Lecture 7: Simple Harmonic Motion

Al-Mustaqbal University
College of Science
Department of Medical Physics

Lecturers
Dr. Anees Ali
Dr. Ahed Hameed

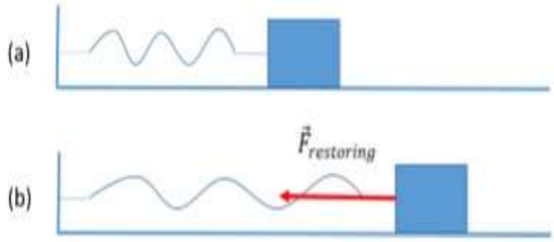
SIMPLE HARMONIC MOTION

- **Simple harmonic motion** (SHM) is a type of periodic motion. Two simple systems of SHM that are mainly discussed in college are **an ideal spring** and **a simple pendulum**. But before discussing these 2 systems, it is essential to go over periodic motion first.

- **Periodic motion** or **oscillation** refers to kinds of motion that repeat themselves over and over. For example, a clock pendulum.

- Understanding periodic motion is critical for understanding more complicated concepts like mechanical waves (e.g. sound) and electromagnetic waves (e.g. light)

- Oscillation is characterized by an **equilibrium** and a **restoring force**. At equilibrium, restoring force on the object is zero. When the object is displaced from equilibrium, a restoring force acts on the object to restore its equilibrium. For example, a spring.



(a) The object is at equilibrium. The spring is neither stretched nor compressed.

(b) The object is displaced and the spring is stretched.

- When the restoring force is directly proportional to displacement from equilibrium, the oscillation is called **simple harmonic motion** (SHM).
- Important characteristics of any periodic motion:
    - **Amplitude** (A) is maximum magnitude of displacement from equilibrium
    - **Period** (T) is the time to complete one cycle (unit: s)
    - **Frequency** (f) is the number of cycles in a unit of time (unit: s^{-1})
    - Period and frequency are related by the following relationship:

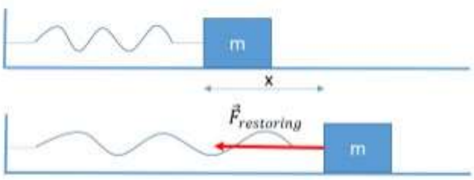
$$T = \frac{1}{f} \text{ or } f = \frac{1}{T}$$

    - **Angular frequency** (ω):

$$\omega = 2\pi f = \frac{2\pi}{T} \text{(rad/s)}$$

A/ Ideal spring

$F_{restoring} = -kx$



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k: spring constant

x: displacement from equilibrium

$$\omega = \sqrt{\frac{k}{m}}; f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}; T = \frac{1}{f} = \frac{f\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

$$x = A\cos(\omega t + \Phi)$$

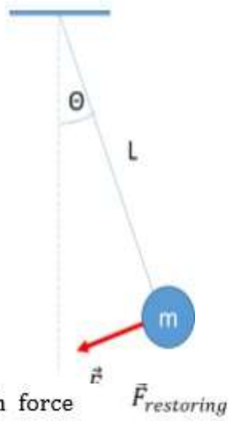
Φ: initial angular displacement

$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = const$$

E: mechanical energy of the system

v_x: velocity of mass m at x (m/s)

B/ Simple pendulum



$F_{restoring} = -mgsin\theta \cong -mg\theta$ (when θ is small)

Θ: angular displacement form equilibrium

$$\omega = \sqrt{\frac{g}{L}}; f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}; T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

L: string length (m)

C/ Examples

1/ When a body of unknown mass is attached to an ideal spring with force constant 120 N/m, it is found to vibrate with a frequency of 6.00 Hz. Find

(a) The period of the motion;

(b) The angular frequency;

(c) The mass of the body.

Solution:

k = 120 N/m; f = 6.00 Hz

(a) $T = \frac{1}{f} = \frac{1}{6.00} = 0.167\ s$

(b) $\omega = 2\pi f = 2\pi \times 6.00 = 37.7$ (rad/s)

(c) $\omega = \sqrt{\frac{k}{m}}$ or $m = \frac{k}{\omega^2} = \frac{120}{37.7^2} = 0.0845\ kg = 84.5\ g$

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Q/An ideal spring with force has the period of the motion (*T*) is ---------.

a) T= f

b) **T= 1/f**

c) T= 1/2f

d) $T=1/f^2$

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Q/An ideal spring with force has the angular frequency (*T*) is ---------.

a) $\boldsymbol{\omega = 2\pi f}$

b) $\omega = 2\pi f^2$

c) $\omega = \pi f$

d) $\omega = \pi^2 f$

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Q/An ideal spring with force has the mass of the body is ---------.

a) $m = k^2\omega^2$

b) $m = k\,\omega^2$

c) $\boldsymbol{m = \frac{k}{\omega^2}}$

d) $m = k\omega$

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SIMPLE HARMONIC MOTION

2/ A building in San Francisco has light fixtures consisting of small 2.35-kg bulbs with shades hanging from the ceiling at the end of light, thin cords 1.50 m long. If a minor earth quake occurs, how many swings per second will these fixtures make?

Solution:

m = 2.35 kg; L = 1.50 m

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.8}{1.50}} = 0.407\ swings/s$$

D/ Practice problems (Answer key is below)

1/ A 1.50-kg mass on a spring has displacement as a function of time given by the equation

$$x(t) = (7.40cm)\cos[(4.16\ s^{-1})t - 2.42]$$

Find

(a) The time for one complete vibration;

(b) The force constant of the spring;

(c) The maximum speed of the mass;

(d) The maximum force on the mass;

(e) The position, speed and acceleration of the mass at t = 1.00 s;

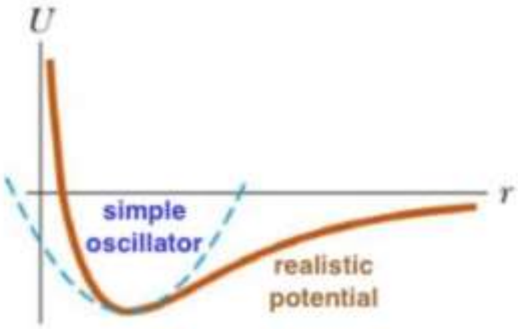
(f) The force on the mass at that time

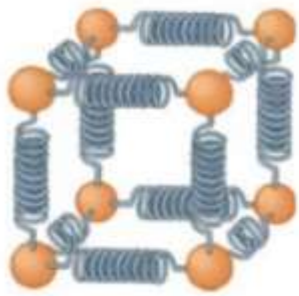
(g) The mechanical energy of the system

2/ After landing on an unfamiliar planet, a space explorer constructs a simple pendulum of length 50.0 cm. She finds that the pendulum makes 100 complete swings in 136 s. What is the value of g on this planet?

Simple Harmonic Oscillators applied to solids

- **Simple harmonic oscillators are good models of a wide variety of physical phenomena**
- **Molecular example**
  - **If the atoms in the molecule do not move too far, the forces between them can be modeled as if there were springs between the atoms**
  - **The potential energy acts similar to that of the SHM oscillator**





Some concepts for oscillations

restoring force:	A force causes the system to return to some equilibrium state periodically and repeat the motion
natural frequency:	Resonant oscillation period, determined by physics of the system alone. Disturb system to start, then let it go. Examples: pendulum clock, violin string
undamped oscillations:	Idealized case, no energy lost, motion persists forever Example: orbit of electrons in atoms and molecules
damped oscillations:	Oscillation dies away due to loss of energy, converted to heat or another form. Example: a swing eventually stops
simple harmonic oscillation:	Undamped natural oscillation with $F = -kx$ (Hooke's Law); i.e. restoring force is proportional to the displacement away from the equilibrium state
forced oscillations:	External periodic force drives the system motion at it's own frequency/period, may not be the resonant frequency

Motion Equations for Simple Harmonic Motion

$$x(t) = A\cos(\omega t + \phi)$$

$\boxed{A = x_{max}}$

$$v = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi)$$

$$a = \frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \phi)$$

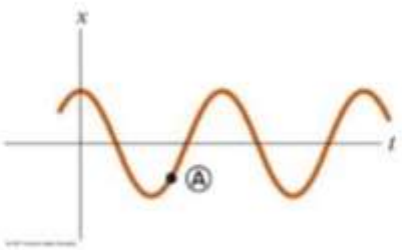
- Simple harmonic motion is one-dimensional - directions can be denoted by + or - sign
- Simple harmonic motion is not uniformly accelerated motion
- The sine and cosine functions oscillate between ±1. The maximum values of velocity and acceleration for an object in SHM are:

$$\boxed{\begin{aligned} v_{max} &= \omega A = \sqrt{\frac{k}{m}}\,A \\ a_{max} &= \omega^2 A = \frac{k}{m}\,A \end{aligned}}$$

Position and velocity of an oscillator

- 11.2. The figure shows the displacement of a harmonic oscillator versus time. When the motion has progressed to point A on the graph, which of the following correctly describe the position and velocity?

A) The position and velocity are both positive
B) The position and velocity are both negative
C) The position is negative, the velocity is zero
D) The position is positive, the velocity is negative
E) The position is negative, the velocity is positive



$$x(t) = x_m\cos(\omega t + \varphi)$$

$$v_x(t) \equiv \frac{dx(t)}{dt} = -\omega x_m \sin(\omega t + \varphi)$$

Example: Spring Oscillator in natural, undamped oscillation

Let k = 65 N/m, $x_m = 0.11$ m at t = 0, m = 0.68 kg

a) Find ω, f, T

$$\omega = \sqrt{k/m} = \sqrt{65/0.68} = 9.78 \text{ rad/s}$$

$$f = \omega/2\pi = 1.56 \text{ Hz} \qquad T = 1/f = 0.64 \text{ s} = 640 \text{ ms}$$

$$\boxed{\begin{aligned} x(t) &= x_m \cos(\omega t + \varphi) \\ v_x(t) &= -\omega x_m \sin(\omega t + \varphi) \\ a_x(t) &= -\omega^2 x_m \cos(\omega t + \varphi) \end{aligned}}$$

b) Find the amplitude of the oscillations

$$x_m = 0.11 \text{ m}$$

c) Find the maximum speed and when it is reached

$$v_m = \text{amplitude of } v(t) = \omega x_m = 9.78 \times 0.11 = 1.1 \text{ m/s} \quad \text{at} \quad t = T/4, 3T/4, \text{etc}$$

d) Find the maximum acceleration and when it is reached

$$a_m = \text{amplitude of } a(t) = \omega^2 x_m = (9.78)^2 \times 0.11 = 11 \text{ m/s}^2 \quad \text{at} \quad t = 0, T/2, \text{same as } x(t)$$

e) Find phase constant

Match initial conditions at $t = 0$: $x(t) = x_m = x_m \cos(\varphi) \Rightarrow \cos(\varphi) = 1$

$\therefore \varphi = 0, +/- 2\pi$, etc

f) The formula for the motion is:

$$x(t) = 0.11 \times \cos(9.78t + 0)$$

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Q/Spring oscillator in natural, undamped oscillation has the formula for the motion is ---------.

a) x(t) = $x_m \cot(\omega t + \emptyset)$

b) x(t) = $x_m \tan(\omega t + \emptyset)$

c) x(t) = $x_m \sin(\omega t + \emptyset)$

d) $\boldsymbol{x(t) = x_m \cos(\omega t + \emptyset)}$

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