CHPTER TWO

Dimensional Analysis

2.1 Introduction

Any phenomenon is physical sciences and engineering can be described by the *fundamentals dimensions* mass, length, time, and temperature. Till the rapid development of science and technology the engineers and scientists depend upon the experimental data. But the rapid development of science and technology has created new mathematical methods of solving complicated problems, which could not have been solved completely by analytical methods and would have consumed enormous time. This mathematical method of obtaining the equations governing certain natural phenomenon by balancing the fundamental dimensions is called *(Dimensional Analysis)*. Of course, the equation obtained by this method is known as *(Empirical Equation).*

2.2 Fundamentals Dimensions

The various physical quantities used by engineer and scientists can be expressed in terms of fundamentals dimensions are: Mass (M), Length (L), Time (T), and Temperature (θ). All other quantities such as area, volume, acceleration, force, energy, etc., are termed as " derived quantities".

2.3 Dimensional Homogeneity

An equation is called *"dimensionally homogeneous"* if the fundamentals dimensions have identical powers of [L T M] (i.e. length, time, and mass) on both sides. Such an equation be independent of the system of measurement (i.e. metric, English, or S.I.). Let consider the common equation of volumetric flow rate,

$$Q = A\ u$$

$$L^3T^{-1} = L^2\ LT^{-1} = L^3T^{-1}.$$

We see, from the above equation that both right and left hand sides of the equation have the same dimensions, and the equation is therefore dimensionally homogeneous.

Example -2.1-

a) Determine the dimensions of the following quantities in M-L-T system 1- force 2- pressure 3- work 4- power 5- surface tension 6- discharge 7- torque 8- momentum.

b) Check the dimensional homogeneity of the following equations

1- $u = \sqrt{\dfrac{2\,g(\rho_m - \rho)\Delta z}{\rho}}$ 2- $Q = \dfrac{8}{15} cd \tan\dfrac{\theta}{2}\sqrt{2\,g}Z_\circ^{\frac{5}{2}}$

Solution:

a)

1- $F = m.g$ (kg.m/s^2) $\equiv [MLT^{-2}]$

2- $P=F/A \equiv [(MLT^{-2})(L^{-2})]$ (Pa) $\equiv [ML^{-1}T^{-2}]$

3- Work = F.L $\equiv [(MLT^{-2})(L)]$ (N.m) $\equiv [ML^2T^{-2}]$

4- Power = Work/time $\equiv [(ML^2T^{-2})(T^{-1})]$ (W) $\equiv [ML^{-1}T^{-2}]$

5- Surface tension = F/L $\equiv [(MLT^{-2})(L^{-1})]$ (N/m) $\equiv [ML^{-1}T^{-2}]$

6- Discharge (Q) m^3/s $\equiv [L^3T^{-1}]$
7- Torque (Γ) = F.L $\equiv$ [(MLT^{-2}) (L)] N.m $\equiv [ML^2T^{-2}]$
8- Moment = m.u L)] N.m $\equiv [ML^2T^{-2}]$

b) 1- $u = \sqrt{\dfrac{2\,g(\rho_m - \rho)\Delta z}{\rho}}$

L.H.S. u $\equiv [LT^{-1}]$

R.H.S. u $\equiv \left[\dfrac{LT^{-2}(ML^3)}{ML^{-3}}\right]^{1/2} \equiv [LT^{-1}]$

Since the dimensions on both sides of the equation are same, therefore the equation is dimensionally homogenous.

2- $Q = \dfrac{8}{15} cd \tan\dfrac{\theta}{2}\sqrt{2\,g}Z_\circ^{\frac{5}{2}}$

L.H.S. u $\equiv [L^3T^{-1}]$

R.H.S. (LT^{-2}) $(L)^{5/2} \equiv [L^3T^{-1}]$

This equation is dimensionally homogenous.

2.4 Methods of Dimensional Analysis

Dimensional analysis, which enables the variables in a problem to be grouped into form of *dimensionless groups*. Thus reducing the effective number of variables. The method of dimensional analysis by providing a smaller number of independent groups is most helpful to experimenter.

Many methods of dimensional analysis are available; two of these methods are given here, which are:

1- Rayleigh's method (or Power series)

2- Buckingham's method (or Π-Theorem)

2.4.1 Rayleigh's method (or Power series)

In this method, the functional relationship of some variable is expressed in the form of an exponential equation, which must be dimensionally homogeneous. If (y) is some function of independent variables (x_1, x_2, x_3, ………etc.), then functional relationship may be written as;

y = f(x_1, x_2, x_3, ………etc.)

The dependent variable (y) is one about which information is required; whereas the independent variables are those, which govern the variation of dependent variables.

The Rayleigh's method is based on the following steps:-

1- First of all, write the functional relationship with the given data.
2- Now write the equation in terms of a constant with exponents i.e. powers a, b, c,...
3- With the help of the principle of dimensional homogeneity, find out the values of a, b, c, … by obtaining simultaneous equation and simplify it.
4- Now substitute the values of these exponents in the main equation, and simplify it.

Example -2.2-

If the capillary rise (h) depends upon the specific weight (sp.wt) surface tension (σ) of the liquid and tube radius (r) show that:

$$h = r\phi\left(\frac{\sigma}{(sp.wt.)\,r^2}\right), \text{ where } \phi \text{ is any function.}$$

Solution:

Capillary rise (h) m ≡ [L]
Specific weight (sp.wt) N/m^3 (MLT^{-2} L^{-3}) ≡ [ML^{-2}T^{-2}]
Surface tension (σ) N/m (MLT^{-2} L^{-1}) ≡ [MT^{-2}]
Tube radius (r) m ≡ [L]

$$h = f\,(sp.wt., \sigma, r)$$

$$h = k\,(sp.wt.^a, \sigma^b, r^c)$$

$$[L] = [ML^{-2}T^{-2}]^a\,[MT^{-2}]^b[L]^c$$

Now by the principle of dimensional homogeneity, equating the power of M, L, T on both sides of the equation

For M $0 = a + b \quad \Rightarrow \quad a = -b$

For L $1 = -2a + c \quad \Rightarrow \quad a = -b$

For T $0 = -2a - 2b \quad \Rightarrow \quad a = -b$

$$h = k\,(sp.wt.^{-b}, \sigma^b, r^{1-2b})$$

$$h = k\,r\left(\frac{\sigma}{sp.wt.\,r^2}\right)^b \qquad \therefore \quad h = r\phi\left(\frac{\sigma}{(sp.wt.)\,r^2}\right)$$

Example -2.3-

Prove that the resistance (F) of a sphere of diameter (d) moving at a constant speed (u) through a fluid density (ρ) and dynamic viscosity (μ) may be expressed as:

$$F = \frac{\mu^2}{\rho}\phi\left(\frac{\rho\,u\,d}{\mu}\right), \text{ where } \phi \text{ is any function.}$$

Solution:

Resistance (F) N ≡ [MLT^{-2}]
Diameter (d) m ≡ [L]
Speed (u) m/s ≡ [LT^{-1}]
Density (ρ) kg/m^3 ≡ [ML^{-3}]
Viscosity (μ) kg/m.s ≡ [ML^{-1} T^{-1}]

$$F = f\,(d, u, \rho, \mu)$$

$$F = k\,(d^a, u^b, \rho^c, \mu^d)$$

$$[MLT^{-2}] = [L]^a\,[LT^{-1}]^b[ML^{-3}]^c[ML^{-1}T^{-1}]^d$$

For M $1 = c + d \quad \Rightarrow \quad c = 1 - b$ ----------------(1)

For L $1 = a + b - 3c - d$ -----------------------(2)

For T $-2 = -b - d \quad \Rightarrow \quad b = 2 - b$ ----------------(3)

By substituting equations (1) and (2) in equation (3) give

$a = 1 – b + 3c + d = 1 – (2 – d) + 3 (1 – d) + d = 2 – d$

$F = k (d^{2-d}, u^{2-d}, \rho^{1-d}, \mu^{d}) = k \{(d^2 u^2 \rho) (\mu / \rho u d)^d\}$ ---------x $\{(\rho/ \mu^2) / (\rho/ \mu^2)\}$

$F = k \{(d^2 u^2 \rho^2 / \mu^2) (\mu / \rho u d)^d (\mu^2 / \rho)\}$

$$\therefore \quad F = \frac{\mu^2}{\rho}\phi\left(\frac{\rho\, u\, d}{\mu}\right)$$

Example -2.4-

The thrust (P) (قوة الدفع) of a propeller depends upon diameter (D); speed (u) through a fluid density (ρ); revolution per minute (N); and dynamic viscosity (μ) Show that:

$$P = (\rho\, D^2\, u^2)\ f\left((\frac{\mu}{\rho\, D\, u}), (\frac{D\, N}{u})\right)$$, where *f* is any function.

Solution:

Thrust (P) N	$\equiv [MLT^{-2}]$
Diameter (D) m	$\equiv [L]$
Speed (u) m/s	$\equiv [LT^{-1}]$
Density (ρ) kg/m^3	$\equiv [ML^{-3}]$
Rev. per min. (N) min^{-1}	$\equiv [T^{-1}]$
Viscosity (μ) kg/m.s	$\equiv [ML^{-1} T^{-1}]$

$P = f (D, u, \rho, N, \mu)$

$P = k (D^a, u^b, \rho^c, N^d, \mu^e)$

$[MLT^{-2}] = [L]^a [LT^{-1}]^b[ML^{-3}]^c[T^{-1}][ML^{-1}T^{-1}]^d$

For M $\quad 1 = c + e \quad \Rightarrow \quad c =1 – e$ --------------(1)

For L $\quad 1 = a + b – 3c – e \quad \Rightarrow \quad a = 1– b + 3c + e$ --------------(2)

For T $\quad – 2 = – b – d– e \quad \Rightarrow \quad b = 2 – e – d$ --------------(3)

By substituting equations (1) and (3) in equation (2) give

$a = 1 – (2 – e – d) + 3 (1 – e) + e = 2 – e + d$

$P = k (D^{2-e+d}, u^{2-e+d}, \rho^{1-e}, N^d, \mu^e)$

$$P = (\rho\, D^2\, u^2)\, k\left[\left(\frac{\mu}{\rho\, D\, u}\right)^e, \left(\frac{D\, N}{u}\right)\right]$$

$$\therefore\ P = (\rho\, D^2\, u^2)\ f\left((\frac{\mu}{\rho\, D\, u}), (\frac{D\, N}{u})\right)$$

Home Work

P.2.1

Show, by dimensional analysis, that the power (P) developed by a hydraulic turbine is given by; $P = (\rho \ N^3 \ D^5) \ f\left((\frac{N^2 \ D^2}{g \ H}) \right)$ where (ρ) is the fluid density, (N) is speed of rotation in r.p.m., (D) is the diameter of runner, (H) is the working head, and (g) is the gravitational acceleration.

P.2.2

The resistance (R) experienced by a partially submerged body depends upon the velocity (u), length of the body (L), dynamic viscosity (μ) and density (ρ) of the fluid, and gravitational acceleration (g). Obtain a dimensionless expression for (R).

Ans. $R = (u^2 L^2 \rho) \ f\left((\frac{\mu}{u \ L \ g}), (\frac{L \ g}{u^2}) \right)$

P.2.3

Using Rayleigh's method to determine the rational formula for discharge (Q) through a sharp-edged orifice freely into the atmosphere in terms of head (h), diameter (d), density (ρ), dynamic viscosity (μ), and gravitational acceleration (g).

Ans. $Q = (d\sqrt{g \ h}) f\left[(\frac{\mu}{\rho \ d^{\frac{3}{2}} \ g^{\frac{1}{2}}}), (\frac{h}{d}) \right]$

2.4.2 Buckingham's method (or Π-Theorem)

It has been observed that the Rayleigh's method of dimensional analysis becomes cumbersome, when a large number of variables are involved. In order to overcome this difficulty, the Buckingham's method may be convenient used. It states that " If there are **(n)** variables in a dimensionally homogeneous equation, and if these variables contain **(m)** fundamental dimensions such as (MLT) they may be grouped into **(n-m)** non-dimensional independent Π-terms".

Mathematically, if a dependent variable X_1 depends upon independent variables (X_2, X_3, X_4, ………. X_n), the functional equation may be written as:

$X_1 = k (X_2, X_3, X_4, \ldots\ldots\ldots X_n)$

This equation may be written in its general form as;

$f (X_1, X_2, X_3, \ldots\ldots\ldots X_n) = 0$

In this equation, there are n variables. If there are m fundamental dimensions, then according to Buckingham's Π-theorem;

$f_1 (\Pi_1, \Pi_2, \Pi_3, \ldots\ldots\ldots \Pi_{n-m}) = 0$

The Buckingham's Π-theorem is based on the following steps:

1. First of all, write the functional relationship with the given data.
2. Then write the equation in its general form.
3. Now choose **m** *repeating variables (or recurring set)* and write separate expressions for each Π-term. Every Π-term will contain the repeating variables and one of the remaining variables. **Just** the repeating variables are written in exponential form.
4. With help of the principle of dimensional homogeneity find out the values of powers a, b, c, …… by obtaining simultaneous equations.
5. Now substitute the values of these exponents in the Π-terms.
6. After the Π-terms are determined, write the functional relation in the required form.

Note:-

Any Π-term may be replaced by any power of it, because the power of a non-dimensional term is also non-dimensional e.g. Π_1 may be replaced by Π_1^2, Π_1^3, $\Pi_1^{0.5}$,……. or by $2\Pi_1$, $3\Pi_1$, $\Pi_1/2$, ……etc.

2.4.2.1 Selection of repeating variables

In the previous section, we have mentioned that we should choose **(m)** repeating variables and write separate expressions for each Π-term. Though there is no hard or fast rule for the selection of repeating variables, yet the following points should be borne in mind while selecting the repeating variables:

1. The variables should be such that none of them is dimension les.
2. No two variables should have the same dimensions.
3. Independent variables should, as far as possible, be selected as repeating variables.
4. Each of the fundamental dimensions must appear in at least one of the **m** variables.
5. It must not possible to form a dimensionless group from some or all the variables within the repeating variables. If it were so possible, this dimensionless group would, of course, be one of the Π-term.
6. In general the selected repeating variables should be expressed as the following: **(1)** representing the flow characteristics, **(2)**, representing the geometry and **(3)** representing the physical properties of fluid.
7. In case of that the example is held up, then one of the repeating variables should be changed.

Example -2.5-

By dimensional analysis, obtain an expression for the drag force (F) on a partially submerged body moving with a relative velocity (u) in a fluid; the other variables being the linear dimension (L), surface roughness (e), fluid density (ρ), and gravitational acceleration (g).

Solution:

Drag force (F) N	$\equiv [MLT^{-2}]$
Relative velocity (u) m/s	$\equiv [LT^{-1}]$
Linear dimension (L) m	$\equiv [L]$
Surface roughness (e) m	$\equiv [L]$
Density (ρ) kg/m^3	$\equiv [ML^{-3}]$
Acceleration of gravity (g) m/s^2	$\equiv [ML^{-1}\,T^{-1}]$

$$F = k\,(u, L, e, \rho, g)$$

$$f\,(F, u, L, e, \rho, g) = 0$$

$$n = 6,\ m = 3,\ \Rightarrow\ \Pi = n - m = 6 - 3 = 3$$

No. of repeating variables = m = 3

The selected repeating variables is (u, L, ρ)

$$\Pi_1 = u^{a1}\,L^{b1}\,\rho^{c1}\,F \quad \text{--------------(1)}$$

$$\Pi_2 = u^{a2}\,L^{b2}\,\rho^{c2}\,e \quad \text{--------------(2)}$$

$$\Pi_3 = u^{a3}\,L^{b3}\,\rho^{c3}\,g \quad \text{--------------(3)}$$

For Π_1 equation (1)

$$[M^0\,L^0\,T^0] = [L\,T^{-1}]^{a1}\,[L]^{b1}[ML^{-3}]^{c1}[MLT^{-2}]$$

Now applied dimensional homogeneity

For M $0 = c1 + 1$ $\Rightarrow$ $c1 = -1$

For T $0 = -a1 - 2$ $\Rightarrow$ $a1 = -2$

For L $0 = a1 + b1 - 3c1 + 1$ $\Rightarrow$ $b1 = -2$

$\Pi_1 = u^{-2} L^{-2} \rho^{-1} F \quad \Rightarrow \quad \Pi_1 = \dfrac{F}{u^2 \, L^2 \, \rho}$

For Π_2 equation (2)

$[M^0 L^0 T^0] = [L\, T^{-1}]^{a2} [L]^{b2} [ML^{-3}]^{c2} [L]$

For M $0 = c2$ $\Rightarrow$ $c2 = 0$

For T $0 = -a2$ $\Rightarrow$ $a2 = 0$

For L $0 = a2 + b2 - 3c2 + 1$ $\Rightarrow$ $b2 = -1$

$\Pi_2 = L^{-1} e \quad \Rightarrow \quad \Pi_2 = \dfrac{e}{L}$

For Π_3 equation (3)

$[M^0 L^0 T^0] = [L\, T^{-1}]^{a3} [L]^{b3} [ML^{-3}]^{c3} [L\, T^{-2}]$

For M $0 = c3$ $\Rightarrow$ $c3 = 0$

For T $0 = -a3 - 2$ $\Rightarrow$ $a3 = -2$

For L $0 = a3 + b3 - 3c3 + 1$ $\Rightarrow$ $b3 = 1$

$\Pi_3 = u^{-2} L\, g \quad \Rightarrow \quad \Pi_3 = \dfrac{L\, g}{u^2}$

$f_1 (\Pi_1, \Pi_2, \Pi_3) = 0 \quad \Rightarrow \quad f_1\left(\dfrac{F}{u^2 \, L^2 \, \rho}, \dfrac{e}{L}, \dfrac{L\, g}{u^2}\right) = 0$

$\therefore \; F = u^2 L^2 \, \rho \; f\left(\dfrac{e}{L}, \dfrac{L\, g}{u^2}\right)$

Example -2.6-

Prove that the discharge (Q) over a spillway (قناة لتصريف فائض المياه من سد او نهر) is given by the relation $Q = u\, D^2 \; f\left(\dfrac{\sqrt{g\, D}}{u}, \dfrac{H}{D}\right)$ where (u) velocity of flow (D) depth at the throat, (H), head of water, and (g) gravitational acceleration.

Solution:

Discharge (Q) m^3/s $\equiv [L^3 T^{-1}]$

Velocity (u) m/s $\equiv [LT^{-1}]$

Depth (D) m $\equiv [L]$

Head of water (H) m $\equiv [L]$

Acceleration of gravity (g) m/s^2 $\equiv [ML^{-1}\, T^{-1}]$

$Q = k\,(u, D, H, g)$

$f\,(Q, u, D, H, g) = 0$

$n = 5,\ m = 2,\ \Rightarrow\ \Pi = n - m = 5 - 2 = 3$

No. of repeating variables = m = 2
The selected repeating variables is (u, D)

$\Pi_1 = u^{a1}\, D^{b1}\, Q$ --------------(1)

$\Pi_2 = u^{a2}\, D^{b2}\, H$ --------------(2)

$\Pi_3 = u^{a3}\, D^{b3}\, g$ --------------(3)

For Π_1 equation (1)

$[M^0\, L^0\, T^0] = [L\, T^{-1}]^{a1}\, [L]^{b1} [L^3 T^{-1}]$

For T $\quad 0 = -a1 - 1 \quad \Rightarrow \quad a1 = -1$

For L $\quad 0 = a1 + b1 + 3 \quad \Rightarrow \quad b1 = -2$

$\Pi_1 = u^{-1}\, D^{-2}\, Q \quad \Rightarrow \quad \Pi_1 = \dfrac{Q}{u\, D^2}$

For Π_2 equation (2)

$[M^0\, L^0\, T^0] = [L\, T^{-1}]^{a2}\, [L]^{b2} [L]$

For T $\quad 0 = -a2 \quad \Rightarrow \quad a2 = 0$

For L $\quad 0 = a2 + b2 + 1 \quad \Rightarrow \quad b2 = -1$

$\Pi_2 = D^{-1}\, H \quad \Rightarrow \quad \Pi_2 = \dfrac{D}{H}$

For Π_3 equation (3)

$[M^0\, L^0\, T^0] = [L\, T^{-1}]^{a3}\, [L]^{b3}\, [L\, T^{-2}]$

For T $\quad 0 = -a3 - 2 \quad \Rightarrow \quad a3 = -2$

For L $\quad 0 = a3 + b3 + 1 \quad \Rightarrow \quad b3 = 1$

$\Pi_3 = u^{-2}\, D\, g \quad \Rightarrow \quad \Pi_3 = \dfrac{D\, g}{u^2} = \dfrac{\sqrt{g\, D}}{u}$

$f_1\,(\Pi_1, \Pi_2, \Pi_3) = 0 \quad \Rightarrow \quad f_1\left(\dfrac{Q}{u\, D^2}, \dfrac{D}{H}, \dfrac{\sqrt{D\, g}}{u}\right)$

$\therefore\ Q = u\, D^2\ f\left(\dfrac{\sqrt{g\, D}}{u}, \dfrac{H}{D}\right)$

Example -2.5-

Show that the discharge of a centrifugal pump is given by $Q = N\ D^2\ f(\frac{g\ H}{N^2 D^2}, \frac{\mu}{ND^2\ \rho})$ where (N) is the speed of the pump in r.p.m., (D) the diameter of impeller, (g) gravitational acceleration, (H) manometric head, (μ), (ρ) are the dynamic viscosity and the density of the fluid.

Solution:

Discharge (Q) m^3/s ≡ $[L^3T^{-1}]$
Pump speed (N) r.p.m. ≡ $[T^{-1}]$
Diameter of impeller (D) m ≡ [L]
Acceleration of gravity (g) m/s^2 ≡ $[ML^{-1}\ T^{-1}]$
Head of manometer (H) m ≡ [L]
Viscosity (μ) kg/m.s ≡ $[ML^{-1}\ T^{-1}]$
Density (ρ) kg/m^3 ≡ $[ML^{-3}]$

$$Q = k\ (N, D, g, H, \mu, \rho)$$

$$f\ (Q, N, D, g, H, \mu, \rho) = 0$$

$$n = 7,\ m = 3,\ \Rightarrow\ \Pi = n - m = 7 - 3 = 4$$

No. of repeating variables = m = 3
The selected repeating variables is (N, D, ρ)

$$\Pi_1 = N^{a1}\ D^{b1}\ \rho^{c1}\ Q \qquad \text{--------------}(1)$$

$$\Pi_2 = N^{a2}\ D^{b2}\ \rho^{c2}\ g \qquad \text{--------------}(2)$$

$$\Pi_3 = N^{a3}\ D^{b3}\ \rho^{c3}\ H \qquad \text{--------------}(3)$$

$$\Pi_4 = N^{a4}\ D^{b4}\ \rho^{c4}\ \mu \qquad \text{--------------}(4)$$

For Π_1 equation (1)

$$[M^0\ L^0\ T^0] = [T^{-1}]^{a1}\ [L]^{b1}[ML^{-3}]^{c1}[L^3\ T^{-1}]$$

For M $\quad 0 = c1 \quad \Rightarrow \quad c1 = 0$

For T $\quad 0 = -\ a1 - 1 \quad \Rightarrow \quad a1 = -\ 1$

For L $\quad 0 = b1 - 3c1 + 3 \quad \Rightarrow \quad b1 = -\ 3$

$$\Pi_1 = N^{-1}\ D^{-3}\ Q \qquad \Rightarrow \quad \Pi_1 = \frac{Q}{N\ D^3}$$

For Π_2 equation (2)

$$[M^0\ L^0\ T^0] = [T^{-1}]^{a2}\ [L]^{b2}[ML^{-3}]^{c2}[LT^{-2}]$$

For M $\quad 0 = c2 \quad \Rightarrow \quad c2 = 0$

For T $\quad 0 = -\ a2 - 2 \quad \Rightarrow \quad a2 = -\ 2$

For L $\quad 0 = b2 - 3c2 + 1 \quad \Rightarrow \quad b2 = -1$

$\Pi_2 = N^{-2} D^{-1} g \quad \Rightarrow \quad \Pi_2 = \dfrac{g}{N^2 D}$

For Π_3 equation (3)

$$[M^0 L^0 T^0] = [T^{-1}]^{a3} [L]^{b3} [ML^{-3}]^{c3} [L]$$

For M $\quad 0 = c3 \quad \Rightarrow \quad c3 = 0$

For T $\quad 0 = -a3 \quad \Rightarrow \quad a3 = 0$

For L $\quad 0 = b3 - 3c3 + 1 \quad \Rightarrow \quad b3 = -1$

$\Pi_3 = D^{-1} H \quad \Rightarrow \quad \Pi_3 = \dfrac{H}{D}$

For Π_3 equation (4)

$$[M^0 L^0 T^0] = [T^{-1}]^{a4} [L]^{b4} [ML^{-3}]^{c4} [ML^{-1}T^{-1}]$$

For M $\quad 0 = c4 + 1 \quad \Rightarrow \quad c4 = -1$

For T $\quad 0 = -a4 - 1 \quad \Rightarrow \quad a4 = -1$

For L $\quad 0 = b4 - 3c4 - 1 \quad \Rightarrow \quad b4 = -2$

$\Pi_4 = N^{-1} D^{-2} \rho^{-1} \mu \quad \Rightarrow \quad \Pi 4 = \dfrac{\mu}{ND^2 \rho}$

$f_1 (\Pi_1, \Pi_2, \Pi_3, \Pi_4) = 0 \quad \Rightarrow \quad f_1\left(\dfrac{Q}{N D^3}, \dfrac{g}{N^2 D}, \dfrac{H}{D}, \dfrac{\mu}{ND^2 \rho}\right) = 0$

Since the product of two Π-terms is dimensionless, therefore replace the term Π_2 and Π_3 by $\dfrac{gH}{N^2 D^2}$

$\Rightarrow \quad f\left(\dfrac{Q}{ND^3}, \dfrac{gH}{N^2 D^2}, \dfrac{\mu}{ND^2 \rho}\right) \qquad \therefore \; Q = ND^3 f\left(\dfrac{gH}{N^2 D^2}, \dfrac{\mu}{ND^2 \rho}\right)$

Note:

The expression outside the bracket may be multiplied or divided by any amount, whereas the expression inside the bracket should not be multiplied or divided. e.g. $\pi/4$, $\sin\theta$, $\tan\theta/2$, ….c.

2.5 **Dimensions of some important variables**

Item	Property	Symbol	SI Units	M.L.T.
1-	Velocity	u	m/s	LT^{-1}
2-	Angular velocity	ω	Rad/s, Deg/s	T^{-1}
3-	Rotational velocity	N	Rev/s	T^{-1}
4-	Acceleration	a, g	m/s^2	LT^{-2}
5-	Angular acceleration	α	s^{-2}	T^{-2}
6-	Volumetric flow rate	Q	m^3/s	L^3T^{-1}
7-	Discharge	Q	m^3/s	L^3T^{-1}
8-	Mass flow rate	$\dot{m}$	kg/s	MT^{-1}
9-	Mass (flux) velocity	G	$kg/m^2.s$	$ML^{-2}T^{-1}$
10-	Density	ρ	kg/m^3	ML^{-3}
11-	Specific volume	υ	m^3/kg	L^3M
12-	Specific weight	sp.wt	N/m^3	$ML^{-2}T^{-2}$
13-	Specific gravity	sp.gr	[-]	[-]
14-	Dynamic viscosity	μ	kg/m.s, Pa.s	$ML^{-1}T^{-1}$
15-	Kinematic viscosity	ν	m^2/s	L^2T^{-1}
16-	Force	F	N	MLT^{-2}
17-	Pressure	P	N/m^2≡Pa	$ML^{-1}T^{-2}$
18-	Pressure gradient	**ΔP/L**	**Pa/m**	$ML^{-2}T^{-2}$
19-	Shear stress	τ	N/m^2	$ML^{-1}T^{-2}$
20-	Shear rate	$\dot{\gamma}$	s^{-1}	T^{-1}
21-	Momentum	M	kg.m/s	MLT^{-1}
22-	Work	W	N.m≡J	ML^2T^{-2}
23-	Moment	M	N.m≡J	ML^2T^{-2}
24-	Torque	Γ	N.m≡J	ML^2T^{-2}
25-	Energy	E	J	ML^2T^{-2}
26-	Power	P	J/s≡W	ML^2T^{-3}
27-	Surface tension	σ	N/m	MT^{-2}
28-	Efficiency	η	[-]	[-]
29-	Head	h	m	L
30-	Modulus of elasticity	ε, K	Pa	$ML^{-1}T^{-2}$

English Units
$g = 32.741\ ft/s^2$
$g_c = 32.741\ lb_m.ft/lb_f.s^2$

SI Units
$g = 9.81\ m/s^2$
$g_c = 1.0\ kg.m/N.s^2$

psi ≡ lb_f/in^2 Pa ≡ Pascal = N/m^2 bar = 10^5 Pa

1.0 atm = 1.01325 bar = $1.01325*10^5$ Pa = 101.325 kPa = 14.7 psi = 760 torr (mmHg) ≈ 1.0 kg/cm^2

R = 8.314 ($Pa.m^3$/mol.K) or (J/mol.K) = 82.06 ($atm.cm^3$/mol.K) =10.73 ($psi.ft^3$/lbmol.R) = 1.987 (cal/mol.K) = 1.986 (Btu/lbmol.R) = 1545 (lb_f.ft/lbmol.R)