The f_T of a transistor is much higher than the f_β. The relation between these two frequencies is

$$f_T \cong \beta \cdot f_\beta \approx h_{fe} \cdot BW$$

f_T: the gain-bandwidth product frequency.

Finally, in data sheet, the CB high-frequency parameters rather than CE parameters are often specified for a transistor. The following equation permits a direct conversion for determining f_β if f_α and α are specified.

$$f_\beta = f_\alpha(1-\alpha)$$

Example 14-1:

For the BJT amplifier circuit shown in Fig. 14-7, with the following parameters:
C_{be} = 36 pF, C_{bc} = 4 pF, C_{ce} = 1 pF, C_{W_i} = 6 pF, C_{W_o} = 8 pF, and $r_o = 1/h_{oe} = \infty$ Ω.

1. Determine f_L, f_H, BW, f_β, and f_T.
2. Sketch the frequency response.

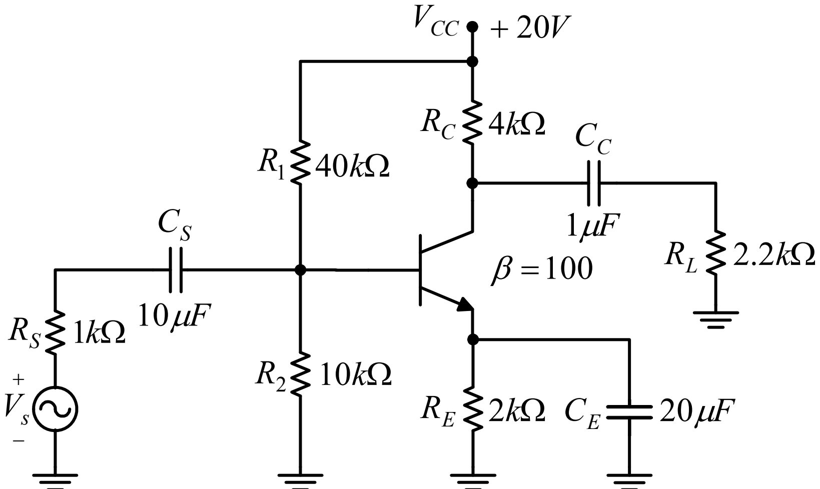


Fig. 14-7

Solution:

Testing: $\beta R_E \geq 10R_2$, $100(2k) \geq 10(10k)$, $200k\Omega > 100k\Omega$ Satisfied.

$$V_B = \frac{V_{CC} \cdot R_2}{R_1 + R_2} = \frac{20(10k)}{40k + 10k} = 4V, \quad I_E = \frac{V_E}{R_E} = \frac{V_B - V_{EB}}{R_E} = \frac{4 - 0.7}{2k} = 1.65mA.$$

$$r_e = \frac{26mV}{I_E} = \frac{26m}{1.65m} = 15.76\Omega, \quad h_{ie} = \beta r_e = 100(15.76) = 1.58k\Omega.$$

$$A_{v_{mid}} = -\frac{R_L \| R_C}{r_e} = -\frac{2.2k \| 4k}{15.76} = -90.$$

$$R_i = R_1 \| R_2 \| h_{ie} = 40k \| 10k \| 1.58k = 1.32k\Omega,$$

$$f_{L_S} = \frac{1}{2\pi(R_S + R_i)C_S} = \frac{1}{2\pi(1k + 1.32k)(10\mu)} = 7Hz.$$

$$R_o = R_C \| 1/h_{oe} = 4k\Omega,$$

$$f_{L_C} = \frac{1}{2\pi(R_L + R_o)C_C} = \frac{1}{2\pi(2.2k + 4k)(1\mu)} = 26Hz.$$

$$R'_S = R_S \| R_1 \| R_2 = 1k \| 40k \| 10k = 0.89k\Omega,$$

$$R_e = R_E \| [\frac{R'_S}{\beta} + r_e] = 2k \| [\frac{0.89k}{100} + 15.76] = 24.35\Omega.$$

$$f_{L_E} = \frac{1}{2\pi R_e C_E} = \frac{1}{2\pi(24.35)(20\mu)} = 327Hz.$$

The lower-cutoff frequency, $f_L = Max.[f_{L_S}, f_{L_C}, f_{L_E}]$
$= Max.[7,26,327] = 327Hz$.

$$R_{Th_i} = R_S \| R_i = 1k \| 1.32k = 0.57k\Omega.$$

$$C_i = C_{W_i} + C_{be} + C_{M_i} = C_{W_i} + C_{be} + (1 - A_v)C_{bc} = 6p + 36p + (1 + 90)(4p) = 406pF.$$

$$f_{H_i} = \frac{1}{2\pi R_{Th_i} C_i} = \frac{1}{2\pi(0.57k)(406p)} = 687.732kHz.$$

$$R_{Th_o} = R_L \| R_o = 2.2k \| 4k = 1.42k\Omega.$$

$$C_o = C_{W_o} + C_{ce} + C_{M_o} = C_{W_o} + C_{ce} + (1 - 1/A_v)C_{bc} = 8p + 1p + (1 + 1/90)(4p) = 13pF.$$

$$f_{H_o} = \frac{1}{2\pi R_{Th_o} C_o} = \frac{1}{2\pi(1.42k)(13p)} = 8.622MHz.$$

The higher-cutoff frequency, $f_H = Min.[f_{H_i}, f_{H_o}]$
$= Min.[687.732k, 8.622M] = 687.732kHz$.

The bandwidth, $BW = f_H - f_L = 687.732k - 327 = 687.405kHz$.

The beta cutoff frequency, $f_\beta = \dfrac{1}{2\pi\beta r_e(C_{be}+C_{bc})}$

$$= \frac{1}{2\pi(100)(15.76)(36p+4p)} = 2.52MHz\,.$$

The gain-bandwidth product, $f_T = \beta \cdot f_\beta = 100(2.52M) = 252MHz$.

The frequency response for the low- and high-frequency regions, bandwidth, beta cutoff frequency, and gain-bandwidth product frequency are shown in Fig. 14-8.

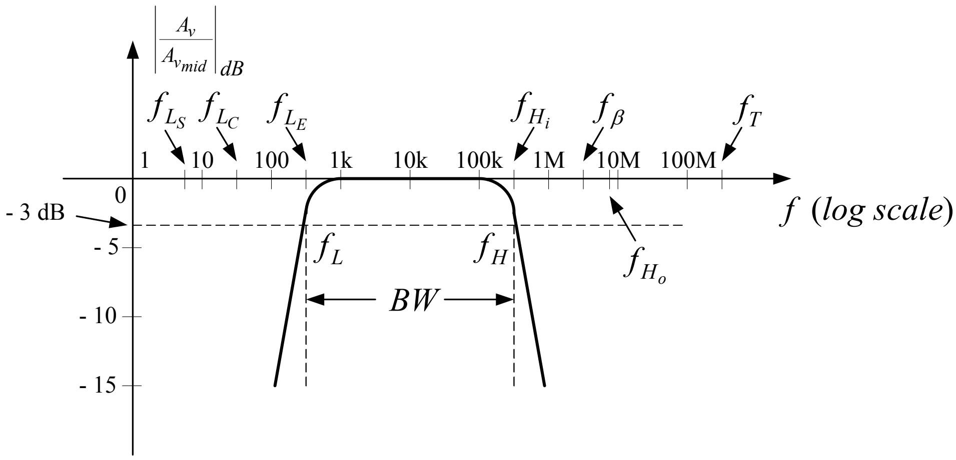


Fig. 14-8

Exercise:

For the BJT amplifier circuit of Fig. 14-9, determine the lower- and higher-cutoff frequencies.

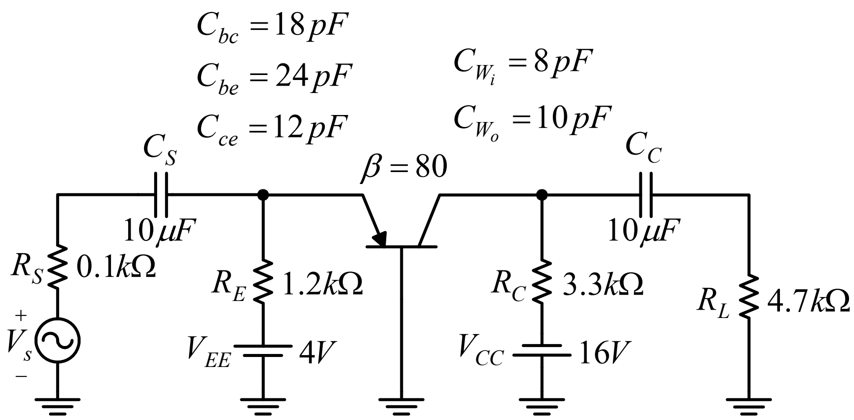


Fig. 14-9