وزارة التعليم العالي والبحث العلمي
جامعة المستقبل

LEC : THREE

Course Name : Fundamentals of Electricity Instructor
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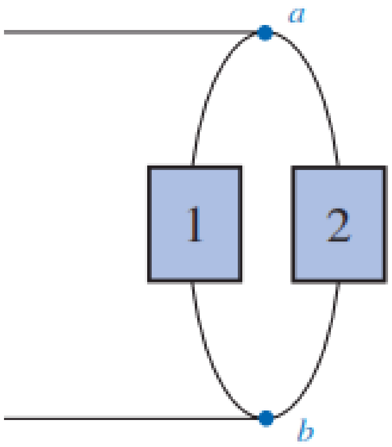
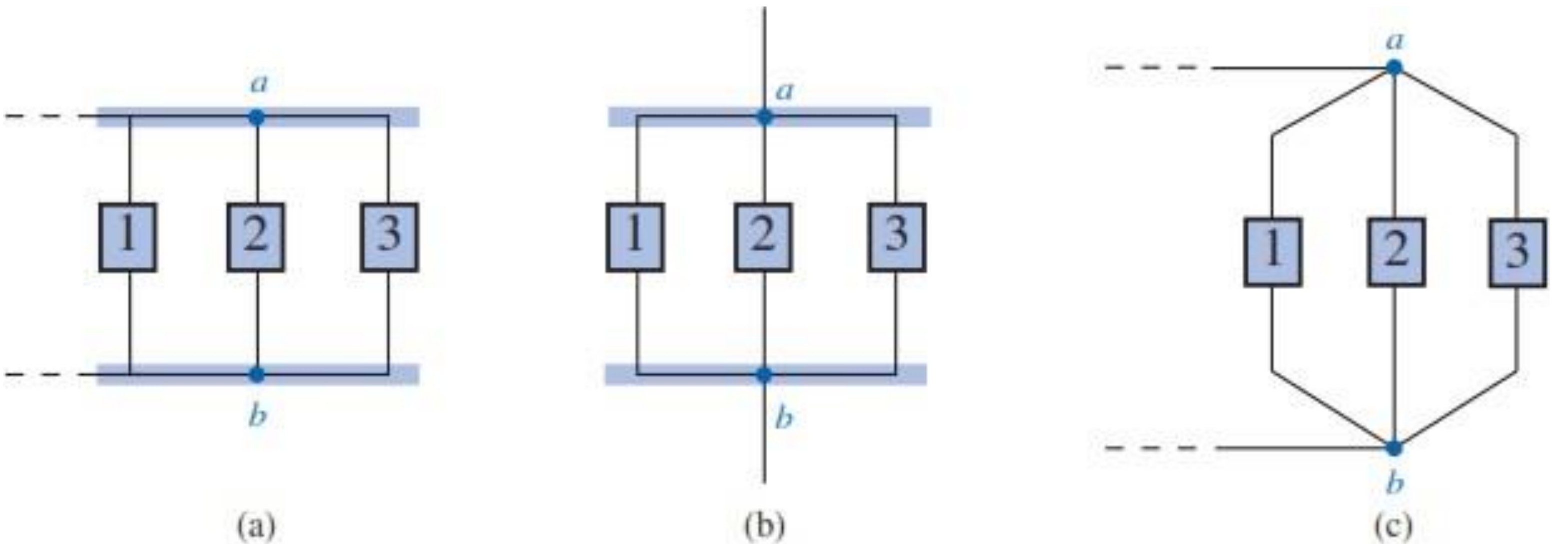
Stage : First
Academic Year : 2023

Lecture Title : **Parallel Circuits - Current Divider Rule**

PARALLEL DC CIRCUITS



Two elements, branches, or networks are in parallel if they have two points in common.



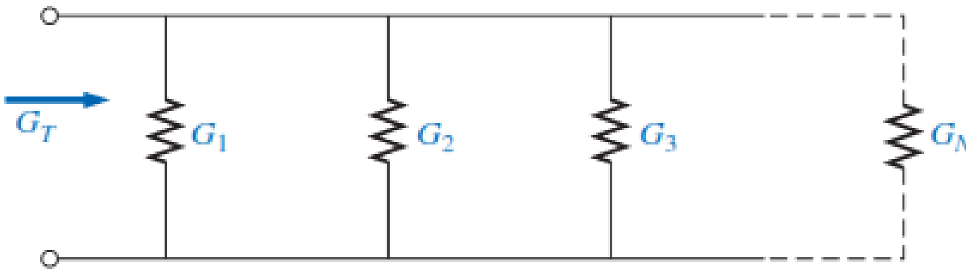
Different ways in which three parallel elements may appear.

TOTAL CONDUCTANCE AND RESISTANCE

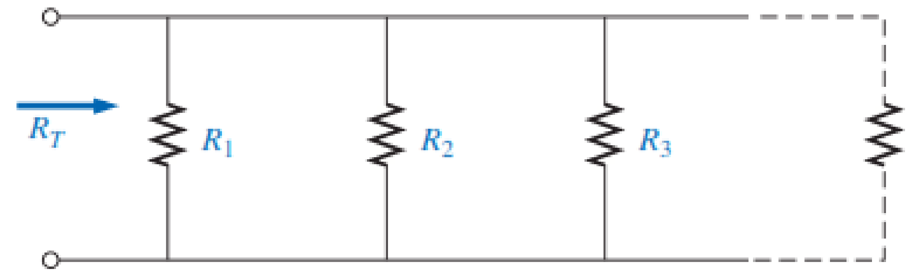
Recall that for series resistors, the total resistance is the sum of the resistor values.

For parallel elements, the total conductance is the sum of the individual conductances.

$$G_T = G_1 + G_2 + G_3 + \cdots + G_N$$



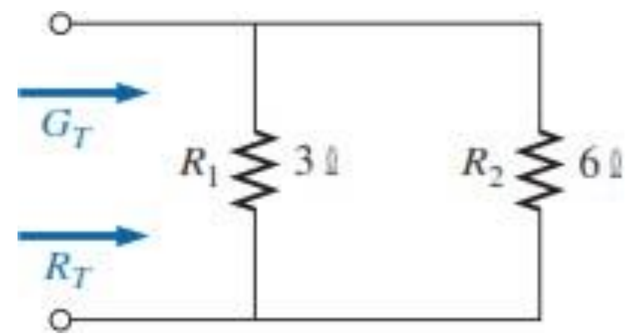
$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}$$



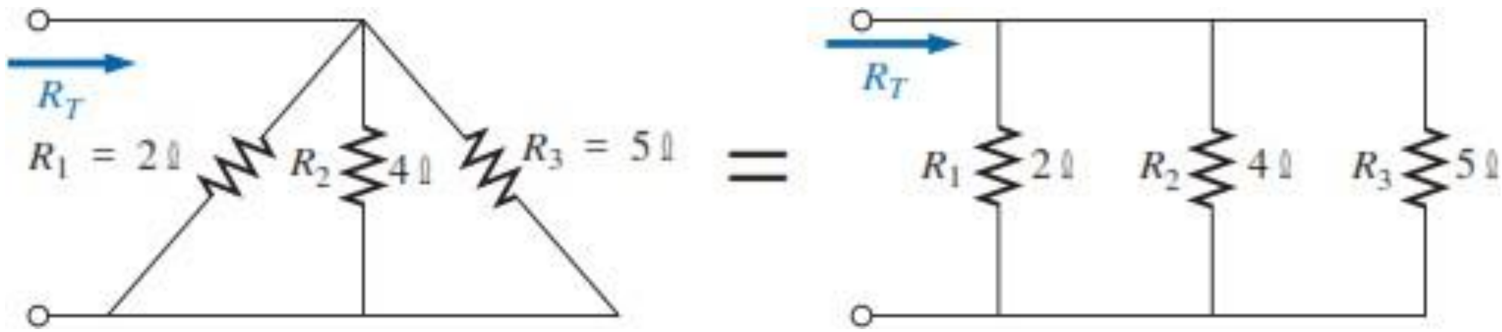
EXAMPLE 3.1 Determine the total conductance and resistance for the parallel network of Fig. shown.

$$G_T = G_1 + G_2 = \frac{1}{3\ \Omega} + \frac{1}{6\ \Omega} = 0.333\ \text{S} + 0.167\ \text{S} = \mathbf{0.5\ S}$$

and $$R_T = \frac{1}{G_T} = \frac{1}{0.5\ \text{S}} = \mathbf{2\ \Omega}$$



EXAMPLE 3.2 Determine the total resistance for the network of Fig. shown.



$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{1}{2\,\Omega} + \frac{1}{4\,\Omega} + \frac{1}{5\,\Omega} = 0.5\text{ S} + 0.25\text{ S} + 0.2\text{ S}$$

$$= 0.95\text{ S}$$

and $$R_T = \frac{1}{0.95\text{ S}} = \mathbf{1.053\,\Omega}$$

The total resistance of parallel resistors is always less than the value of the smallest resistor.

For *equal resistors in parallel, the equation becomes significantly* easier to apply. For *N equal resistors in parallel,*

$$\frac{1}{R_T} = \underbrace{\frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \cdots + \frac{1}{R}}_{N}$$

$$= N\left(\frac{1}{R}\right)$$

and $$\boxed{R_T = \frac{R}{N}}$$

and $$\boxed{G_T = NG}$$

For two parallel resistors, we write

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_T} = \left(\frac{R_2}{R_2}\right)\frac{1}{R_1} + \left(\frac{R_1}{R_1}\right)\frac{1}{R_2} = \frac{R_2}{R_1R_2} + \frac{R_1}{R_1R_2}$$

$$= \frac{R_2 + R_1}{R_1R_2}$$

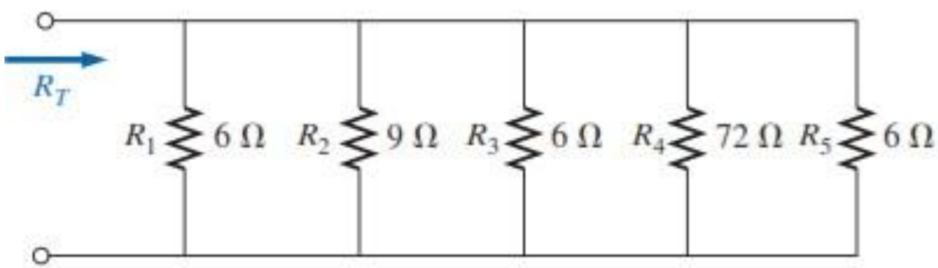
and $$\boxed{R_T = \frac{R_1R_2}{R_1 + R_2}}$$

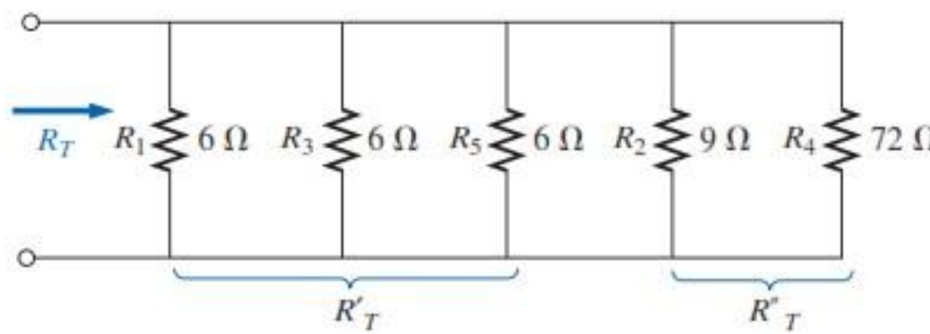
In words, *the total resistance of two parallel resistors is the product of the two divided by their sum.*

For three parallel resistors,

$$\boxed{R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}} \qquad \boxed{R_T = \frac{R_1 R_2 R_3}{R_1R_2 + R_1R_3 + R_2R_3}}$$

EXAMPLE 3.3 Calculate the total resistance of the parallel network of Fig. shown.





$$R'_T = \frac{R}{N} = \frac{6\ \Omega}{3} = 2\ \Omega$$

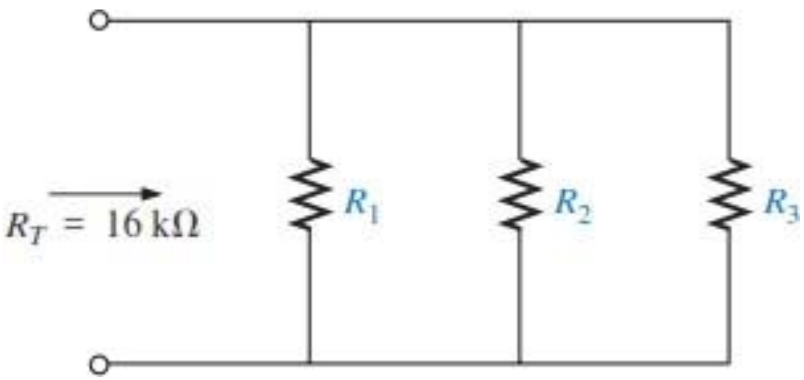
$$R''_T = \frac{R_2R_4}{R_2 + R_4} = \frac{(9\ \Omega)(72\ \Omega)}{9\ \Omega + 72\ \Omega} = \frac{648\ \Omega}{81} = 8\ \Omega$$

$$R_T = R'_T \parallel R''_T$$

In parallel with

$$= \frac{R'_TR''_T}{R'_T + R''_T} = \frac{(2\ \Omega)(8\ \Omega)}{2\ \Omega + 8\ \Omega} = \frac{16\ \Omega}{10} = \mathbf{1.6\ \Omega}$$

EXAMPLE 3.4 Determine the values of *R*1, *R*2, and *R3* in Fig. shown if $R_2 = 2R_1$ and $R_3 = 2R_2$ and the total resistance is 16 kΩ.



$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{16\ \text{k}\Omega} = \frac{1}{R_1} + \frac{1}{2R_1} + \frac{1}{4R_1}$$

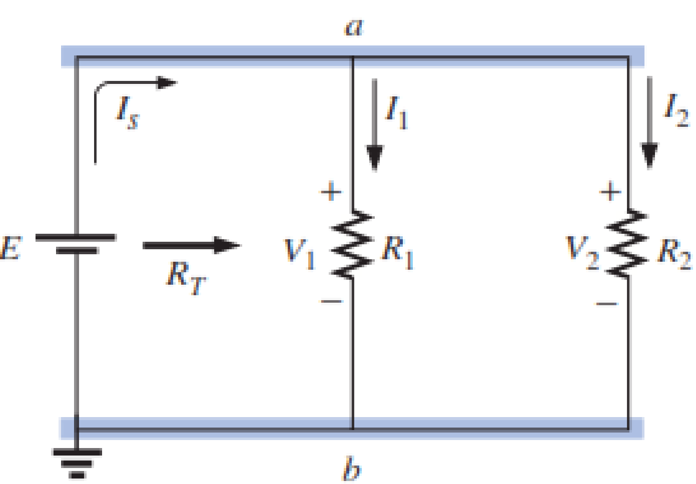
since $\quad R_3 = 2R_2 = 2(2R_1) = 4R_1$

and $\quad \dfrac{1}{16\ \text{k}\Omega} = \dfrac{1}{R_1} + \dfrac{1}{2}\left(\dfrac{1}{R_1}\right) + \dfrac{1}{4}\left(\dfrac{1}{R_1}\right)$

$$\frac{1}{16\ \text{k}\Omega} = 1.75\left(\frac{1}{R_1}\right)$$

with $\quad R_1 = 1.75(16\ \text{k}\Omega) = \mathbf{28\ k\Omega}$

PARALLEL ELEMENTS



$$V_1 = V_2 = E$$

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1}$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2}$$

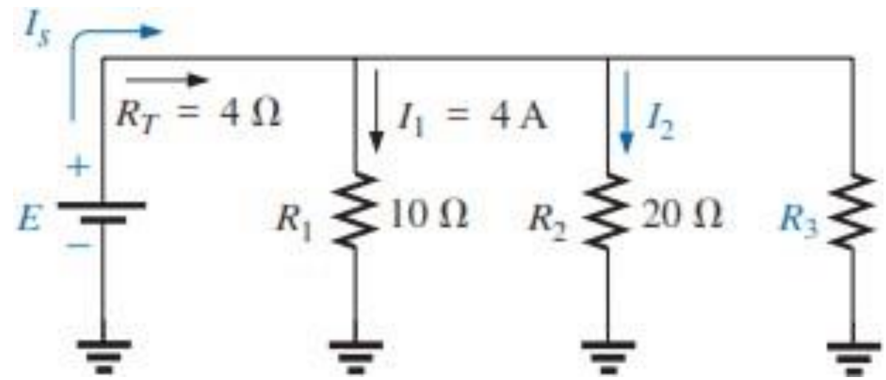
$$E\left(\frac{1}{R_T}\right) = E\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{E}{R_T} = \frac{E}{R_1} + \frac{E}{R_2}$$

$$I_s = I_1 + I_2$$

EXAMPLE 3.5 Given the information provided in Fig. shown:

a. Determine R_3.
b. Calculate E.
c. Find I_s.
d. Find I_2.
e. Determine P_2.



a.
$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$\frac{1}{4\ \Omega} = \frac{1}{10\ \Omega} + \frac{1}{20\ \Omega} + \frac{1}{R_3}$$
$$0.25\ \text{S} = 0.1\ \text{S} + 0.05\ \text{S} + \frac{1}{R_3}$$
$$0.25\ \text{S} = 0.15\ \text{S} + \frac{1}{R_3}$$
$$\frac{1}{R_3} = 0.1\ \text{S}$$
$$R_3 = \frac{1}{0.1\ \text{S}} = \mathbf{10\ \Omega}$$

b. $E = V_1 = I_1R_1 = (4\ \text{A})(10\ \Omega) = \mathbf{40\ V}$

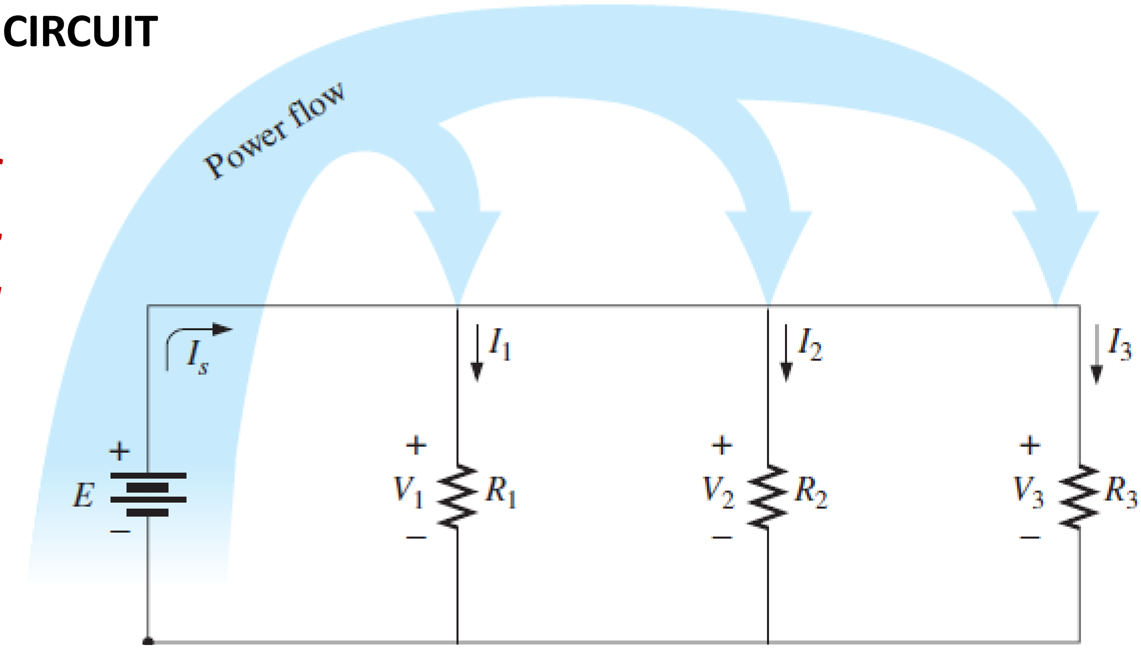
c. $I_s = \dfrac{E}{R_T} = \dfrac{40\ \text{V}}{4\ \Omega} = \mathbf{10\ A}$

d. $I_2 = \dfrac{V_2}{R_2} = \dfrac{E}{R_2} = \dfrac{40\ \text{V}}{20\ \Omega} = \mathbf{2\ A}$

e. $P_2 = I_2^2R_2 = (2\ \text{A})^2(20\ \Omega) = \mathbf{80\ W}$

POWER DISTRIBUTION IN A PARALLEL CIRCUIT

for any network composed of resistive elements, the power applied by the battery will equal that dissipated by the resistive elements.



$$\boxed{P_E = P_{R_1} + P_{R_2} + P_{R_3}}$$

The power delivered by the source

$$\boxed{P_E = EI_s} \qquad \text{(watts, W)}$$

as is the equation for the power to each resistor (shown for R_1 only):

$$\boxed{P_1 = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1}} \qquad \text{(watts, W)}$$

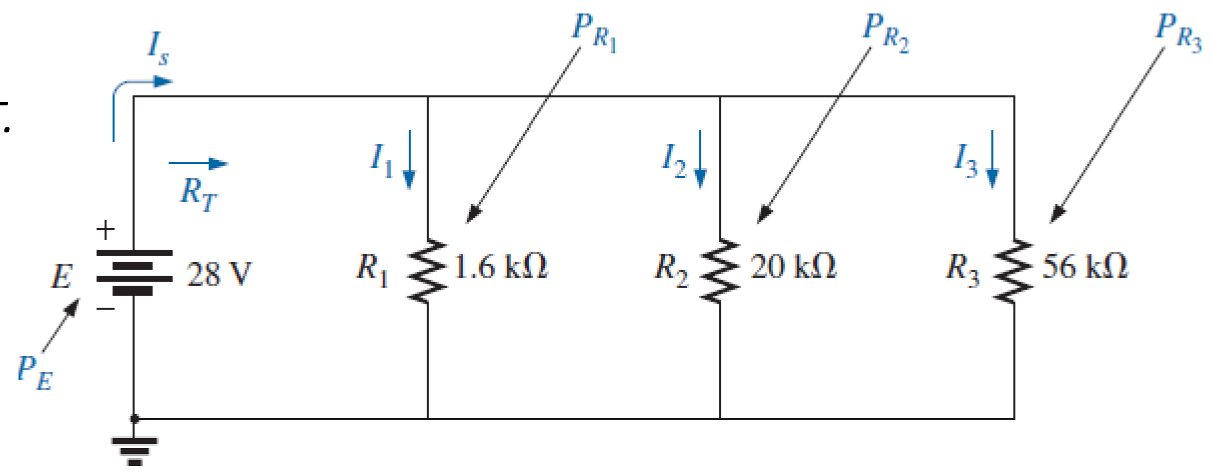
EXAMPLE 3.7 For the parallel network shown:

a. Determine the total resistance *RT.*

b. Find the source current and the current through each resistor.

c. Calculate the power delivered by the source.

d. Determine the power absorbed by each parallel resistor.



a. **It should now be apparent from previous examples that the total resistance is less than 1.6 kΩ.**

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{1}{\frac{1}{1.6\ \text{k}\Omega} + \frac{1}{20\ \text{k}\Omega} + \frac{1}{56\ \text{k}\Omega}}$$

$$= \frac{1}{625 \times 10^{-6} + 50 \times 10^{-6} + 17.867 \times 10^{-6}} = \frac{1}{692.867 \times 10^{-6}}$$

and $R_T = \mathbf{1.44\ k\Omega}$

b. Applying Ohm's law:

$$I_s = \frac{E}{R_T} = \frac{28\text{ V}}{1.44\text{ k}\Omega} = \mathbf{19.44\text{ mA}}$$

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{28\text{ V}}{1.6\text{ k}\Omega} = \mathbf{17.5\text{ mA}}$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{28\text{ V}}{20\text{ k}\Omega} = \mathbf{1.4\text{ mA}}$$

$$I_3 = \frac{V_3}{R_3} = \frac{E}{R_3} = \frac{28\text{ V}}{56\text{ k}\Omega} = \mathbf{0.5\text{ mA}}$$

c.

$$P_E = EI_s = (28\text{ V})(19.4\text{ mA}) = \mathbf{543.2\text{ mW}}$$

d. Applying each form of the power equation:

$$P_1 = V_1I_1 = EI_1 = (28\text{ V})(17.5\text{ mA}) = \mathbf{490\text{ mW}}$$

A review of the results clearly shows the fact that the larger the resistor, the less the power absorbed.

KIRCHHOFF'S CURRENT LAW

Kirchhoff's voltage law provides an important relationship among voltage levels around any closed loop of a network.

We now consider **Kirchhoff's current law (KCL),** which provides an equally important relationship among current levels at any junction.

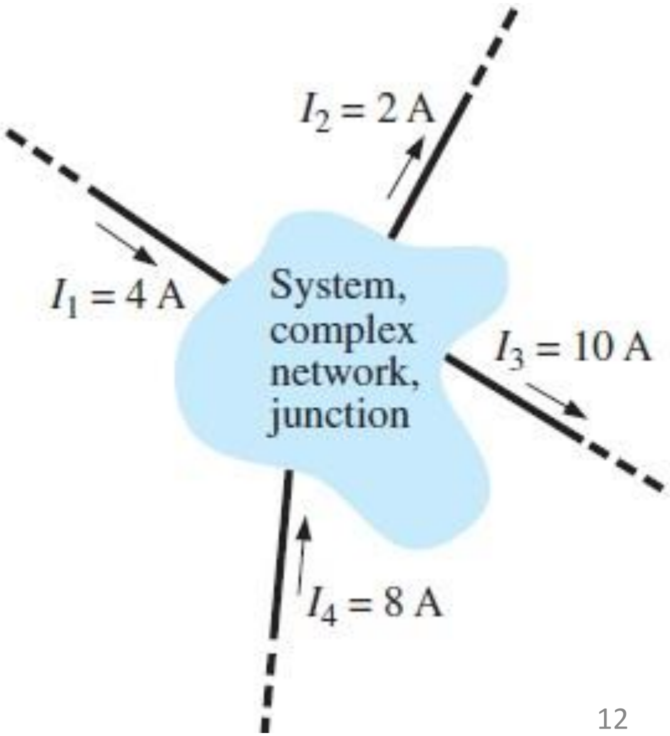
Kirchhoff's current law (KCL) ***states that the algebraic sum of the currents entering and leaving an area, system, or junction is zero.***

In other words,

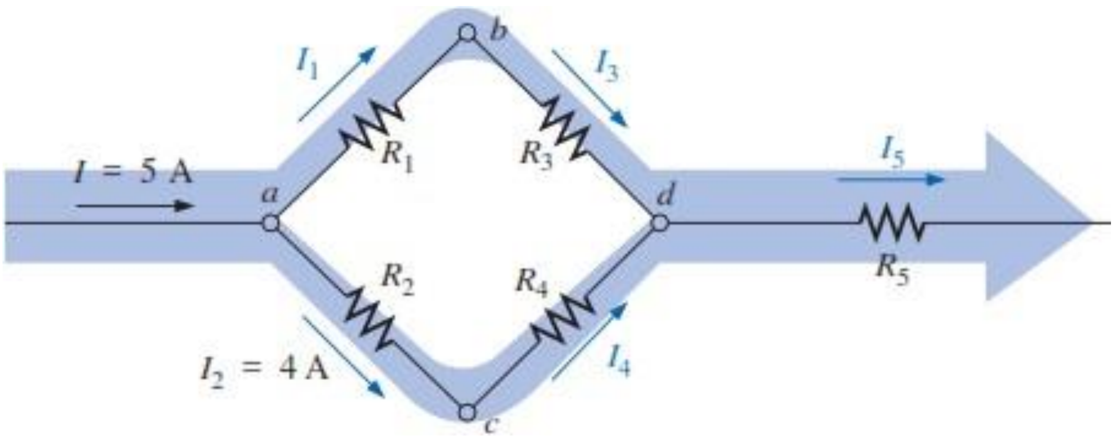
the sum of the currents entering an area, system, or junction must equal the sum of the currents leaving the area, system, or junction.

$$\boxed{\Sigma I_i = \Sigma I_o}$$

$$\begin{aligned} \Sigma I_i &= \Sigma I_o \\ I_1 + I_4 &= I_2 + I_3 \\ 4\text{ A} + 8\text{ A} &= 2\text{ A} + 10\text{ A} \\ \mathbf{12\text{ A}} &= \mathbf{12\text{ A}} \quad \text{(checks)} \end{aligned}$$



EXAMPLE 3.6 Determine I_1, I_3, I_4, and I_5 for the network of Fig. shown.



At node *a:*

$$\Sigma I_i = \Sigma I_o$$
$$I = I_1 + I_2$$
$$5\text{ A} = I_1 + 4\text{ A}$$
$$I_1 = 5\text{ A} - 4\text{ A} = \mathbf{1\ A}$$

At node *b:*

$$\Sigma I_i = \Sigma I_o$$
$$I_1 = I_3$$
$$I_3 = I_1 = \mathbf{1\ A}$$

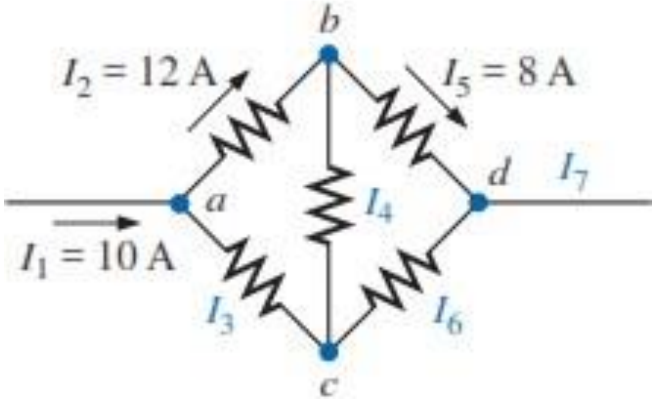
At node *c:*

$$\Sigma I_i = \Sigma I_o$$
$$I_2 = I_4$$
$$I_4 = I_2 = \mathbf{4\ A}$$

At node *d:*

$$\Sigma I_i = \Sigma I_o$$
$$I_3 + I_4 = I_5$$
$$1\text{ A} + 4\text{ A} = I_5 = \mathbf{5\ A}$$

EXAMPLE 3.7 Find the magnitude and direction of the currents I_3, I_4, I_6, and I_7 for the network of Fig. shown. Even though the elements are not in series or parallel, Kirchhoff's current law can be applied to determine all the unknown currents.



Considering the overall system, we know that the current entering must equal that leaving. Therefore,

$$I_7 = I_1 = \mathbf{10\ A}$$

Applying Kirchhoff's current law at node *a,*

$$I_1 + I_3 = I_2$$

$$10\text{ A} + I_3 = 12\text{ A}$$

$$I_3 = 12\text{ A} - 10\text{ A} = \mathbf{2\ A}$$

At node *b, since 12 A are entering and 8 A are leaving,* I_4 must be leaving. Therefore,

$$I_2 = I_4 + I_5$$

$$12\text{ A} = I_4 + 8\text{ A}$$

$$I_4 = 12\text{ A} - 8\text{ A} = \mathbf{4\ A}$$

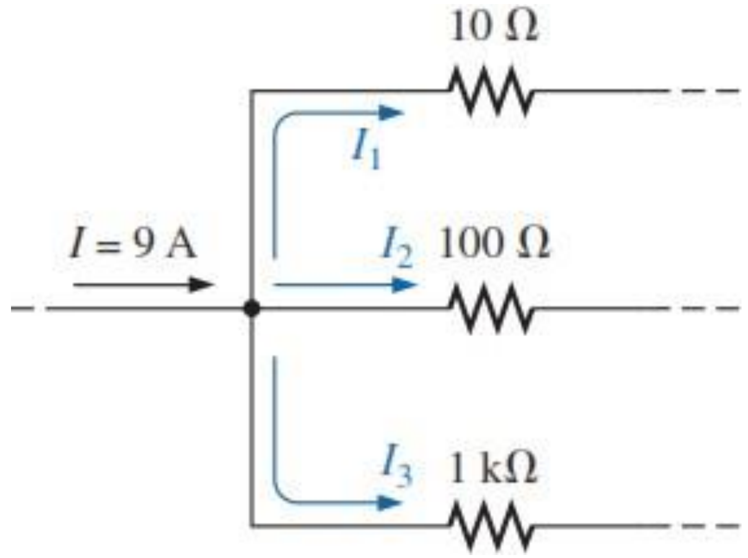
At node *c,* I 3 is leaving at 2 A and I4 is entering at 4 A, requiring that I 6be leaving. Applying Kirchhoff's current law at node *c,*

$$I_4 = I_3 + I_6$$

$$4\text{ A} = 2\text{ A} + I_6$$

$$I_6 = 4\text{ A} - 2\text{ A} = \mathbf{2\ A}$$

CURRENT DIVIDER RULE



-The majority of the current will pass through the smallest resistor of 10 Ω , and the least current will pass through the 1 kΩ resistor.

-In fact, the current through the 100 Ω resistor will also exceed that through the 1 k Ω resistor.

-By recognizing that the resistance of the 100 Ω is 10 times that of the 10 Ω resistor. The result is a current through the 10 Ω resistor that is 10 times that of the 100 Ω resistor.

-Similarly, the current through the 100 Ω resistor is 10 times that through the 1 kΩ resistor.

For two parallel elements of equal value, the current will divide equally.

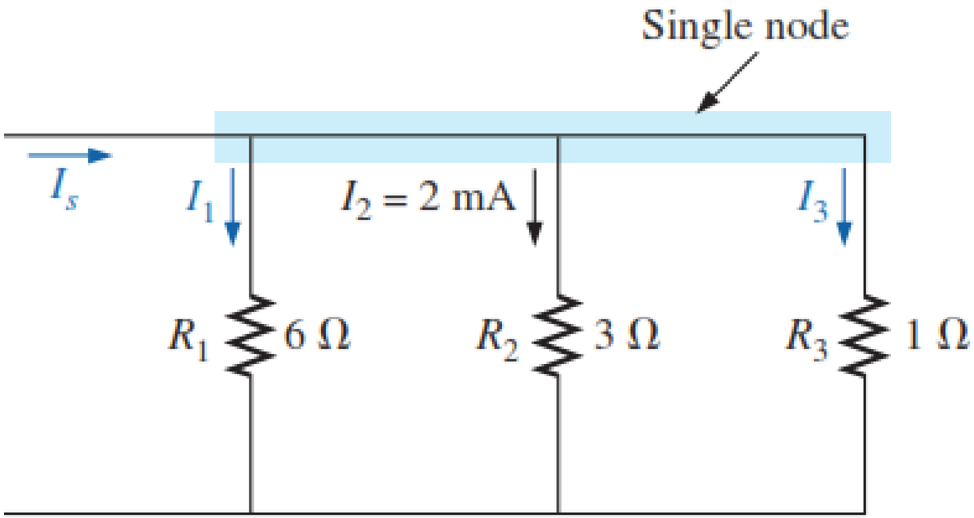
For parallel elements with different values, the smaller the resistance, the greater the share of input current.

For parallel elements of different values, the current will split with a ratio equal to the inverse of their resistor values.

EXAMPLE 3.8

a. Determine currents $I_1$and I_3 for the network in Fig.

b. Find the source current Is



a. Since R_1 is twice R_2, the current I_1 must be one-half I_2, and

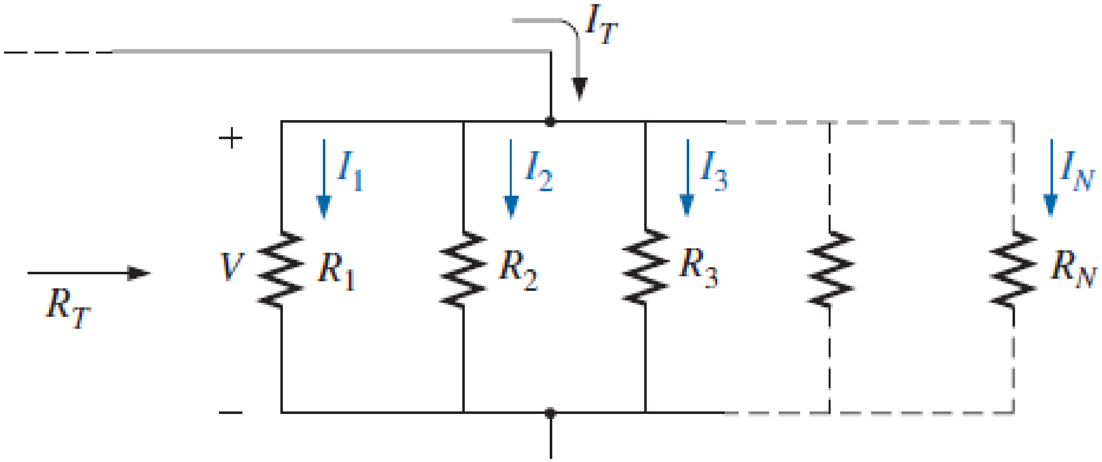
$$I_1 = \frac{I_2}{2} = \frac{2\text{ mA}}{2} = \mathbf{1\ mA}$$

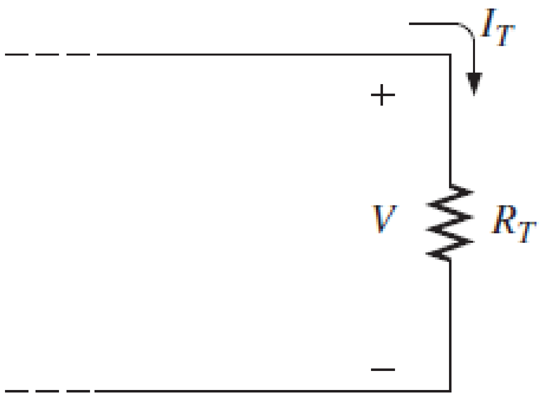
Since R_2 is three times R_3, the current I_3 must be three times I_2, and

$$I_3 = 3I_2 = 3(2\text{ mA}) = \mathbf{6\ mA}$$

b. Applying Kirchhoff's current law:

$$\Sigma I_i = \Sigma I_o$$
$$I_s = I_1 + I_2 + I_3$$
$$I_s = 1\text{ mA} + 2\text{ mA} + 6\text{ mA} = \mathbf{9\ mA}$$





The current through any branch of a parallel resistive network is equal to the total resistance of the parallel network divided by the resistor of interest and multiplied by the total current entering the parallel configuration.

The current I_T *can then be determined using Ohm's law:*

$$I_T = \frac{V}{R_T}$$

Since the voltage *V is the same across parallel elements, the following* is true:

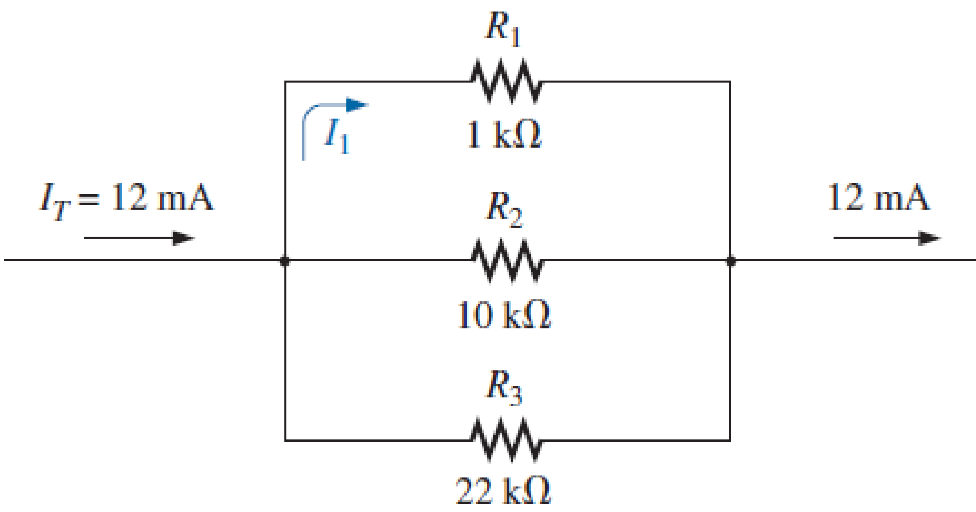
$$V = I_1R_1 = I_2R_2 = I_3R_3 = \cdots = I_xR_x$$

$$I_T = \frac{I_xR_x}{R_T}$$

Solving for *Ix , the final result is the* ***current divider rule:***

$$\boxed{I_x = \frac{R_T}{R_x}I_T}$$

EXAMPLE 3.9 For the parallel network shown, determine current I_1.



$$R_T = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$$

$$= \frac{1}{\frac{1}{1\text{ k}\Omega}+\frac{1}{10\text{ k}\Omega}+\frac{1}{22\text{ k}\Omega}}$$

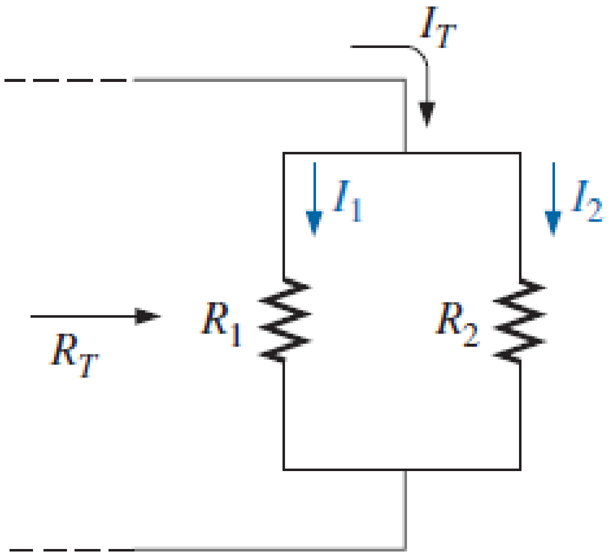
$$= \frac{1}{1\times 10^{-3}+100\times 10^{-6}+45.46\times 10^{-6}}$$

$$= \frac{1}{1.145\times 10^{-3}} = \mathbf{873.01\ \Omega}$$

$$I_1 = \frac{R_T}{R_1}I_T$$

$$= \frac{(873.01\ \Omega)}{1\text{ k}\Omega}(12\text{ mA}) = (0.873)(12\text{ mA}) = \mathbf{10.48\ mA}$$

Special Case: Two Parallel Resistors



$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

$$I_1 = \frac{R_T}{R_1} I_T = \frac{\left(\dfrac{R_1 R_2}{R_1 + R_2}\right)}{R_1} I_T$$

$$\boxed{I_1 = \left(\frac{R_2}{R_1 + R_2}\right) I_T} \qquad \boxed{I_2 = \left(\frac{R_1}{R_1 + R_2}\right) I_T}$$

For two parallel resistors, the current through one is equal to the other resistor times the total entering current divided by the sum of the two resistors.

GOOD LUCK