وزارة التعليم العالي والبحث العلمي
جامعة المستقبل

LEC : FIVE

Course Name : Fundamentals of Electricity Instructor
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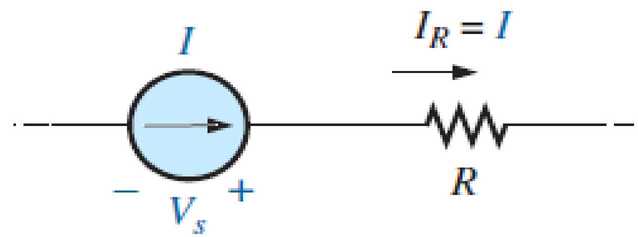
Stage : First
Academic Year : 2023

Lecture Title : **Conversion Delta To Star Connection And**

Conversion Star To Delta Connection

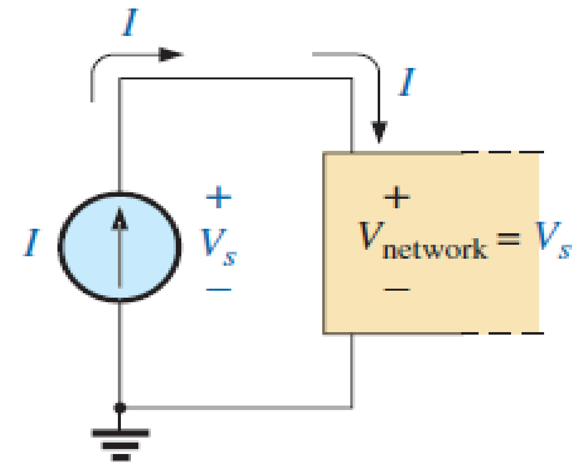
Analysis Methods and Y-D (Star-Delta)

8.2 CURRENT SOURCES



A current source determines the direction and magnitude of the current in the branch where it is located.

The magnitude and the polarity of the voltage across a current source are each a function of the network to which the voltage is applied.



EXAMPLE 8.1 Find the source voltage, the voltage *V_1, and current I_1* for the circuit in Fig.

Since the current source establishes the current in the branch in which it is located, the current I_1 must equal I, and

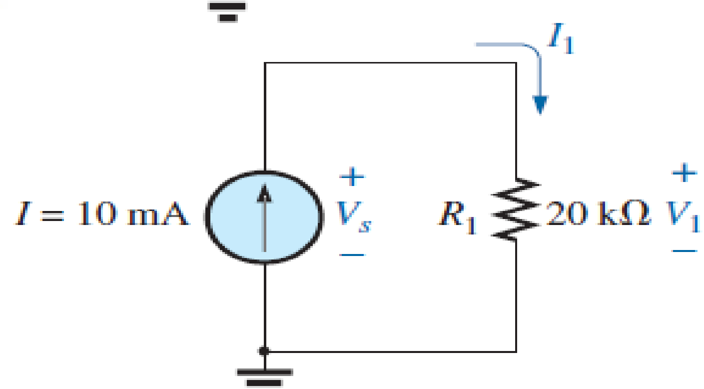
$$I_1 = I = \mathbf{10\ mA}$$

The voltage across R_1 is then determined by Ohm's law:

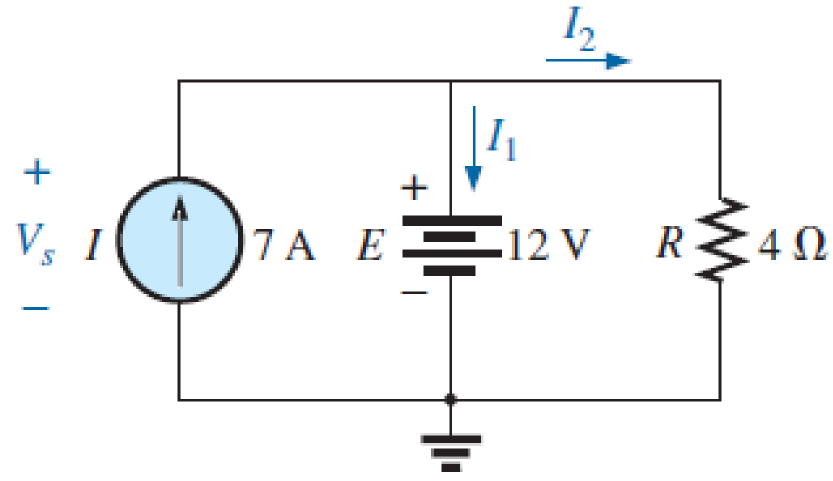
$$V_1 = I_1R_1 = (10\ \text{mA})(20\ \text{k}\Omega) = \mathbf{200\ V}$$

Since resistor R_1 and the current source are in parallel,

$$V_s = V_1 = \mathbf{200\ V}$$



EXAMPLE 8.2 Find the voltage *Vs and currents I1 and I2 for the network* in Fig.



for the current source, *Vs must be determined, and for the voltage source, Is must be* determined.

Since the current source and voltage source are in parallel,

$$V_s = E = \mathbf{12\ V}$$

Further, since the voltage source and resistor R are in parallel,

$$V_R = E = 12\ \text{V}$$

and

$$I_2 = \frac{V_R}{R} = \frac{12\ \text{V}}{4\ \Omega} = \mathbf{3\ A}$$

The current I_1 of the voltage source can then be determined by applying Kirchhoff's current law at the top of the network as follows:

$$\Sigma I_i = \Sigma I_o$$

$$I = I_1 + I_2$$

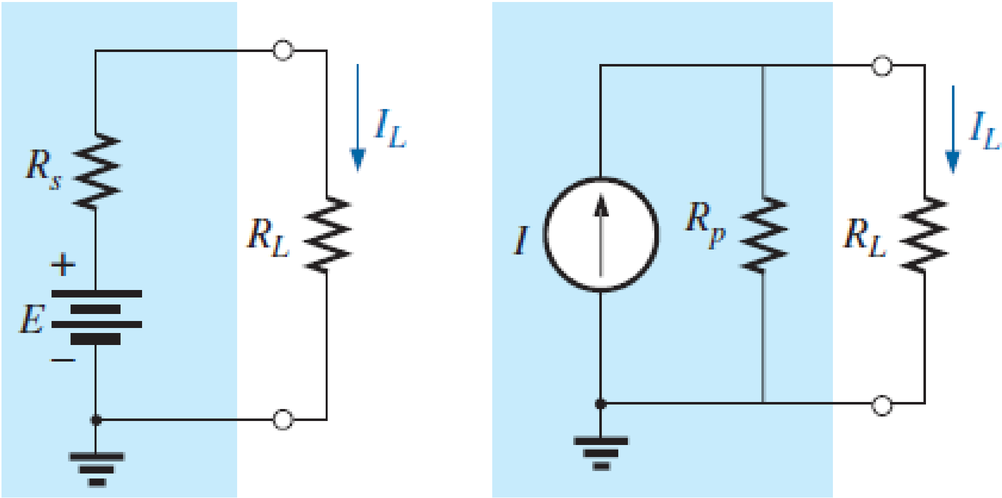
and

$$I_1 = I - I_2 = 7\ \text{A} - 3\ \text{A} = \mathbf{4\ A}$$

8.3 SOURCE CONVERSIONS

For the voltage source, if *Rs=0 , or if it is so small compared to any series resistors* that it can be ignored, then we have an "**ideal**" **voltage source** for all practical purposes.

For the current source, since the resistor *Rp is in parallel, if* $R_P = \infty$ *, or if it is large enough compared to any parallel resistive elements* that it can be ignored, then we have an "**ideal" current source**.

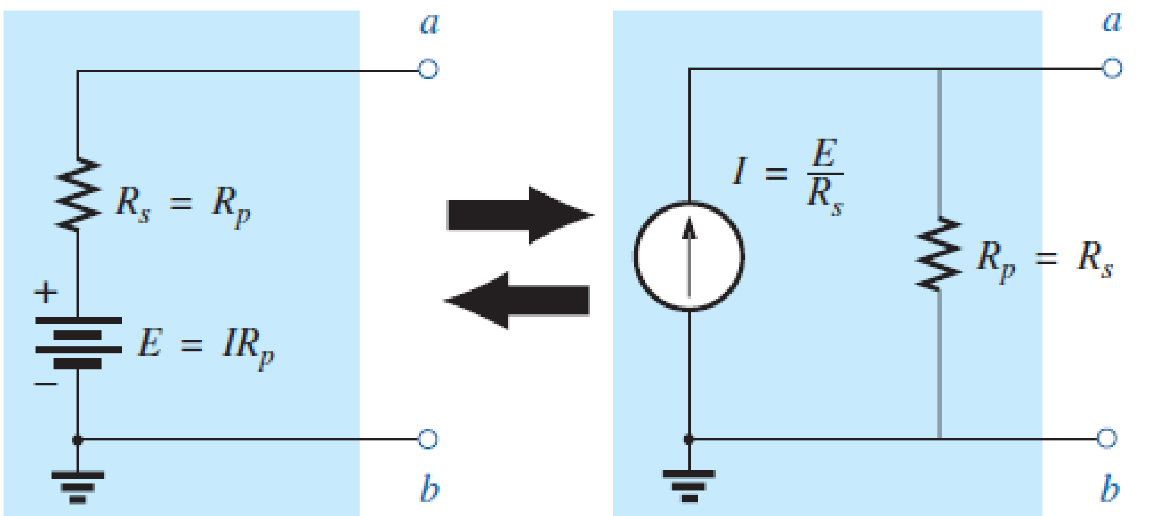


(a) (b)

Note that:

1Ideal sources *cannot be converted from one* type to another.

2The equivalence between a current source and a voltage source exists only at their external terminals.



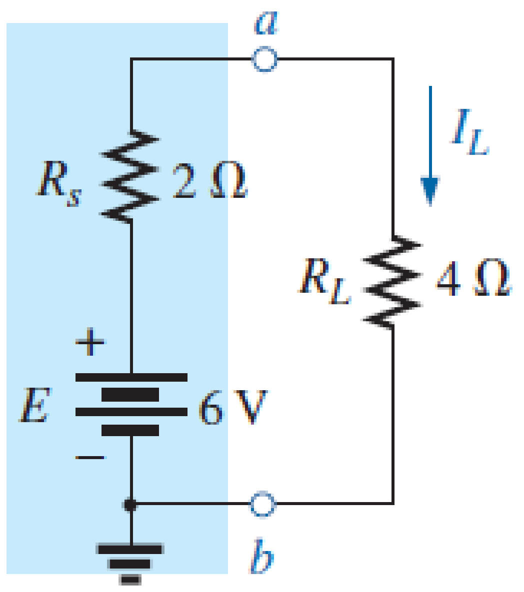
Source conversion.

EXAMPLE 8.4 For the circuit in Fig.

a. Determine the current IL.

b. Convert the voltage source to a current source.

c. Using the resulting current source of part (b), calculate the current through the load resistor, and compare your answer to the result of part (a).



a. Applying Ohm's law:

$$I_L = \frac{E}{R_s + R_L} = \frac{6\text{ V}}{2\,\Omega + 4\,\Omega} = \frac{6\text{ V}}{6\,\Omega} = \mathbf{1\text{ A}}$$

b. Using Ohm's law again:

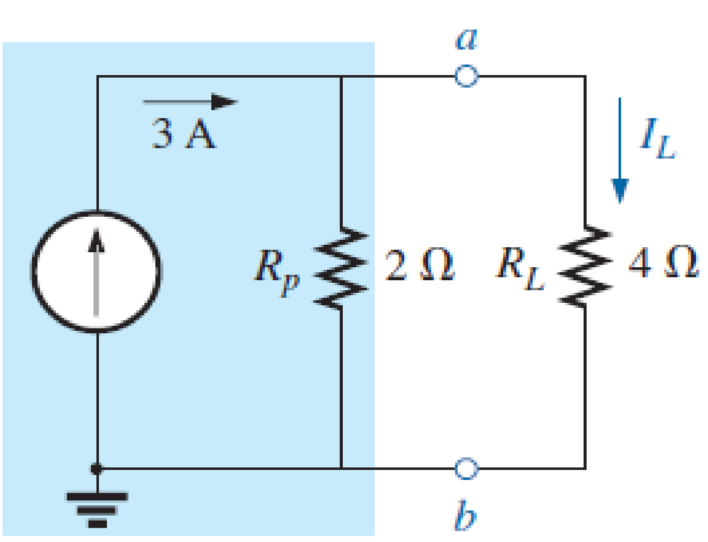
$$I = \frac{E}{R_s} = \frac{6\text{ V}}{2\,\Omega} = \mathbf{3\text{ A}}$$

c. Using the current divider rule:

$$I_L = \frac{R_p I}{R_p + R_L} = \frac{(2\,\Omega)(3\text{ A})}{2\,\Omega + 4\,\Omega}$$

$$= \frac{1}{3}(3\text{ A}) = \mathbf{1\text{ A}}$$

$$I = \frac{E}{R_s} = 3\text{ A}$$



8.4 CURRENT SOURCES IN PARALLEL

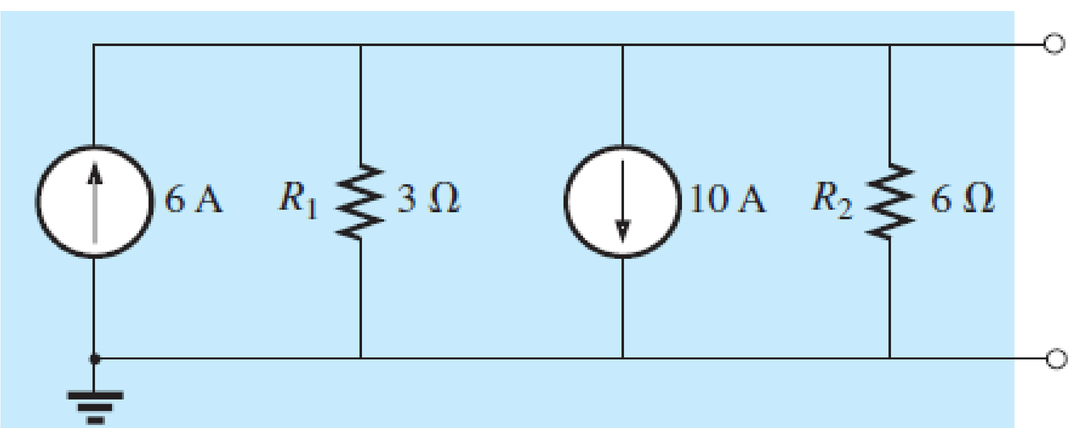
Current sources of different values cannot be placed in series due to a violation of Kirchhoff's current law.

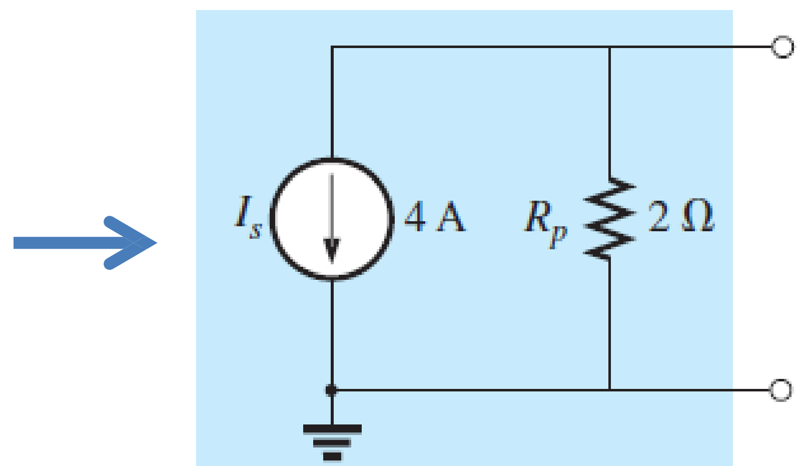
However, current sources can be placed in parallel just as voltage sources can be placed in series. In general,

Two or more current sources in parallel can be replaced by a single current source having a magnitude determined by the difference of the sum of the currents in one direction and the sum in the opposite direction. The new parallel internal resistance is the total resistance of the resulting parallel resistive elements.

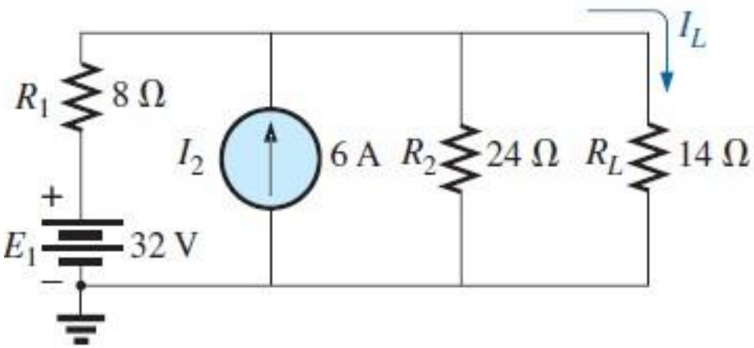
Current sources of different current ratings cannot be connected in series.

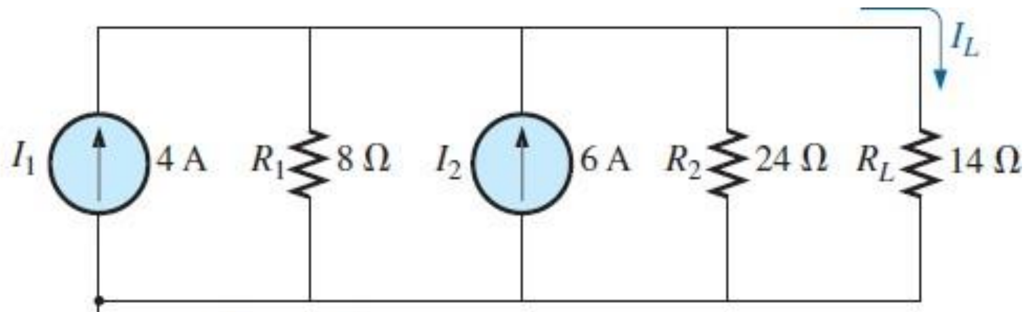
EXAMPLE 8.6 Reduce the parallel current sources in Fig. to a single current source.





EXAMPLE 8.8 Reduce the network in Fig. to a single current source, and calculate the current through R_L .

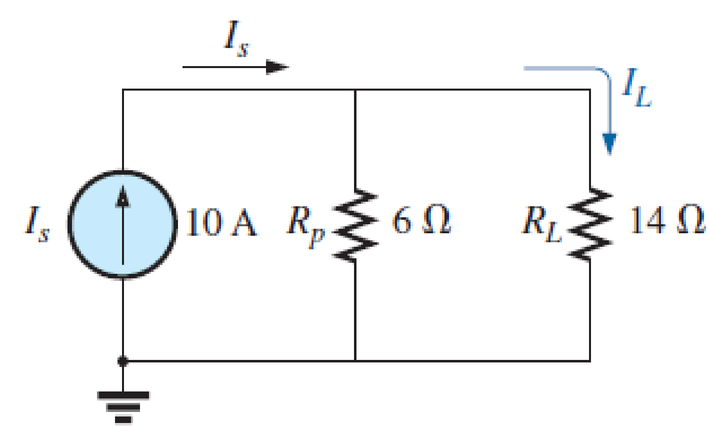




$$I_1 = \frac{E_1}{R_1} = \frac{32\text{ V}}{8\ \Omega} = 4\text{ A}$$

$$I_s = I_1 + I_2 = 4\text{ A} + 6\text{ A} = \mathbf{10\ A}$$

$$R_s = R_1 \| R_2 = 8\ \Omega \| 24\ \Omega = \mathbf{6\ \Omega}$$

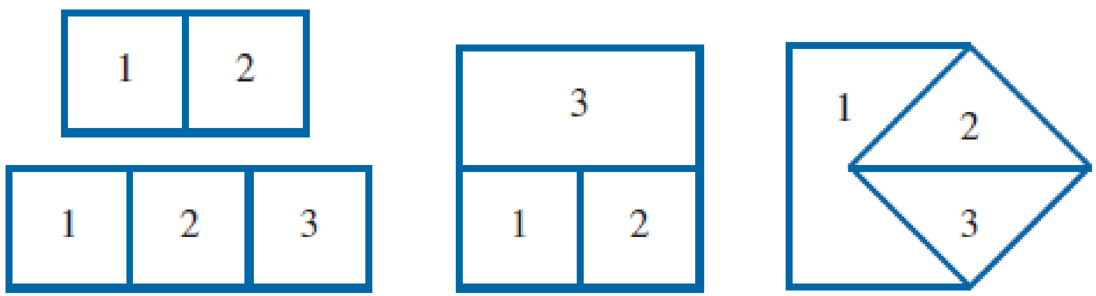


Applying the current divider rule to the resulting network in Fig.

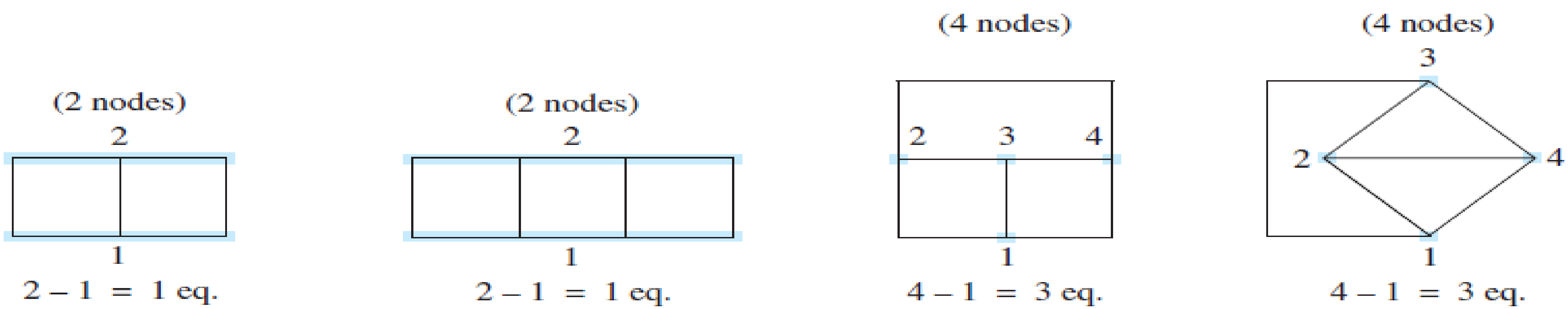
$$I_L = \frac{R_p I_s}{R_p + R_L} = \frac{(6\ \Omega)(10\text{ A})}{6\ \Omega + 14\ \Omega} = \frac{60\text{ A}}{20} = \mathbf{3\ A}$$

8.6 BRANCH-CURRENT ANALYSIS

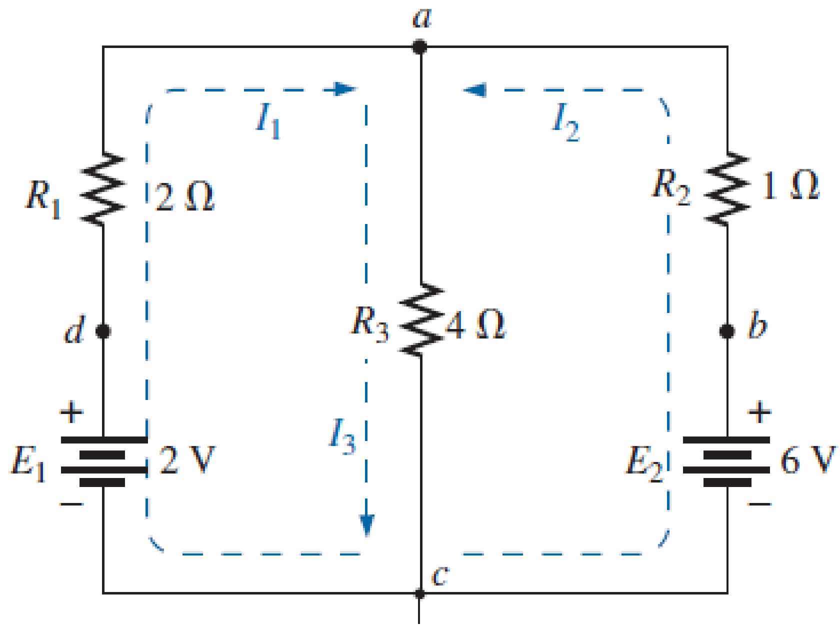
1. Assign a distinct current of arbitrary direction to each branch of the network.
2. Indicate the polarities for each resistor as determined by the assumed current direction.
3. Apply Kirchhoff's voltage law around each closed, independent loop of the network.
4.Apply Kirchhoff's current law at the minimum number of nodes that will include all the branch currents of the network.
5. Solve the resulting simultaneous linear equations for assumed branch currents.



Determining the number of independent closed loops.

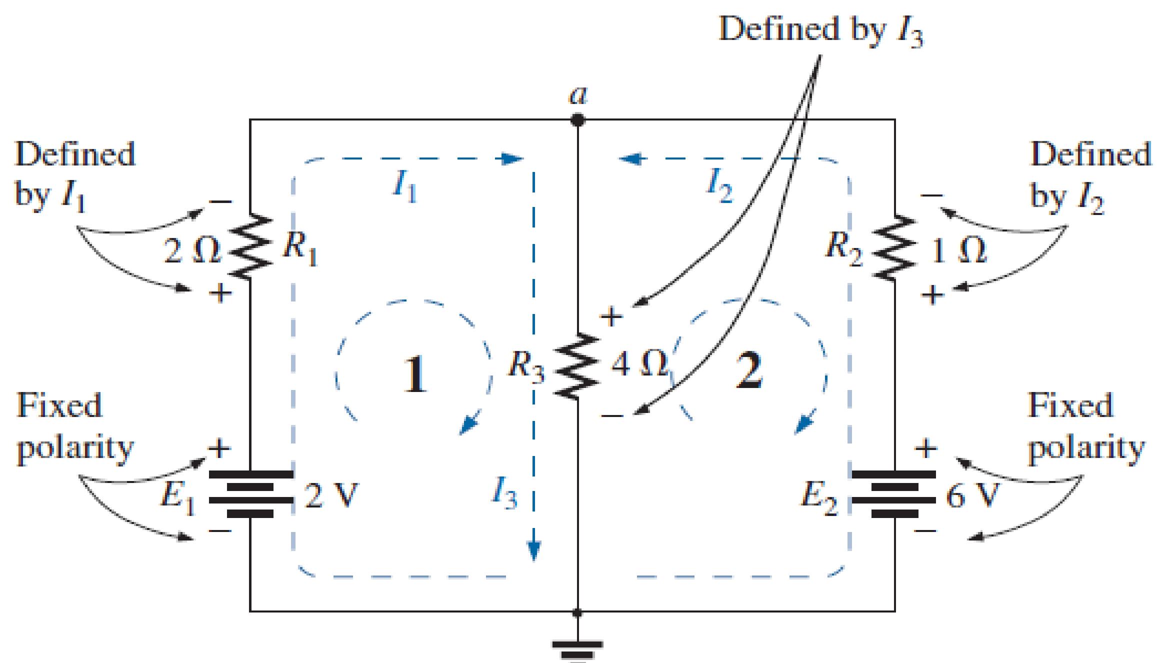


EXAMPLE 8.9 Apply the branch-current method to the network in Fig.



Step 1: Since there are three distinct branches (cda, cba, ca), three currents of arbitrary directions (I_1, I_2, I_3) are chosen, as shown.
The current directions for I_1 and I_2 were chosen to match the “pressure” applied by sources E_1 and E_2, respectively.
Since both I_1 and I_2 enter node a, I_3 is leaving.

Step 2: Polarities for each resistor are drawn to agree with assumed current directions, as shown.



Step 3: Kirchhoff's voltage law is applied around each closed loop (1 and 2) in the clockwise direction:

Rise in potential

$$\text{loop 1:}\quad \Sigma_{\circlearrowright} V = +E_1 - V_{R_1} - V_{R_3} = 0$$

Drop in potential

Rise in potential

$$\text{loop 2:}\quad \Sigma_{\circlearrowright} V = +V_{R_3} + V_{R_2} - E_2 = 0$$

and

Drop in potential

$$\text{loop 1:}\quad \Sigma_{\circlearrowright} V = \underbrace{+2\text{ V}}_{\substack{\text{Battery}\\ \text{potential}}} - \underbrace{(2\ \Omega)I_1}_{\substack{\text{Voltage drop}\\ \text{across } 2\ \Omega\\ \text{resistor}}} - \underbrace{(4\ \Omega)I_3}_{\substack{\text{Voltage drop}\\ \text{across } 4\ \Omega\\ \text{resistor}}} = 0$$

$$\text{loop 2:}\quad \Sigma_{\circlearrowright} V = (4\ \Omega)I_3 + (1\ \Omega)I_2 - 6\text{ V} = 0$$

Step 4: Applying Kirchhoff's current law at node a (in a two-node network, the law is applied at only one node),

$$I_1 + I_2 = I_3$$

Step 5: There are three equations and three unknowns (units removed for clarity):

$$\begin{aligned} 2 - 2I_1 - 4I_3 &= 0 \\ 4I_3 + 1I_2 - 6 &= 0 \\ I_1 + I_2 &= I_3 \end{aligned} \qquad \text{Rewritten:} \quad \begin{aligned} 2I_1 + 0 + 4I_3 &= 2 \\ 0 + I_2 + 4I_3 &= 6 \\ I_1 + I_2 - \ I_3 &= 0 \end{aligned}$$

Using third-order determinants **(Appendix D),** we have

$$I_1 = \frac{\begin{vmatrix} 2 & 0 & 4 \\ 6 & 1 & 4 \\ 0 & 1 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & 0 & 4 \\ 0 & 1 & 4 \\ 1 & 1 & -1 \end{vmatrix}} = -\mathbf{1\ A}$$

A negative sign in front of a branch current indicates only that the actual current is in the direction opposite to that assumed.

$$D = \begin{vmatrix} 2 & 0 & 4 \\ 0 & 1 & 4 \\ 1 & 1 & -1 \end{vmatrix}$$

$$I_2 = \frac{\begin{vmatrix} 2 & 2 & 4 \\ 0 & 6 & 4 \\ 1 & 0 & -1 \end{vmatrix}}{D} = \mathbf{2\ A}$$

$$I_3 = \frac{\begin{vmatrix} 2 & 0 & 2 \\ 0 & 1 & 6 \\ 1 & 1 & 0 \end{vmatrix}}{D} = \mathbf{1\ A}$$

Solution 2: *Instead of using third-order determinants as in Solution 1,* we can reduce the three equations to two by substituting the third equation in the first and second equations:

$$\left.\begin{array}{r} 2 - 2I_1 - 4\overbrace{(I_1 + I_2)}^{I_3} = 0 \\ 4\overbrace{(I_1 + I_2)}^{I_3} + I_2 - 6 = 0 \end{array}\right\} \begin{array}{r} 2 - 2I_1 - 4I_1 - 4I_2 = 0 \\ 4I_1 + 4I_2 + I_2 - 6 = 0 \end{array}$$

or

$$\begin{aligned} -6I_1 - 4I_2 &= -2 \\ +4I_1 + 5I_2 &= +6 \end{aligned}$$

Multiplying through by -1 in the top equation yields

$$\begin{aligned} 6I_1 + 4I_2 &= +2 \\ 4I_1 + 5I_2 &= +6 \end{aligned}$$

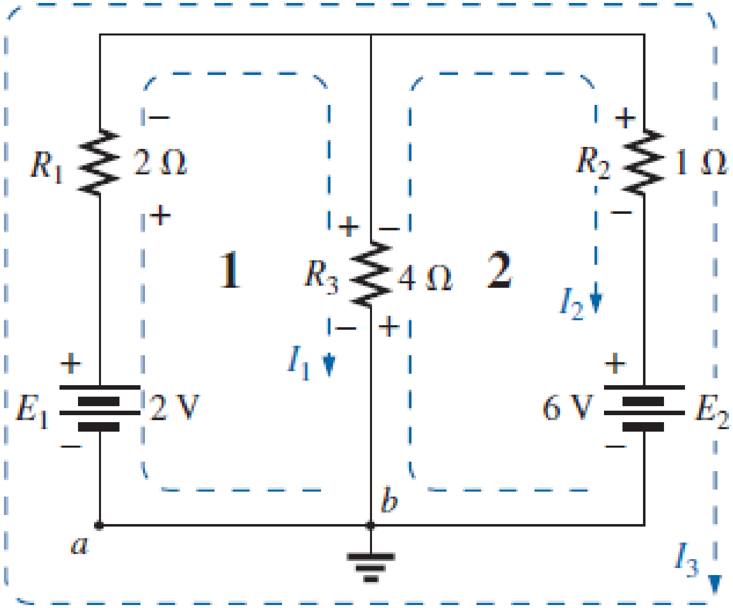
and using determinants,

$$I_1 = \frac{\begin{vmatrix} 2 & 4 \\ 6 & 5 \end{vmatrix}}{\begin{vmatrix} 6 & 4 \\ 4 & 5 \end{vmatrix}} = \frac{10 - 24}{30 - 16} = \frac{-14}{14} = -1\ \mathbf{A}$$

Mesh Analysis Procedure

1. ***Assign a distinct current in the clockwise direction to each independent, closed loop of the network. It is not absolutely necessary to choose the clockwise direction for each loop current. However, by choosing the clockwise direction as a standard, we can develop a shorthand method (Section 8.8) for writing the required equations that will save time and possibly prevent some common errors.***
2. ***Indicate the polarities within each loop for each resistor as determined by the assumed direction of loop current for that loop. Note the requirement that the polarities be placed within each loop. This requires, as shown in Fig., that the 4 Ω resistor have two sets of polarities across it.***
3. ***Apply Kirchhoff's voltage law around each closed loop in the clockwise direction. Again, the clockwise direction was chosen to establish uniformity and prepare us for the method to be introduced in the next section.***
   a. ***If a resistor has two or more assumed currents through it, the total current through the resistor is the assumed current of*** **the loop in which Kirchhoff's voltage law is being applied, plus** ***the assumed currents of the other loops passing through in the same direction, minus the assumed currents through in the opposite direction.***
   b. ***The polarity of a voltage source is unaffected by the direction of the assigned loop currents.***
4. ***Solve the resulting simultaneous linear equations for the assumed loop currents.***

EXAMPLE 8.11 Consider the same basic network as in **Example 8.9,** now appearing as shown



loop 1: $+E_1 - V_1 - V_3 = 0$ (clockwise starting at a)

$$+2\,\text{V} - (2\,\Omega)\,I_1 - \overbrace{(4\,\Omega)\underbrace{(I_1 - I_2)}_{\substack{\text{Total current}\\ \text{through}\\ 4\,\Omega\text{ resistor}}}}^{\substack{\text{Voltage drop across}\\ 4\,\Omega\text{ resistor}}} = 0$$

Subtracted since I_2 is opposite in direction to I

loop 2: $-V_3 - V_2 - E_2 = 0$ (clockwise starting at b)

$$-(4\,\Omega)(I_2 - I_1) - (1\,\Omega)I_2 - 6\,\text{V} = 0$$

The equations are then rewritten as follows

$$\begin{aligned} \text{loop 1:}\quad & +2 - 2I_1 - 4I_1 + 4I_2 = 0 \\ \text{loop 2:}\quad & -4I_2 + 4I_1 - 1I_2 - 6 = 0 \end{aligned}$$

and

$$\text{loop 1:} \quad +2 - 6I_1 + 4I_2 = 0$$
$$\text{loop 2:} \quad -5I_2 + 4I_1 - 6 = 0$$

or

$$\text{loop 1:} \quad -6I_1 + 4I_2 = -2$$
$$\text{loop 2:} \quad +4I_1 - 5I_2 = +6$$

Applying determinants results in

$$I_1 = -1\,\text{A} \quad \text{and} \quad I_2 = -2\,\text{A}$$

The actual current through the 2 V source and 2 Ω resistor is therefore 1 A in the other direction, and the current through the 6 V source and 1 Ω resistor is 2 A in the opposite direction indicated on the circuit.

The current through the 4 Ω resistor is determined by the following equation from the original network:

$$\text{loop 1:} \quad I_{4\Omega} = I_1 - I_2 = -1\,\text{A} - (-2\,\text{A}) = -1\,\text{A} + 2\,\text{A}$$
$$= 1\,\text{A} \quad \text{(in the direction of } I_1\text{)}$$

8.9 NODAL ANALYSIS (GENERAL APPROACH)

The number of nodes for which the voltage must be determined using nodal analysis is 1 less than the total number of nodes.

Nodal Analysis Procedure

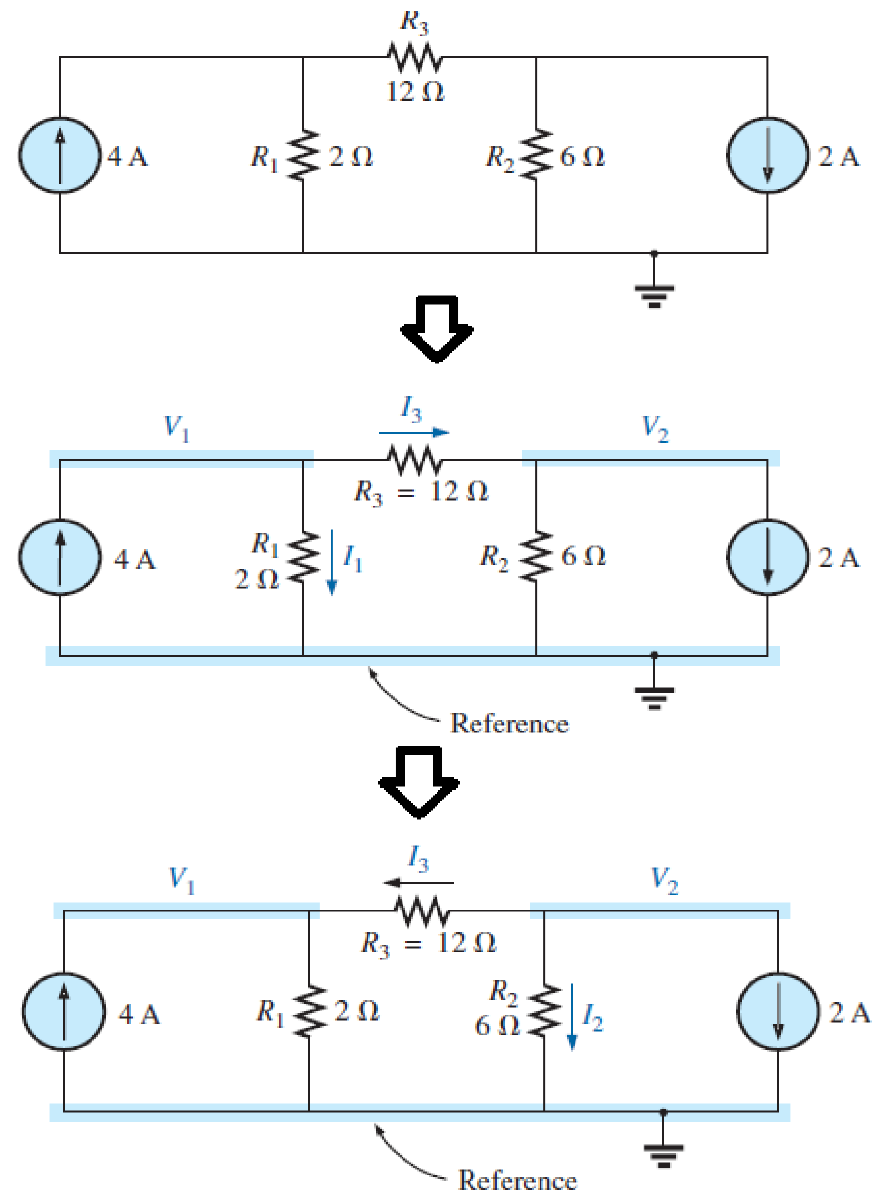
1. Determine the number of nodes within the network.

2.Pick a reference node, and label each remaining node with a subscripted value of voltage: V_1, V_2, and so on.

3.Apply Kirchhoff's current law at each node except the reference. Assume that all unknown currents leave the node for each application of Kirchhoff's current law. In other words, for each node, don't be influenced by the direction that an unknown current for another node may have had. Each node is to be treated as a separate entity, independent of the application of Kirchhoff's current law to the other nodes.

4. Solve the resulting equations for the nodal voltages.

EXAMPLE 8.21 Determine the nodal voltages for the network shown.



For the node V1. Applying Kirchhoff's current law:

$$4\text{ A} = I_1 + I_3$$

and $$4\text{ A} = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_3} = \frac{V_1}{2\,\Omega} + \frac{V_1 - V_2}{12\,\Omega}$$

Expanding and rearranging:

$$V_1\left(\frac{1}{2\,\Omega} + \frac{1}{12\,\Omega}\right) - V_2\left(\frac{1}{12\,\Omega}\right) = 4\text{ A}$$

For node V2, the currents are defined as

$$0 = I_3 + I_2 + 2\text{ A}$$

and $$\frac{V_2 - V_1}{R_3} + \frac{V_2}{R_2} + 2\text{ A} = 0$$

$$\frac{V_2 - V_1}{12\,\Omega} + \frac{V_2}{6\,\Omega} + 2\text{ A} = 0$$

Expanding and rearranging:

$$V_2\left(\frac{1}{12\,\Omega} + \frac{1}{6\,\Omega}\right) - V_1\left(\frac{1}{12\,\Omega}\right) = -2\text{ A}$$

resulting in Eq. (8.1), which consists of two equations and two unknowns:

$$\left.\begin{aligned} V_1\left(\frac{1}{2\,\Omega}+\frac{1}{12\,\Omega}\right)-V_2\left(\frac{1}{12\,\Omega}\right) &= +4\text{ A} \\ V_2\left(\frac{1}{12\,\Omega}+\frac{1}{6\,\Omega}\right)-V_1\left(\frac{1}{12\,\Omega}\right) &= -2\text{ A} \end{aligned}\right\} \qquad \textbf{(8.1)}$$

producing

$$\left.\begin{aligned} \frac{7}{12}V_1-\frac{1}{12}V_2 &= +4 \\ -\frac{1}{12}V_1+\frac{3}{12}V_2 &= -2 \end{aligned}\right\} \qquad \begin{aligned} 7V_1-V_2 &= 48 \\ -1V_1+3V_2 &= -24 \end{aligned}$$

and

$$V_1=\frac{\begin{vmatrix} 48 & -1 \\ -24 & 3 \end{vmatrix}}{\begin{vmatrix} 7 & -1 \\ -1 & 3 \end{vmatrix}}=\frac{120}{20}=\mathbf{+6\ V}$$

$$V_2=\frac{\begin{vmatrix} 7 & 48 \\ -1 & -24 \end{vmatrix}}{20}=\frac{-120}{20}=\mathbf{-6\ V}$$

Since V_1 is greater than V_2, the current through R_3 passes from V_1 to V_2. Its value is

$$I_{R_3} = \frac{V_1 - V_2}{R_3} = \frac{6\text{ V} - (-6\text{ V})}{12\ \Omega} = \frac{12\text{ V}}{12\ \Omega} = \mathbf{1\ A}$$

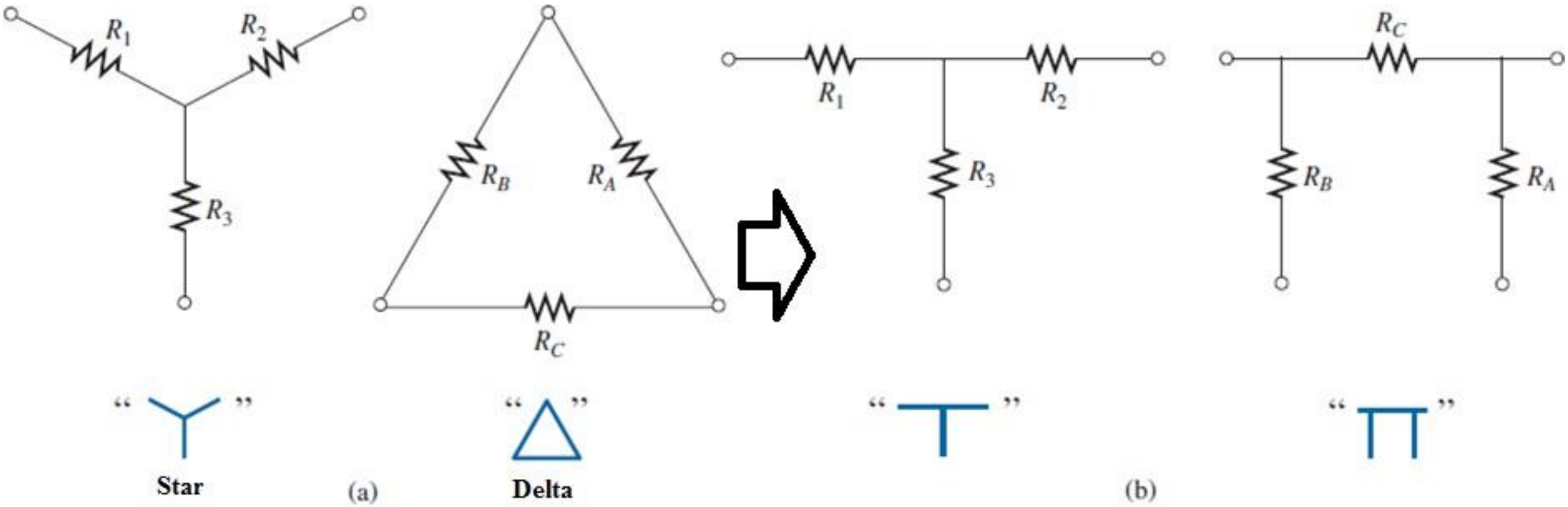
The fact that V_1 is positive results in a current I_{R_1} from V_1 to ground equal to

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{V_1}{R_1} = \frac{6\text{ V}}{2\ \Omega} = \mathbf{3\ A}$$

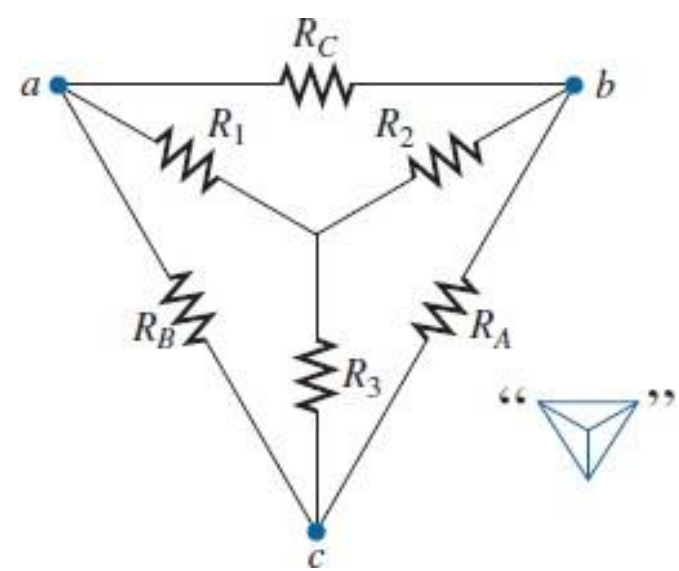
Finally, since V_2 is negative, the current I_{R_2} flows from ground to V_2 and is equal to

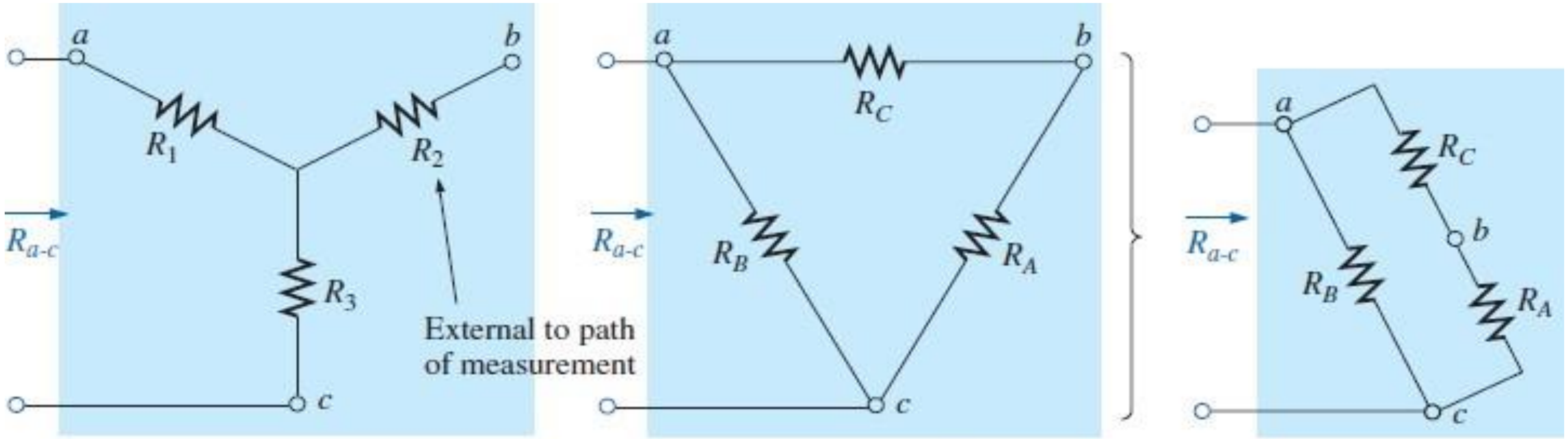
$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{V_2}{R_2} = \frac{6\text{ V}}{6\ \Omega} = \mathbf{1\ A}$$

8.12 (Star-Delta) Y- Δ (T- π) AND Δ -Y (π -T) CONVERSIONS



It is our purpose to find some expression for R_1, R_2, and R_3 in terms of R_A, R_B, and R_C , and vice versa, that will ensure that the resistance between any two terminals of the Y configuration will be the same with the Δ configuration inserted in place of the Y configuration (and vice versa).





Finding the resistance $R_{a\text{-}c}$ for the Y and Δ configurations.

$$R_{a\text{-}c}\,(Y) = R_{a\text{-}c}\,(\Delta)$$

$$\boxed{R_{a\text{-}c} = R_1 + R_3 = \frac{R_B(R_A + R_C)}{R_B + (R_A + R_C)}} \qquad \textbf{(8.3a)}$$

$$\boxed{R_{a\text{-}b} = R_1 + R_2 = \frac{R_C(R_A + R_B)}{R_C + (R_A + R_B)}} \qquad \textbf{(8.3b)}$$

$$\boxed{R_{b\text{-}c} = R_2 + R_3 = \frac{R_A(R_B + R_C)}{R_A + (R_B + R_C)}} \qquad \textbf{(8.3c)}$$

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C}$$ $$R_2 = \frac{R_A R_C}{R_A + R_B + R_C}$$ $$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

Note that each resistor of the Y is equal to the product of the resistors in the two closest branches of the Δ divided by the sum of the resistors in the Δ.

$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$ $$R_B = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2}$$ $$R_C = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_3}$$

Note that the value of each resistor of the Δ is equal to the sum of the possible product combinations of the resistances of the Y divided by the resistance of the Y farthest from the resistor to be determined.

Let us consider what would occur if all the values of a Δ or Y were the same. If R_A R_B R_C, (using R_A only) the following:

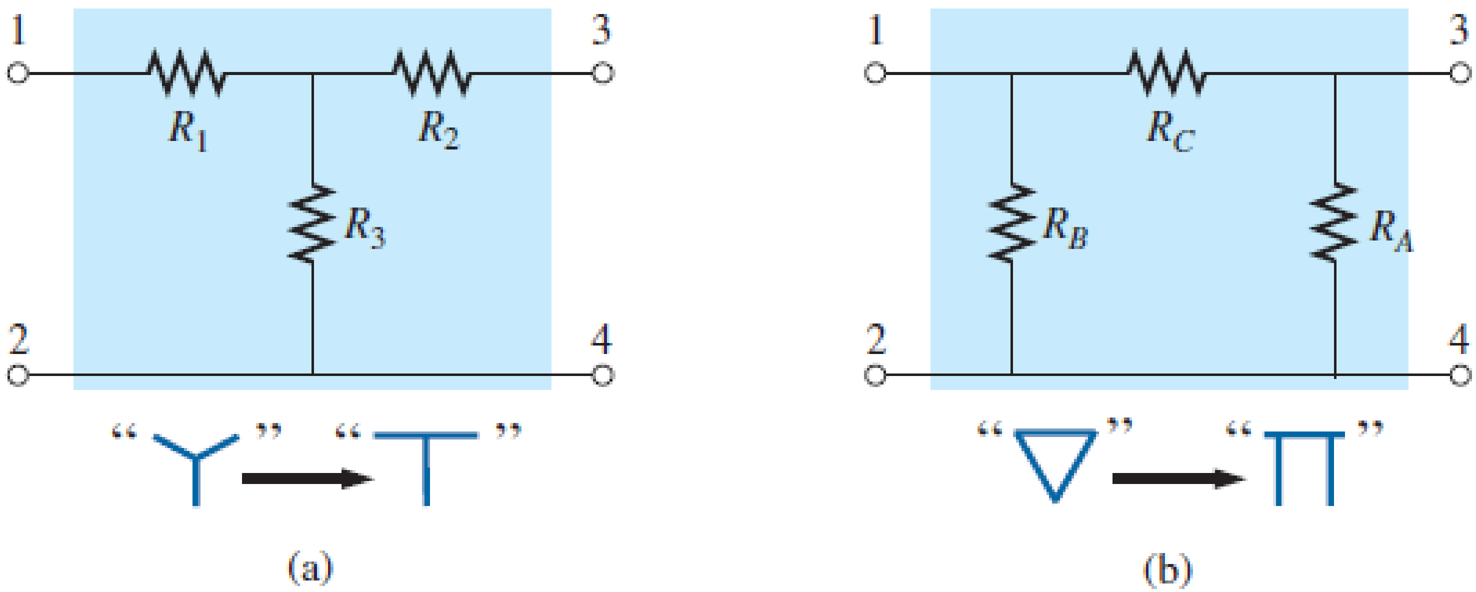
$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{R_A R_A}{R_A + R_A + R_A} = \frac{R_A^2}{3R_A} = \frac{R_A}{3}$$

and, following the same procedure,

$$R_1 = \frac{R_A}{3} \qquad R_2 = \frac{R_A}{3}$$

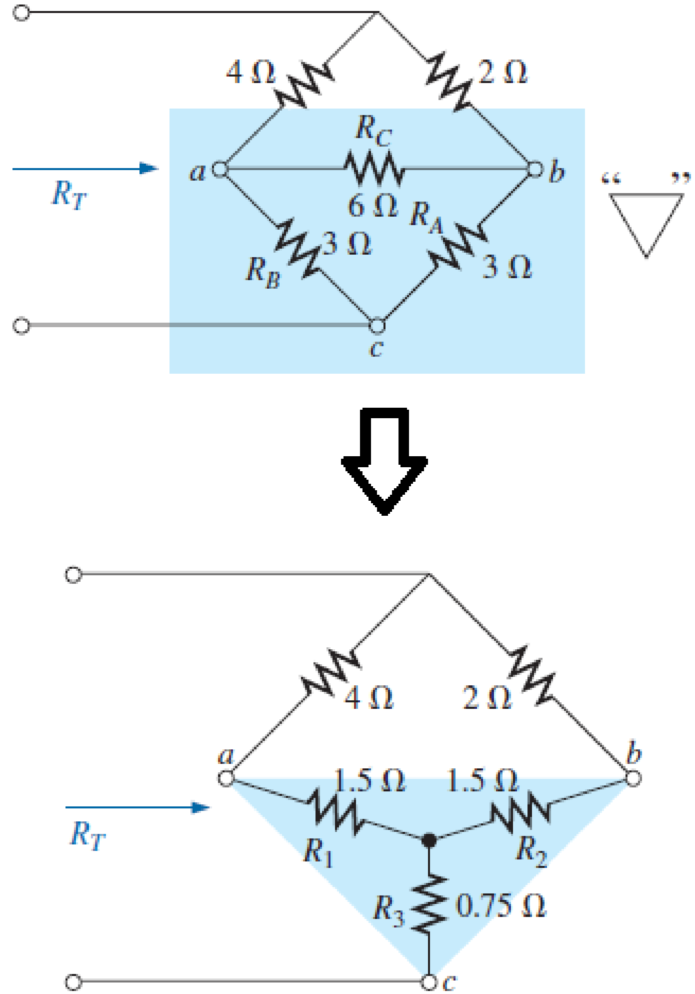
In general, therefore, $$\boxed{R_Y = \frac{R_\Delta}{3}}$$ $$\boxed{R_\Delta = 3R_Y}$$

The Y and the Δ often appear as shown below. They are then referred to as a tee (T) and a pi (π) network, respectively. The equations used to convert from one form to the other are exactly the same as those developed for the Y and Δ transformation.



The relationship between the Y and T configurations and the Δ and π configurations.

EXAMPLE 8.29 Find the total resistance of the network shown, where $R_A = 3\Omega$, $R_B = 3\ \Omega$, and $R_C = 6\ \Omega$.



Two resistors of the Δ were equal; therefore, two resistors of the Y will be equal.

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

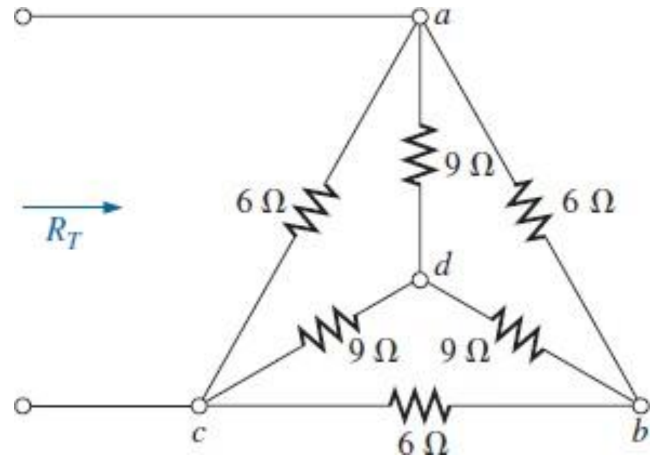
$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(3\ \Omega)(6\ \Omega)}{12\ \Omega} = \frac{18\ \Omega}{12} = \mathbf{1.5\ \Omega}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(3\ \Omega)(3\ \Omega)}{12\ \Omega} = \frac{9\ \Omega}{12} = \mathbf{0.75\ \Omega}$$

Replacing the Δ by the Y, as shown in Fig., yields

$$\begin{aligned} R_T &= 0.75\ \Omega + \frac{(4\ \Omega + 1.5\ \Omega)(2\ \Omega + 1.5\ \Omega)}{(4\ \Omega + 1.5\ \Omega) + (2\ \Omega + 1.5\ \Omega)} \\ &= 0.75\ \Omega + \frac{(5.5\ \Omega)(3.5\ \Omega)}{5.5\ \Omega + 3.5\ \Omega} \\ &= 0.75\ \Omega + 2.139\ \Omega \\ R_T &= \mathbf{2.89\ \Omega} \end{aligned}$$

EXAMPLE 8.30 Find the total resistance of the network shown.

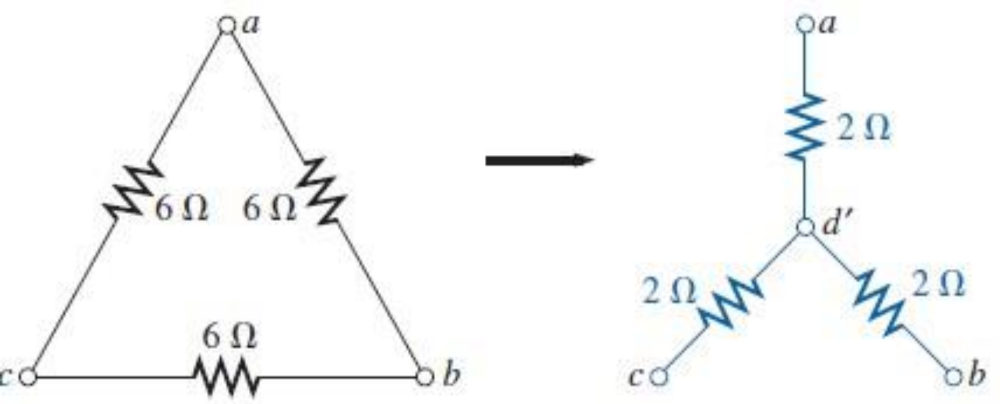


a. Converting the Δ to a Y:

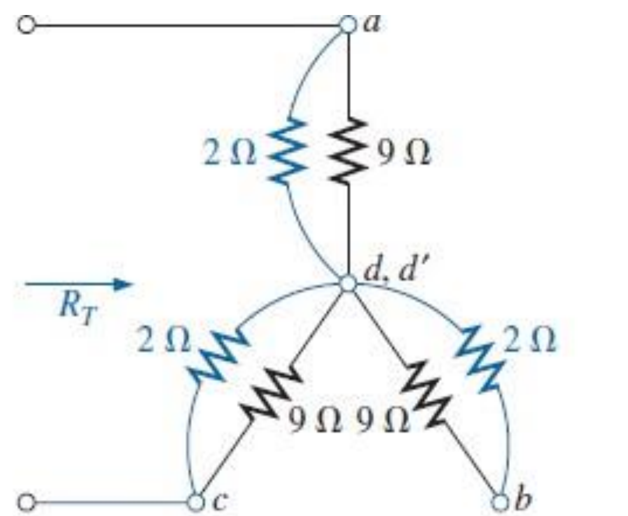
$$R_Y = \frac{R_\Delta}{3} = \frac{6\ \Omega}{3} = 2\ \Omega$$

The network then appears as shown

$$R_T = 2\left[\frac{(2\ \Omega)(9\ \Omega)}{2\ \Omega + 9\ \Omega}\right] = \mathbf{3.27\ \Omega}$$



Converting the Δ configuration of Fig. 8.91 to a Y configuration.



Substituting the Y configuration for the converted Δ into the network

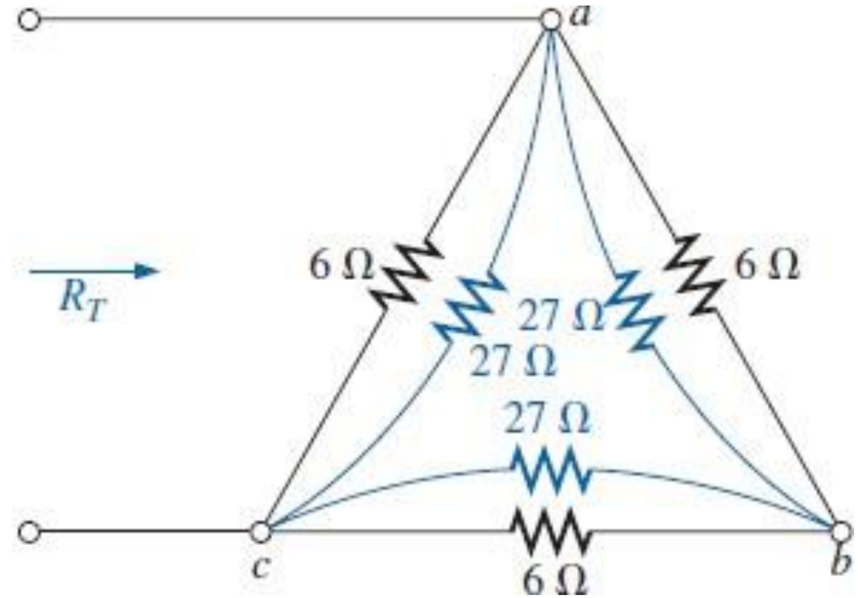
b. *Converting the Y to a Δ :*

$$R_\Delta = 3R_Y = (3)(9\ \Omega) = 27\ \Omega$$

$$R'_T = \frac{(6\ \Omega)(27\ \Omega)}{6\ \Omega + 27\ \Omega} = \frac{162\ \Omega}{33} = 4.91\ \Omega$$

$$R_T = \frac{R'_T(R'_T + R'_T)}{R'_T + (R'_T + R'_T)} = \frac{R'_T 2R'_T}{3R'_T} = \frac{2R'_T}{3}$$

$$= \frac{2(4.91\ \Omega)}{3} = \mathbf{3.27\ \Omega}$$



PROBLEMS

SECTION 8.2 Current Sources: 1, 2, 4, 6

SECTION 8.3 Source Conversions: 7, 8, 9, 10

SECTION 8.4 Current Sources in Parallel: 11, 12, 14

SECTION 8.6 Branch-Current Analysis: 15, 16, 18, 24

SECTION 8.9 Nodal Analysis: 35, 36, 38, 40

SECTION 8.12 Y – Δ (T-π) and Δ - Y (π-T) Conversions: 51, 52, 54, 56