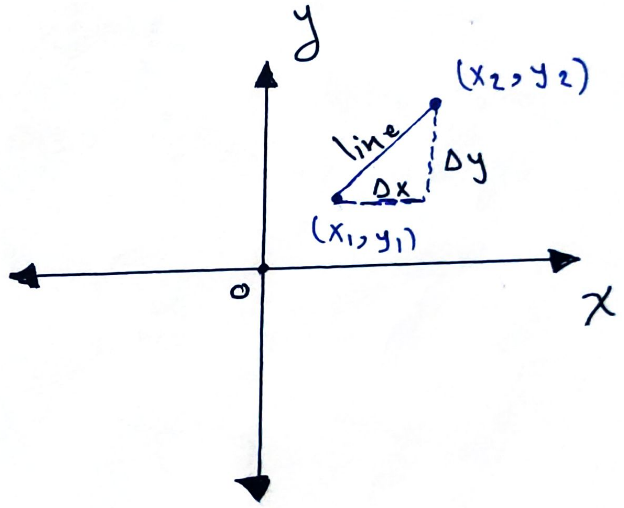
Lecture Three :

Slope of the straight line

Increments (net change) : when a particle moves from p to Q, the increments Δx & Δy are

$\Delta x = x_2 - x_1$, $\Delta y = y_2 - y_1$.

The slope of a line $= m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$



Ex: Find the increments if a particle moves from (4,-3) to (2,5) ?

Sol:

$\Delta x = x_2 - x_1 = 2 - 4 = -2$

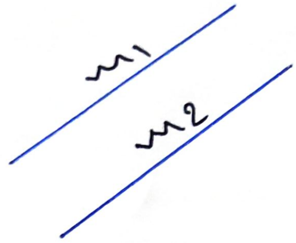
$\Delta y = y_2 - y_1 = 5 + 3 = 8$

* The slope of horizontal line is Zero

Since $m = \frac{\Delta y}{\Delta x} = \frac{0}{\Delta x} = 0$.

* The slope of parallel lines are equal

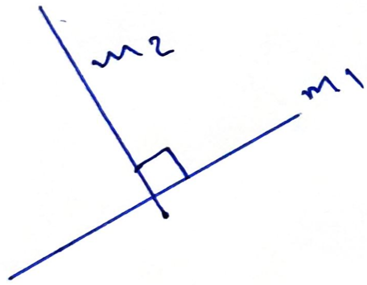
$m_1 = m_2$



* The slope of perpendicular lines are

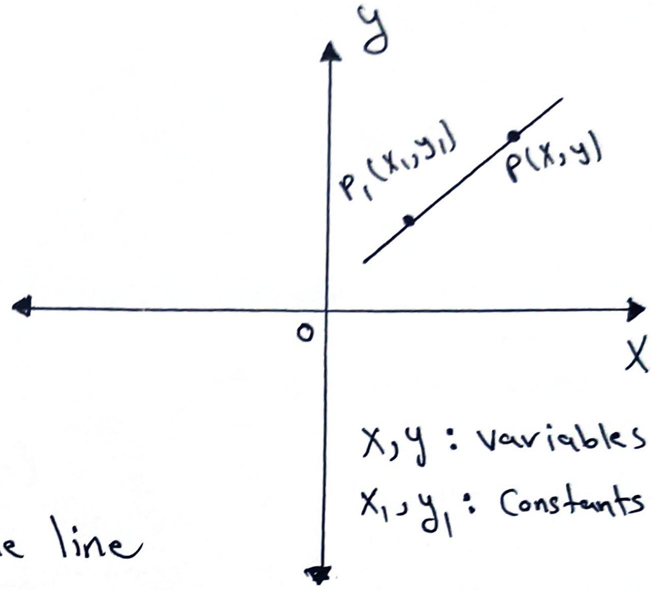
$m_1 * m_2 = -1$. or, $m_1 = -1/m_2$

for example if $m_1 = -3 \Rightarrow m_2 = \frac{1}{3}$.



Point - slope equation :-

To find the equation for The line L with given slope m and passes through the point $P_1(x_1, y_1)$



$$m = \frac{\Delta y}{\Delta x} \Rightarrow m = \frac{y - y_1}{x - x_1}$$

$$\Rightarrow y - y_1 = m(x - x_1)$$

x, y : variables
x_1, y_1 : Constants

Ex 1 :- Write the equation for the line that passes through the point $(-2, 3)$ with slope 4 ?

Sol:

$$y - y_1 = m(x - x_1)$$

$$y - 3 = 4(x - (-2))$$

$$y - 3 = 4(x + 2)$$

$$y - 3 = 4x + 8 \Rightarrow y = 4x + 11$$

Ex 2 :- Write the equation for the line passes through the points $(-2, -1)$ & $(3, 4)$?

Sol:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-1)}{3 - (-2)} = \frac{5}{5} = 1$$

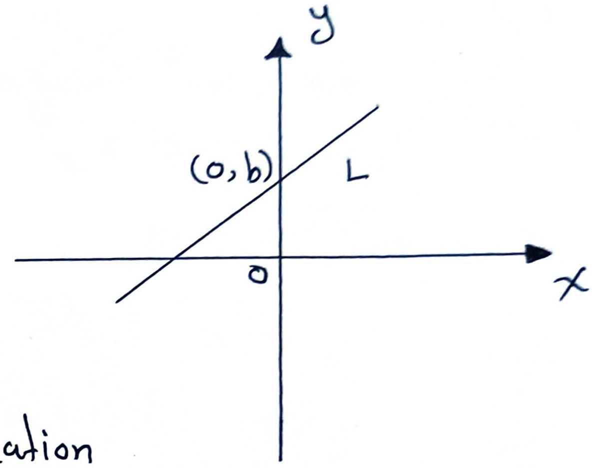
take the point $(-2,-1)$

$y - y_1 = m(x - x_1)$

$y - (-1) = 1\,(x - (-2)) \Rightarrow y + 1 = x + 2 \Rightarrow y = x + 1$

Slope - intercept equation :

To find the equation for the line L with given slope m and given y-intercept (b)



$y - y_1 = m(x - x_1) \Rightarrow y - b = m(x - 0)$

$y = mx + b \Rightarrow$ Slope - intercept equation

Ex : find the equation for the line with slope -2 and y-intercept equals 3 ?

Sol :

$y = mx + b$ $\qquad m = -2$

$y = -2x + 3$ $\qquad b = 3$

Ex : find the equation for the line with slope $-1/2$ and y-intercept equals -3 ?

Sol :

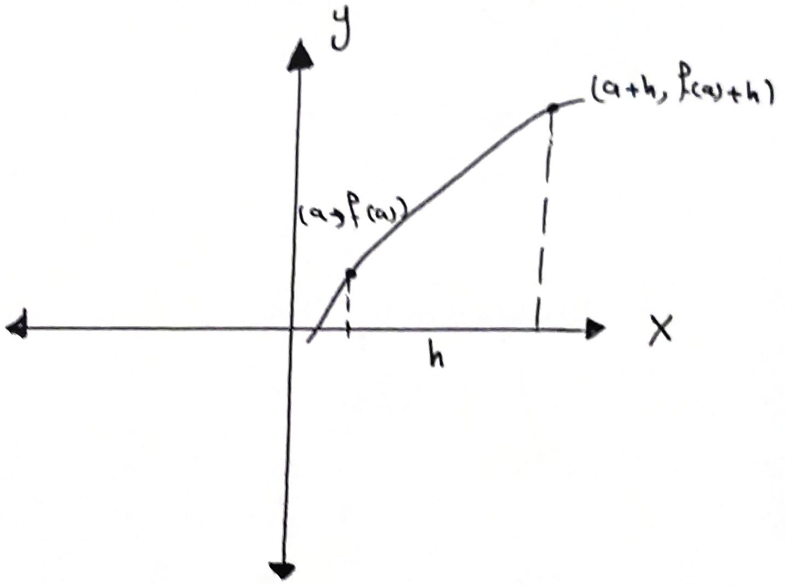
$y = mx + b$

$y = -\frac{1}{2}x + (-3) \Rightarrow y = -1/2\,x - 3$

Slope of the Curve:

The slope of a curve $y = f(x)$ at the point $(a, f(a))$ on it is defined to be the number:

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$



Ex: let's use this limit to check our earlier experimental result that the slope of the curve $y = x^2$ at the point $(1,1)$ is 2. Here $a = 1$ and $f(x) = x^2$

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{(1+h)^2 - 1^2}{h}$$

$$= \lim_{h \to 0} \frac{2h + h^2}{h}$$

$$= \lim_{h \to 0} (2+h)$$

$$= 2$$

Ex: we find the slope of the curve $y=\sqrt{x}$ at the point $(4,2)$. Here $a=4$ and $f(x)=\sqrt{x}$

$$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{\sqrt{4+h}-\sqrt{4}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{4+h}-2}{h} * \frac{\sqrt{4+h}+2}{\sqrt{4+h}+2}$$

$$= \lim_{h\to 0} \frac{(4+h)-4}{h(\sqrt{4+h}+2)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{4+h}+2} = \frac{1}{\sqrt{4}+2}$$

$$= \frac{1}{4}$$

Ex: let's use the definition to calculate the derivative of $f(x)=(x+3)^2$ at $x=0$

$$\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

$$= \lim_{h\to 0} \frac{f(0+h)-f(0)}{h}$$

$$= \lim_{h\to 0} \frac{(0+h+3)^2-(0+3)^2}{h} = \lim_{h\to 0} \frac{(h^2+6h+9)-9}{h}$$

$$= \lim_{h\to 0} (h+6) = 6$$