EXAMPLE PROBLEMS AND SOLUTIONS

A–2–1. Simplify the block diagram shown in Figure 2–17.

Solution. First, move the branch point of the path involving H_1 outside the loop involving H_2, as shown in Figure 2–18(a). Then eliminating two loops results in Figure 2–18(b). Combining two blocks into one gives Figure 2–18(c).

A–2–2. Simplify the block diagram shown in Figure 2–19. Obtain the transfer function relating $C(s)$ and $R(s)$.

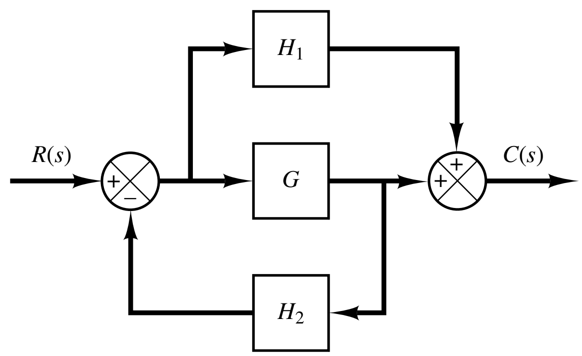

Figure 2–17 Block diagram of a system.

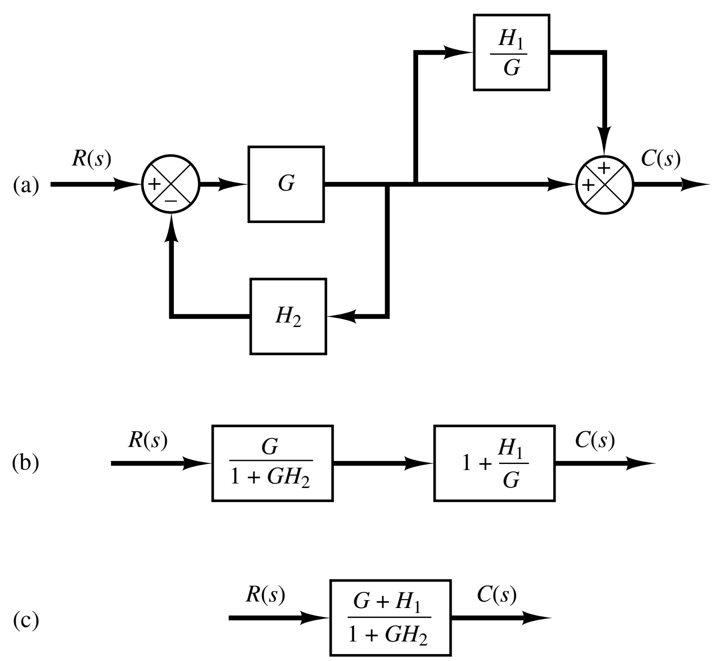

Figure 2–18 Simplified block diagrams for the system shown in Figure 2–17.

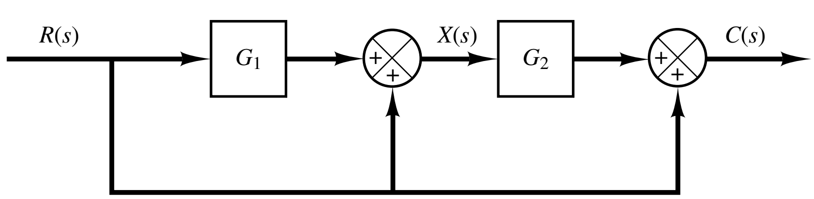

Figure 2–19 Block diagram of a system.

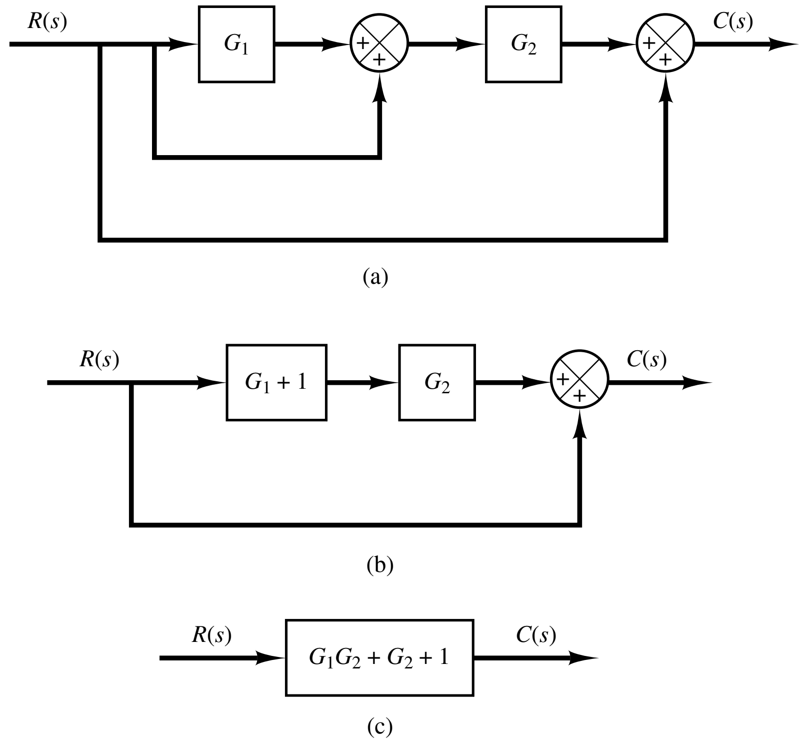

Figure 2–20 Reduction of the block diagram shown in Figure 2–19.

Solution. The block diagram of Figure 2–19 can be modified to that shown in Figure 2–20(a). Eliminating the minor feedforward path, we obtain Figure 2–20(b), which can be simplified to Figure 2–20(c). The transfer function $C(s)/R(s)$ is thus given by

$$\frac{C(s)}{R(s)} = G_1G_2 + G_2 + 1$$

The same result can also be obtained by proceeding as follows: Since signal $X(s)$ is the sum of two signals $G_1R(s)$ and $R(s)$, we have

$$X(s) = G_1R(s) + R(s)$$

The output signal $C(s)$ is the sum of $G_2X(s)$ and $R(s)$. Hence

$$C(s) = G_2X(s) + R(s) = G_2[G_1R(s) + R(s)] + R(s)$$

And so we have the same result as before:

$$\frac{C(s)}{R(s)} = G_1G_2 + G_2 + 1$$

A–2–3. Simplify the block diagram shown in Figure 2–21. Then obtain the closed-loop transfer function $C(s)/R(s)$.

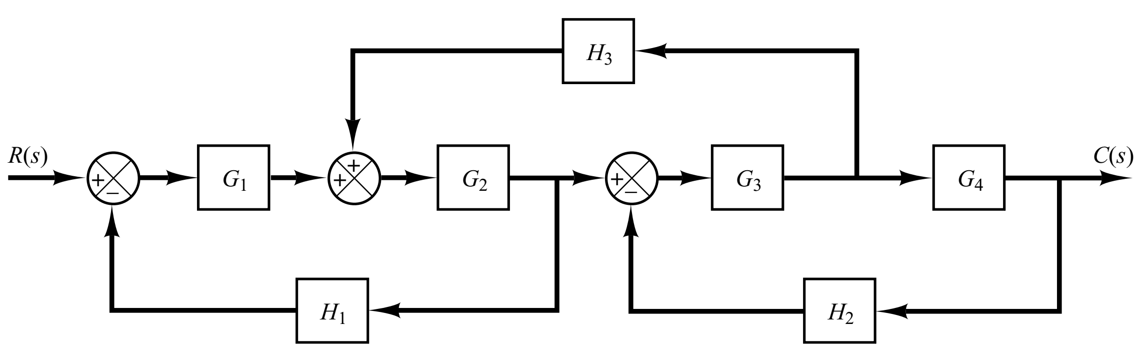

Figure 2–21 Block diagram of a system.

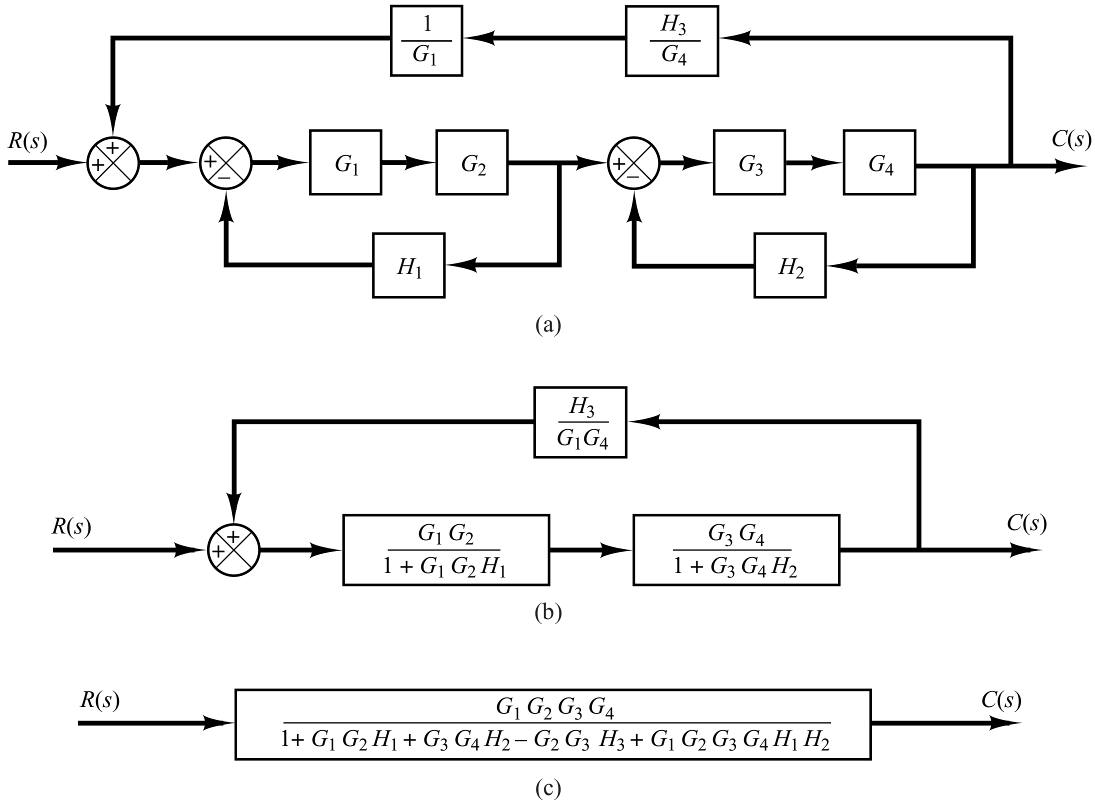

Figure 2–22 Successive reductions of the block diagram shown in Figure 2–21.

Solution. First move the branch point between G_3 and G_4 to the right-hand side of the loop containing G_3, G_4, and H_2. Then move the summing point between G_1 and G_2 to the left-hand side of the first summing point. See Figure 2–22(a). By simplifying each loop, the block diagram can be modified as shown in Figure 2–22(b). Further simplification results in Figure 2–22(c), from which the closed-loop transfer function $C(s)/R(s)$ is obtained as

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4}{1 + G_1G_2H_1 + G_3G_4H_2 - G_2G_3H_3 + G_1G_2G_3G_4H_1H_2}$$

A–2–4. Obtain transfer functions $C(s)/R(s)$ and $C(s)/D(s)$ of the system shown in Figure 2–23.

Solution. From Figure 2–23 we have

$$U(s) = G_f R(s) + G_c E(s) \tag{2–47}$$

$$C(s) = G_p\left[D(s) + G_1U(s)\right] \tag{2–48}$$

$$E(s) = R(s) - HC(s) \tag{2–49}$$

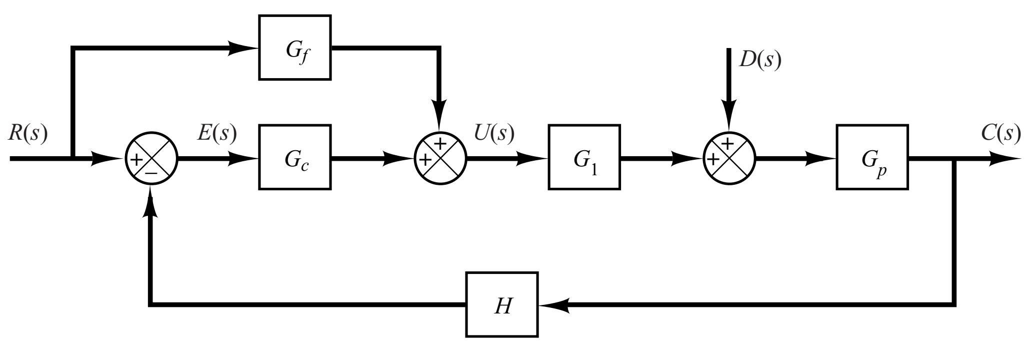

Figure 2–23 Control system with reference input and disturbance input.

By substituting Equation (2–47) into Equation (2–48), we get

$$C(s) = G_p D(s) + G_1 G_p \left[G_f R(s) + G_c E(s)\right] \tag{2–50}$$

By substituting Equation (2–49) into Equation (2–50), we obtain

$$C(s) = G_p D(s) + G_1 G_p \left\{G_f R(s) + G_c\left[R(s) - HC(s)\right]\right\}$$

Solving this last equation for $C(s)$, we get

$$C(s) + G_1 G_p G_c HC(s) = G_p D(s) + G_1 G_p \left(G_f + G_c\right) R(s)$$

Hence

$$C(s) = \frac{G_p D(s) + G_1 G_p \left(G_f + G_c\right) R(s)}{1 + G_1 G_p G_c H} \tag{2–51}$$

Note that Equation (2–51) gives the response $C(s)$ when both reference input $R(s)$ and disturbance input $D(s)$ are present.

To find transfer function $C(s)/R(s)$, we let $D(s) = 0$ in Equation (2–51). Then we obtain

$$\frac{C(s)}{R(s)} = \frac{G_1 G_p \left(G_f + G_c\right)}{1 + G_1 G_p G_c H}$$

Similarly, to obtain transfer function $C(s)/D(s)$, we let $R(s) = 0$ in Equation (2–51). Then $C(s)/D(s)$ can be given by

$$\frac{C(s)}{D(s)} = \frac{G_p}{1 + G_1 G_p G_c H}$$

A–2–5. Figure 2–24 shows a system with two inputs and two outputs. Derive $C_1(s)/R_1(s)$, $C_1(s)/R_2(s)$, $C_2(s)/R_1(s)$, and $C_2(s)/R_2(s)$. (In deriving outputs for $R_1(s)$, assume that $R_2(s)$ is zero, and vice versa.)

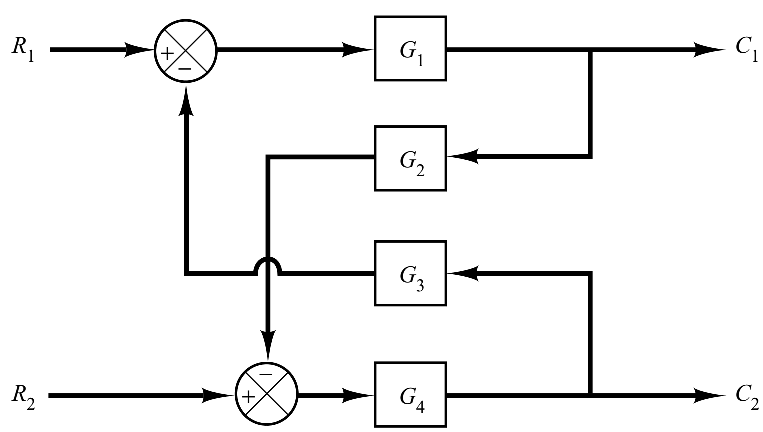

Figure 2–24
System with two inputs and two outputs.