COLLEGE OF ENGINEERING AND TECHNOLOGIES

ALMUSTAQBAL UNIVERSITY

Digital Signal Processing (DSP)

CTE 306

Lecture 8

- Periodic & Aperiodic Signals -

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Classification of Signals

One of the most important classify of a signals are:

- Periodic signals.
- Non periodic signals.

Revision on Numbers

- Real numbers are the numbers that we normally use and apply in real-world applications.

- Real Numbers include:

❖ Whole Numbers (like 0, 1, 2, 3, 4, etc)

❖ Rational Numbers (like 3/4, 0.125, 0.333, 1.1, etc)

❖ Irrational Numbers (like π, √2, etc)

- Real Numbers can also be positive, negative or Zero.

Periodic Signals – Continuous time

- A continuous-time signal, $x(t)$ is a periodic signal if $x(t + nT) = x(t)$, where T is the period of the signal and n is an integer.

- Sinusoidal, square and triangular waves are periodic signals.

- For $x(t) = x_1(t) + x_2(t)$, where $x_1(t)$ and $x_2(t)$ are two periodic signals with fundamental T_1 and T_2 respectively, $x(t)$ is a periodic signal if T_1/T_2 = a rational number.

- The fundamental period, T for $x(t)$ is the least common multiples (LCM) of T_1 and T_2.

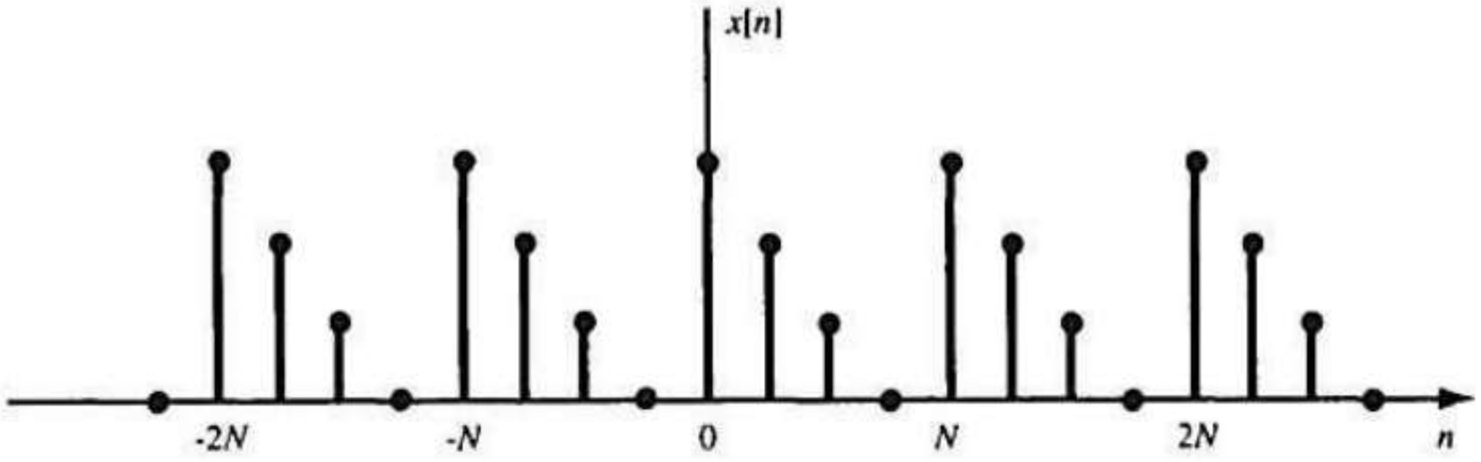
Least Common Multiples (LCM)

- The smallest positive number that is a multiple of two or more numbers.
- The multiples of 4 are: 4, 8, 12, 16, 20, 24, 28, 32,36, 40, 44,...
- The multiples of 5 are: 5, 10, 15, 20, 25, 30, 35, 40,45, 50, ...
- So, the common multiples of 4 and 5 are: 20, 40, (and 60, 80, etc ..., too).
- The smallest of the common multiples is 20, so the least common multiple (LCM) of 4 and 5 is 20.

Periodic Signals – Discrete time

A discrete-time signal may always be classified as either being periodic or aperiodic. A signal x(n) is said to be periodic if, for some positive real integer N,

$x(n) = x(n + N)$ for all n.



Periodic Signals – Discrete time

This is equivalent to saying that the sequence repeats itself every N samples. If is not satisfied for any integer N, x(n) is said to be an aperiodic signal.

The sinusoidal signal of the form

$x(n) = A sin 2\pi f o n$

Is periodic when $f0$ is a rational number, that is, if $f0$ can be expressed as

$$f_0 = \frac{k}{N}$$

Where k and N are integers

Example

Determine whether each of the following signal is periodic. If a signal is periodic, determine its fundamental period.

$$x(t) = \cos(t + \pi/4)$$

Sol:

$x(t) = \cos(t + \pi/4)$ is in the form

$A \cos(2\pi f_0 t)$

Where f_0 is the fundamental frequency.

In this case, $f_0 = 1/(2\pi)$.

Therefore, the fundamental frequency, $T_0 = 1/f_0 = 2\pi$

Example

Determine whether or not the following signals are periodic. In case a signal is periodic, specify the fundamental frequency.

$$x(t) = 8 \sin(0.8\pi t + \pi/4) + 5 \cos(0.6\pi t + \pi/6)$$

$W_1 = 0.8\pi \Rightarrow T_1 = 2\pi / W_1 \Rightarrow 2\pi / 0.8 p \Rightarrow 5/2$

$W_2 = 0.8\pi \Rightarrow T_2 = 2\pi / W_2 \Rightarrow 2\pi / 0.6\pi \Rightarrow 10/3$

$T_1 / T_2 = 5/2 * 3/10 = 3/4 \Rightarrow$ Rational Number $\Rightarrow$ Periodic.

$3T_2 = 4T_1 = T$

$T = 3T_2 \Rightarrow 3 * 10/3 \Rightarrow 10$ Sec.

$T = 4T_1 \Rightarrow 4 * 5/2 \Rightarrow 10$ Sec.

The fundamental frequency, T_0 is the least common multiples (LCM) which is 10 seconds.

Example

Determine whether or not the following signals are periodic. In case a signal is periodic, specify the fundamental frequency.

$$x(t) = \cos(\pi/3)t + \sin(\pi/4)t$$

Sol:

This is the sum of two functions that are both periodic.

Their fundamental periods are $T_1 = 6$ seconds and $T_2 = 8$ seconds respectively.

T1/T2 = 6/8 is a rational number.

Therefore x(t) is a periodic signal.

The fundamental frequency, T_0 is the least common multiples (LCM) which is 24 seconds.

Example

Is $x(n) = \cos(\frac{\pi n}{8})$ *periodic? If so, what is the period?*

Sol: The sequence can be expressed as

$$x(n) = \cos(2\pi \frac{1}{16} n)$$

So in this case, fo = 1/16 is a rational number and the sinusoidal sequence is periodic with a period N = 16.

Example

Determine whether or not each of the following signals is periodic. If a signal is periodic, determine its fundamental period.

$$x[n] = e^{j(\pi/4)n}$$

Sol:

$$x[n] = e^{j(\frac{\pi}{4})n} = e^{j2\pi f_0 n} \quad \rightarrow \quad 2\pi f_0 = \frac{\pi}{4}$$

Since $f_0 = 1/8$ is a rational number, x[n] is periodic, the fundamental period is N=8

Example

Determine whether or not each of the following signals is periodic. If a signal is periodic, determine its fundamental period.

$$x[n] = \cos\frac{\pi}{3}n + \sin\frac{\pi}{4}n$$

Sol:

$$x_1[n] = cos\frac{\pi}{3}n \quad \rightarrow \quad 2\pi f_0 = \frac{\pi}{3}$$

$$x_2[n] = cos\frac{\pi}{4}n \quad \rightarrow \quad 2\pi f_0 = \frac{\pi}{4}$$

Since $f1 = 1/6$ is a rational number, $x1$[n] is periodic, the fundamental period is N1=6, and Since $f2 = 1/8$ is a rational number,

Example

x[n] is periodic, the fundamental period is N=8. Thus, x[n] is periodic and its fundamental period is given by the least common multiple of 6 and 8, that is N=24.

Determine whether or not each of the following signals is periodic. If a signal is periodic, determine its fundamental period.

$$x[n] = \cos \tfrac{1}{4}n$$

Sol:

$$x[n] = \cos \frac{1}{4} n \qquad \rightarrow \qquad 2\pi f_0 = \frac{1}{4}$$

Since $fo = 1/8\pi$ is not a rational number, x[n] is aperiodic.