وزارة التعليم العالي والبحث العلمي
جامعة المستقبل

LEC : SEVEN

Course Name : Fundamentals of Electricity Instructor
Name : Zahraa Hazim Obaid
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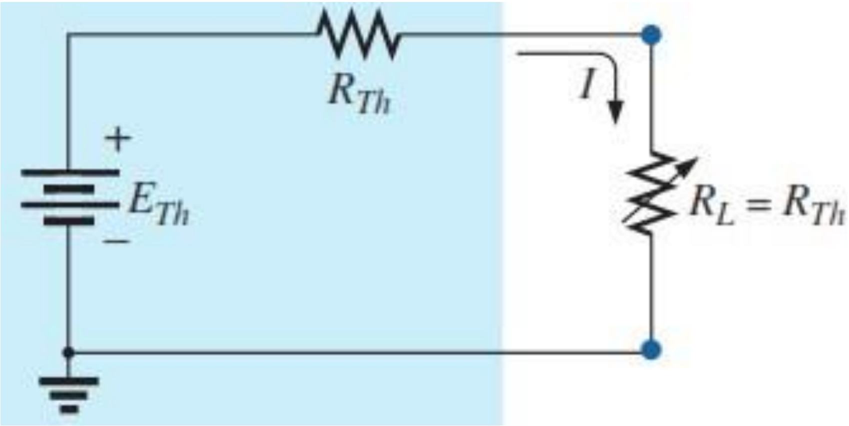
Lecture Title : **MAXIMUM POWER TRANSFER THEOREM**

9.5 MAXIMUM POWER TRANSFER THEOREM

A load will receive maximum power from a network when its resistance is exactly equal to the Thévenin resistance of the network applied to the load. That is,

$$\boxed{R_L = R_{Th}}$$



The maximum power delivered to the load can be determined by first finding the current:

$$I_L = \frac{E_{Th}}{R_{Th} + R_L} = \frac{E_{Th}}{R_{Th} + R_{Th}} = \frac{E_{Th}}{2R_{Th}}$$

Then substitute into the power equation:

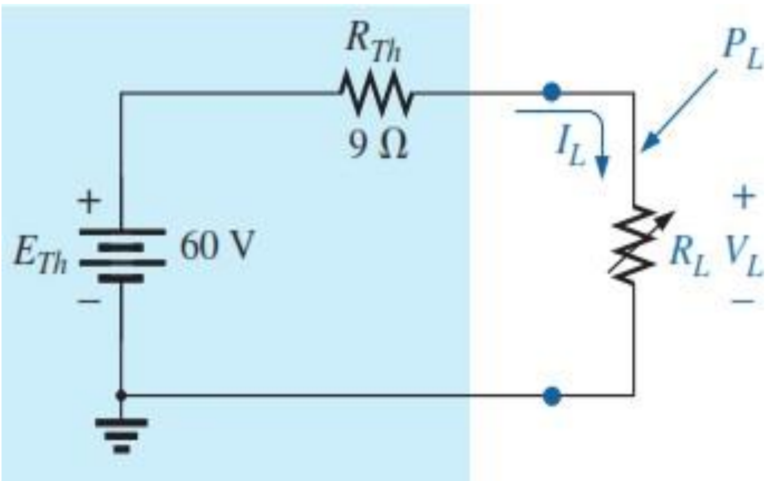
$$P_L = I_L^2 R_L = \left(\frac{E_{Th}}{2R_{Th}}\right)^2 (R_{Th}) = \frac{E_{Th}^2 R_{Th}}{4R_{Th}^2}$$

and

$$\boxed{P_{L_{max}} = \frac{E_{Th}^2}{4R_{Th}}}$$

Maximum power transfer occurs when the load voltage and current are one-half of their maximum possible values.

For the circuit in Fig., the current through the load is determined by



$$I_L = \frac{E_{Th}}{R_{Th} + R_L} = \frac{60\text{ V}}{9\ \Omega + R_L}$$

The voltage is determined by

$$V_L = \frac{R_L E_{Th}}{R_L + R_{Th}} = \frac{R_L(60\text{ V})}{R_L + R_{Th}}$$

and the power by $$P_L = I_L^2 R_L = \left(\frac{60\text{ V}}{9\ \Omega + R_L}\right)^2 (R_L) = \frac{3600 R_L}{(9\ \Omega + R_L)^2}$$

R_L (Ω)	P_L (W)		I_L (A)		V_L (V)	
5	91.84		4.29		21.43	
6	96.00		4.00		24.00	
7	98.44	Increase	3.75	Decrease	26.25	Increase
8	99.65		3.53		28.23	
9 (R_{Th})	100.00 (Maximum)		3.33 ($I_{max}/2$)		30.00 ($E_{Th}/2$)	
10	99.72		3.16		31.58	
11	99.00		3.00		33.00	
12	97.96		2.86		34.29	
13	96.69		2.73		35.46	

Under maximum power conditions, only half the power delivered by the source gets to the load. On an efficiency basis, we are working at only a **50% level**, but we are content because we are getting **maximum power** out of our system.

The dc operating efficiency is defined as the ratio of the power delivered to the load (P_L) to the power delivered by the source (P_s). That is:

$$\boxed{\eta\% = \frac{P_L}{P_s} \times 100\%}$$

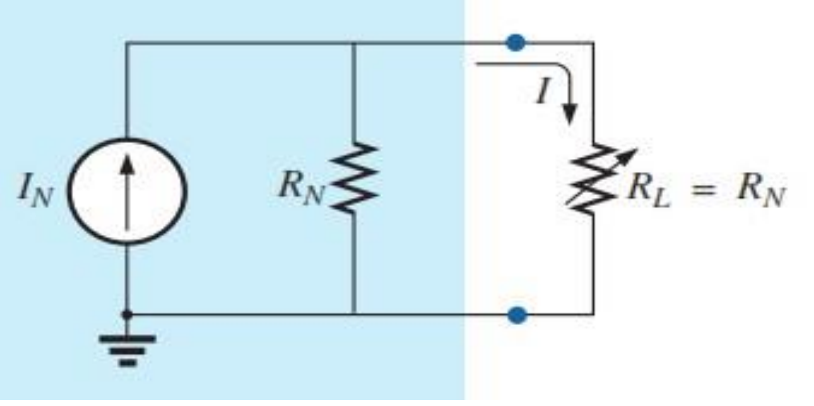
For the situation where $R_L = R_{Th}$,

$$\eta\% = \frac{I_L^2 R_L}{I_L^2 R_T} \times 100\% = \frac{R_L}{R_T} \times 100\% = \frac{R_{Th}}{R_{Th} + R_{Th}} \times 100\%$$

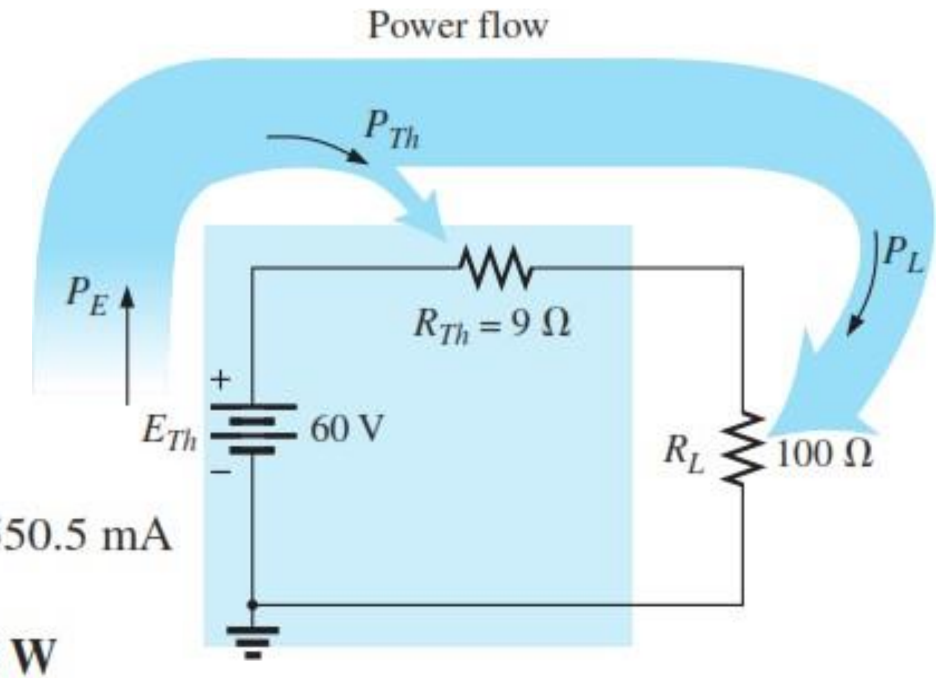
$$= \frac{R_{Th}}{2R_{Th}} \times 100\% = \frac{1}{2} \times 100\% = \mathbf{50\%}$$

$$\boxed{R_L = R_N}$$

$$\boxed{P_{L_{max}} = \frac{I_N^2 R_N}{4}} \quad \text{(W)}$$



Consider the circuit in Fig. where the load resistance is set at 100 Ω and the power to the Thévenin resistance and to the load are calculated as follows:



$$I_L = \frac{E_{Th}}{R_{Th} + R_L} = \frac{60\text{ V}}{9\ \Omega + 100\ \Omega} = \frac{60\text{ V}}{109\ \Omega} = 550.5\text{ mA}$$

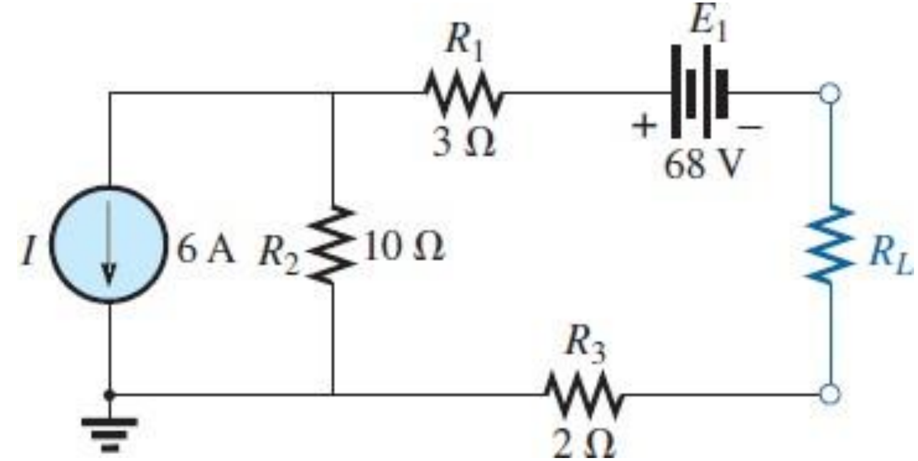
with $P_{R_{Th}} = I_L^2 R_{Th} = (550.5\text{ mA})^2(9\ \Omega) \cong \mathbf{2.73\ W}$

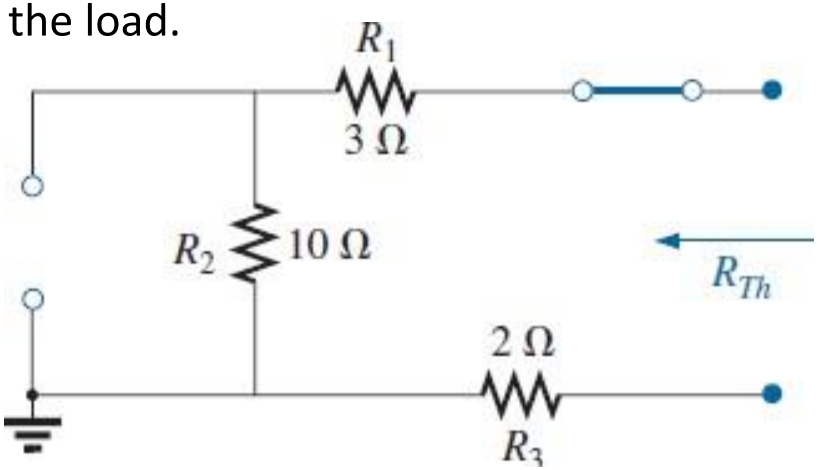
and $P_L = I_L^2 R_L = (550.5\text{ mA})^2(100\ \Omega) \cong \mathbf{30.3\ W}$

The results clearly show that most of the power supplied by the battery is getting to the load—a desirable attribute on an efficiency basis. However, the power getting to the load is only 30.3 W compared to the 100 W obtained under maximum power conditions.

If efficiency is the overriding factor, then the load should be much larger than the internal resistance of the supply. If maximum power transfer is desired and efficiency less of a concern, then the conditions dictated by the maximum power transfer theorem should be applied.

EXAMPLE 9.17 Given the network in Fig., find the value of RL for maximum power to the load, and find the maximum power to the load.





$$R_{Th} = R_1 + R_2 + R_3 = 3\ \Omega + 10\ \Omega + 2\ \Omega = 15\ \Omega$$

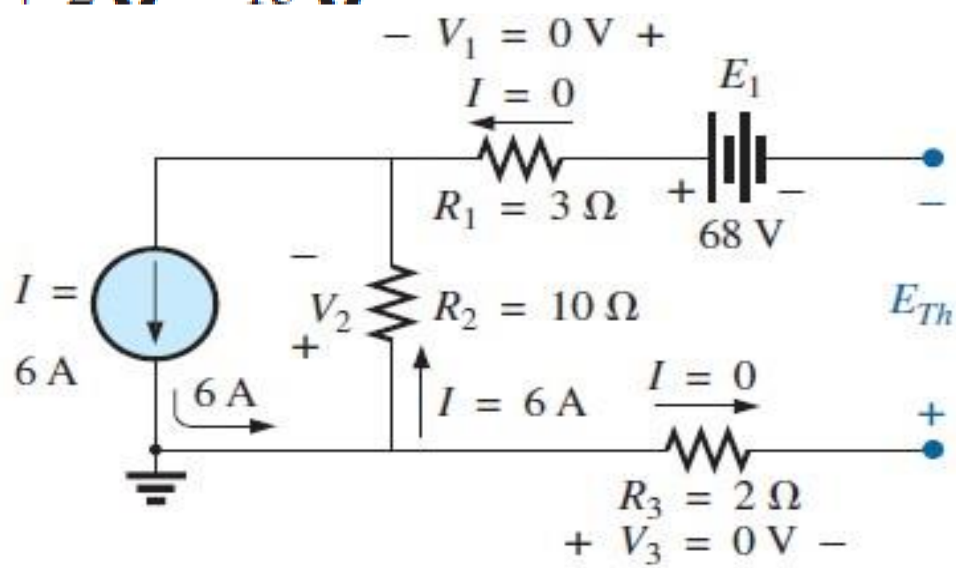
$$R_L = R_{Th} = \mathbf{15\ \Omega}$$

$$V_1 = V_3 = 0\ \text{V}$$

$$V_2 = I_2R_2 = IR_2 = (6\ \text{A})(10\ \Omega) = 60\ \text{V}$$

$$E_{Th} = V_2 + E = 60\ \text{V} + 68\ \text{V} = \mathbf{128\ V}$$

$$P_{L_{\max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(128\ \text{V})^2}{4(15\ \Omega)} = \mathbf{273.07\ W}$$



PROBLEMS

SECTION 9.2 Superposition Theorem: 1, 2, 3, 5

SECTION 9.3 Thévenin's Theorem: 8, 9, 10, 11, 12, 16

SECTION 9.4 Norton's Theorem: 20, 21, 23

SECTION 9.5 Maximum Power Transfer Theorem: 24, 26, 27, 28, 30