equations are then solved for the output variable of interest in terms of the input variable from which the transfer function is evaluated. Example 2.17 demonstrates this problem-solving technique.

Example 2.17

Transfer Function—Two Degrees of Freedom

PROBLEM: Find the transfer function, $X_2(s)/F(s)$, for the system of Figure 2.17(a).

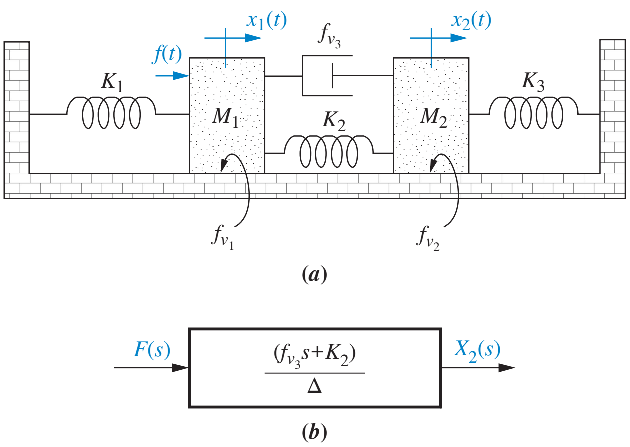
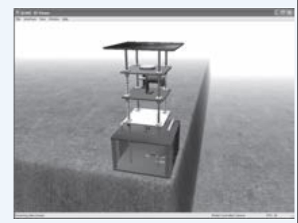


FIGURE 2.17 **a.** Two-degrees-of-freedom translational mechanical system;[8] **b.** block diagram

Virtual Experiment 2.1
Vehicle Suspension

Put theory into practice exploring the dynamics of another two-degrees-of-freedom system—a vehicle suspension system driving over a bumpy road and demonstrated with the Quanser Active Suspension System modeled in LabVIEW.



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Virtual experiments are found on Learning Space.

SOLUTION: The system has two degrees of freedom, since each mass can be moved in the horizontal direction while the other is held still. Thus, two simultaneous equations of motion will be required to describe the system. The two equations come from free-body diagrams of each mass. Superposition is used to draw the free-body diagrams. For example, the forces on M_1 are due to (1) its own motion and (2) the motion of M_2 transmitted to M_1 through the system. We will consider these two sources separately.

If we hold M_2 still and move M_1 to the right, we see the forces shown in Figure 2.18(a). If we hold M_1 still and move M_2 to the right, we see the forces shown in Figure 2.18(b). The total force on M_1 is the superposition, or sum, of the forces just discussed. This result is shown in Figure 2.18(c). For M_2, we proceed in a similar fashion: First we move M_2 to the right while holding M_1 still; then we move M_1 to the right and hold M_2 still. For each case we evaluate the forces on M_2. The results appear in Figure 2.19.

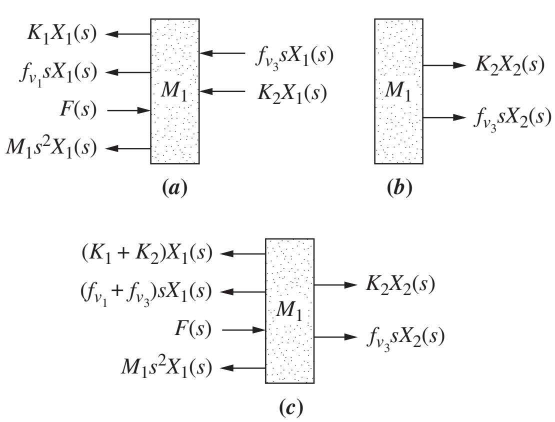


FIGURE 2.18 **a.** Forces on M_1 due only to motion of M_1; **b.** forces on M_1 due only to motion of M_2; **c.** all forces on M_1

[8] Friction shown here and throughout the book, unless otherwise indicated, is viscous friction. Thus, f_{v1} and f_{v2} are not Coulomb friction, but arise because of a viscous interface.

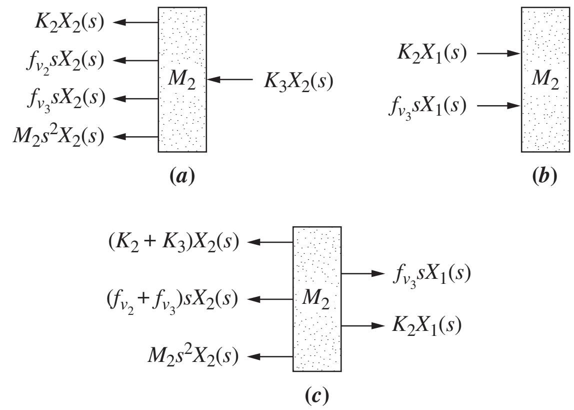


FIGURE 2.19 **a.** Forces on M_2 due only to motion of M_2; **b.** forces on M_2 due only to motion of M_1; **c.** all forces on M_2

The Laplace transform of the equations of motion can now be written from Figures 2.18(c) and 2.19(c) as

$$[M_1s^2(F_{v_1}+f_{v_3})s+(K_1+K_2)]X_1(s)-(f_{v_3}s+K_2)X_2(s)=F(s) \tag{2.118a}$$

$$-(f_{v_3}s+K_2)X_1(s)+[M_2s^2+(f_{v_2}+f_{v_3})s+(K_2+K_3)]X_2(s)=0 \tag{2.118b}$$

From this, the transfer function, $X_2(s)/F(s)$, is

$$\frac{X_2(s)}{F(s)}=G(s)=\frac{(f_{v_3}s+K_2)}{\Delta} \tag{2.119}$$

as shown in Figure 2.17(b) where

$$\Delta=\begin{vmatrix} [M_1s^2+(f_{v_1}+f_{v_3})s+(K_1+K_2)] & -(f_{v_3}s+K_2) \\ -(f_{v_3}s+K_2) & [M_2s^2+(f_{v_2}+f_{v_3})s+(K_2+K_3)] \end{vmatrix}$$

Notice again, in Eq. (2.118), that the form of the equations is similar to electrical mesh equations:

$$\begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{connected}\\ \text{to the motion}\\ \text{at } x_1\end{bmatrix} X_1(s) - \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{between}\\ x_1 \text{ and } x_2\end{bmatrix} X_2(s) = \begin{bmatrix}\text{Sum of}\\ \text{applied forces}\\ \text{at } x_1\end{bmatrix} \tag{2.120a}$$

$$-\begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{between}\\ x_1 \text{ and } x_2\end{bmatrix} X_1(s) + \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{connected}\\ \text{to the motion}\\ \text{at } x_2\end{bmatrix} X_2(s) = \begin{bmatrix}\text{Sum of}\\ \text{applied forces}\\ \text{at } x_2\end{bmatrix} \tag{2.120b}$$

The pattern shown in Eq. (2.120) should now be familiar to us. Let us use the concept to write the equations of motion of a three-degrees-of-freedom mechanical network by inspection, without drawing the free-body diagram.

Example 2.18

Equations of Motion by Inspection

PROBLEM: Write, but do not solve, the equations of motion for the mechanical network of Figure 2.20.

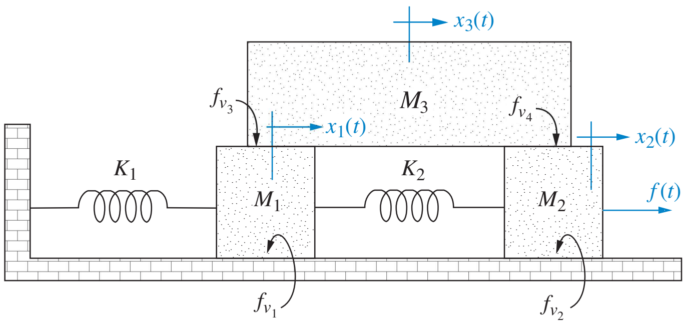


FIGURE 2.20 Three-degrees-of-freedom translational mechanical system

SOLUTION: The system has three degrees of freedom, since each of the three masses can be moved independently while the others are held still. The form of the equations will be similar to electrical mesh equations. For M_1,

$$\begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{connected}\\ \text{to the motion}\\ \text{at } x_1\end{bmatrix} X_1(s) - \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{between}\\ x_1 \text{ and } x_2\end{bmatrix} X_2(s)$$
$$- \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{between}\\ x_1 \text{ and } x_3\end{bmatrix} X_3(s) = \begin{bmatrix}\text{Sum of}\\ \text{applied forces}\\ \text{at } x_1\end{bmatrix} \tag{2.121}$$

Similarly, for M_2 and M_3, respectively,

$$- \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{between}\\ x_1 \text{ and } x_2\end{bmatrix} X_1(s) + \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{connected}\\ \text{to the motion}\\ \text{at } x_2\end{bmatrix} X_2(s)$$
$$- \begin{bmatrix}\text{Sum of}\\ \text{impedances}\\ \text{between}\\ x_2 \text{ and } x_3\end{bmatrix} X_3(s) = \begin{bmatrix}\text{Sum of}\\ \text{applied forces}\\ \text{at } x_2\end{bmatrix} \tag{2.122}$$

$$-\begin{bmatrix}\text{Sum of}\\\text{impedances}\\\text{between}\\x_1\text{ and }x_3\end{bmatrix}X_1(s)-\begin{bmatrix}\text{Sum of}\\\text{impedances}\\\text{between}\\x_2\text{ and }x_3\end{bmatrix}X_2(s)$$
$$+\begin{bmatrix}\text{Sum of}\\\text{impedances}\\\text{connected}\\\text{to the motion}\\\text{at }x_3\end{bmatrix}X_3(s)=\begin{bmatrix}\text{Sum of}\\\text{applied forces}\\\text{at }x_3\end{bmatrix} \tag{2.123}$$

M_1 has two springs, two viscous dampers, and mass associated with its motion. There is one spring between M_1 and M_2 and one viscous damper between M_1 and M_3. Thus, using Eq. (2.121),

$$[M_1s^2+(f_{v_1}+f_{v_3})s+(K_1+K_2)]X_1(s)-K_2X_2(s)-f_{v_3}sX_3(s)=0 \tag{2.124}$$

Similarly, using Eq. (2.122) for M_2,

$$-K_2X_1(s)+[M_2s^2+(f_{v_2}+f_{v_4})s+K_2]X_2(s)-f_{v_4}sX_3(s)=F(s) \tag{2.125}$$

and using Eq. (2.123) for M_3,

$$-f_{v_3}sX_1(s)-f_{v_4}sX_2(s)+[M_3s^2+(f_{v_3}+f_{v_4})s]X_3(s)=0 \tag{2.126}$$

Equations (2.124) through (2.126) are the equations of motion. We can solve them for any displacement, $X_1(s)$, $X_2(s)$, or $X_3(s)$, or transfer function.

Skill-Assessment Exercise 2.8

PROBLEM: Find the transfer function, $G(s)=X_2(s)/F(s)$, for the translational mechanical system shown in Figure 2.21.

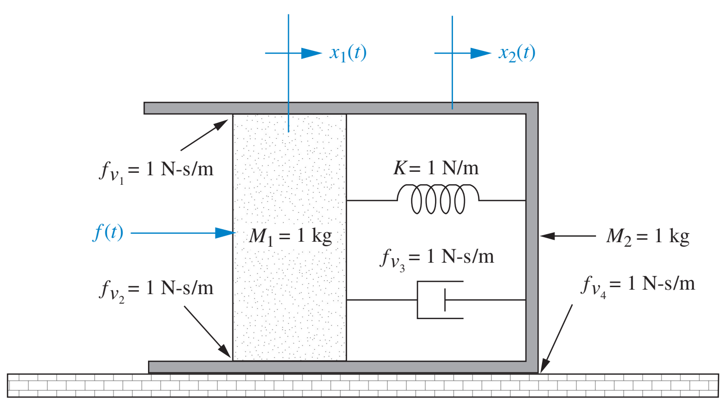


FIGURE 2.21 Translational mechanical system for Skill-Assessment Exercise 2.8

ANSWER: $G(s)=\dfrac{3s+1}{s(s^3+7s^2+5s+1)}$

The complete solution is at www.wiley.com/college/nise.

2.6 Rotational Mechanical System Transfer Functions

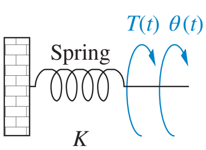
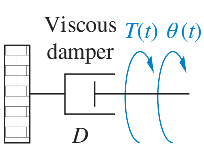
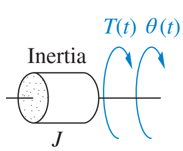
Having covered electrical and translational mechanical systems, we now move on to consider rotational mechanical systems. Rotational mechanical systems are handled the same way as translational mechanical systems, except that torque replaces force and angular displacement replaces translational displacement. The mechanical components for rotational systems are the same as those for translational systems, except that the components undergo rotation instead of translation. Table 2.5 shows the components along with the relationships between torque and angular velocity, as well as angular displacement. Notice that the symbols for the components look the same as translational symbols, but they are undergoing rotation and not translation.

Also notice that the term associated with the mass is replaced by inertia. The values of K, D, and J are called *spring constant*, *coefficient of viscous friction*, and *moment of inertia*, respectively. The impedances of the mechanical components are also summarized in the last column of Table 2.5. The values can be found by taking the Laplace transform, assuming zero initial conditions, of the torque-angular displacement column of Table 2.5.

The concept of degrees of freedom carries over to rotational systems, except that we test a point of motion by *rotating* it while holding still all other points of motion. The number of points of motion that can be rotated while all others are held still equals the number of equations of motion required to describe the system.

Writing the equations of motion for rotational systems is similar to writing them for translational systems; the only difference is that the free-body diagram consists of torques rather than forces. We obtain these torques using superposition. First, we rotate a body while holding all other points still and place on its free-body diagram all torques due to the body's own motion. Then, holding the body still, we rotate adjacent points of motion one at a time and add the torques due to the adjacent motion to the free-body diagram. The process is repeated for each point of motion. For each free-body diagram, these torques are summed and set equal to zero to form the equations of motion.

TABLE 2.5 Torque-angular velocity, torque-angular displacement, and impedance rotational relationships for springs, viscous dampers, and inertia

Component	Torque-angular velocity	Torque-angular displacement	Impedence $Z_M(s) = T(s)/\theta(s)$
	$T(t) = K\int_0^t \omega(\tau)d\tau$	$T(t) = K\theta(t)$	K
	$T(t) = D\omega(t)$	$T(t) = D\dfrac{d\theta(t)}{dt}$	Ds
	$T(t) = J\dfrac{d\omega(t)}{dt}$	$T(t) = J\dfrac{d^2\theta(t)}{dt^2}$	Js^2

Note: The following set of symbols and units is used throughout this book: $T(t)$ – N-m (newton-meters), $\theta(t)$ – rad (radians), $\omega(t)$ – rad/s (radians/second), K – N-m/rad (newton-meters/radian), D – N-m-s/rad (newton-meters-seconds/radian). J – kg-m^2(kilograms-meters^2 – newton-meters-seconds^2/radian).