Lecture Four :

Derivatives :-

- $f(x) = k \Rightarrow f'(x) = 0$

$f(x) = 5 \Rightarrow f'(x) = 0$

- $f(x) = x^n \Rightarrow f'(x) = n x^{n-1}$

$f(x) = x^6 \Rightarrow f'(x) = 6x^5$

- $f(x) = k x^n \Rightarrow f'(x) = k * n x^{n-1}$

$f(x) = 5x^3 + 5 \Rightarrow f'(x) = 5 * 3 x^2 + 0 = 15 x^2$

- $f(x) = h(x)\, g(x) \Rightarrow f'(x) = h(x)\, g'(x) + g(x)\, h'(x)$

$f(x) = (3x^2 - 1)(2x^3 + 3) \Rightarrow f'(x) = (3x^2 - 1)(6x^2 + 0) + (2x^3 + 3)(6x)$

$= 18x^4 - 6x^2 + 12x^4 + 18x$

$= 30x^4 - 6x^2 + 18x$

- $f(x) = \dfrac{g(x)}{h(x)} \Rightarrow \dfrac{h(x)\, g'(x) - g(x) \cdot h'(x)}{(h(x))^2} = f'(x)$

$f(x) = \dfrac{3x^2 + x - 2}{x^2 + 1} \Rightarrow f'(x) = \dfrac{(x^2+1)(6x+1-0) - (3x^2+x-2)(2x+0)}{(x^2+1)^2}$

$$= \frac{(x^2+1)(6x+1)-(3x^2+x-2)(2x)}{(x^2+1)^2}$$

$$= \frac{6x^3+x^2+6x+1-6x^3-2x^2+4x}{(x^2+1)^2}$$

$$= \frac{-x^2+10x+1}{(x^2+1)^2}$$

- $f(x) = (h(x))^n \Rightarrow f'(x) = n\,(h(x))^{n-1}\,(h'(x))$

$$f(x) = (x^3-5)^2 \Rightarrow f'(x) = 2\,(x^3-5)\,(3x^2)$$

- $f(x) = K^{g(x)} \Rightarrow f'(x) = K^{g(x)}\; g'(x)\; \ln K$

$$f(x) = 5^{(2x^3+1)} \Rightarrow f'(x) = 5^{2x^3+1}\,(6x^2+0)\cdot \ln 5$$

$$= 5^{2x^3+1}\,(6x^2)\;\ln 5$$

- $f(x) = e^{g(x)} \Rightarrow f'(x) = e^{g(x)} \cdot g'(x)$

$$f(x) = e^{-3x} \Rightarrow f'(x) = e^{-3x}\,(-3) = -3e^{-3x}$$

- $f(x) = \ln(g(x)) \Rightarrow f'(x) = \dfrac{g'(x)}{g(x)}$

$$f(x) = \ln(5x^2+3x) \Rightarrow f'(x) = \frac{10x+3}{5x^2+3x}$$

Differentiation of Trigonometric Functions:

- $f(x) = \sin x \Rightarrow f'(x) = \cos x$

  $f(x) = \sin 5x \Rightarrow f'(x) = \cos 5x * 5 = 5 \cos 5x$

- $f(x) = \cos x \Rightarrow f'(x) = -\sin x$

  $f(x) = \cos(3x^2) \Rightarrow f'(x) = -\sin(3x^2) * 6x = -6x \sin(3x^2)$

- $f(x) = \tan x \Rightarrow f'(x) = \sec^2 x$

  $f(x) = \tan x \Rightarrow f'(x) = \sec^2 x * 1 = \sec^2 x$

- $f(x) = \cot x \Rightarrow f'(x) = -\csc^2 x$

  $f(x) = \cot x^2 \Rightarrow f'(x) = -\csc^2 x^2 * 2x = -2x \csc^2 x^2$

- $f(x) = \sec x \Rightarrow f'(x) = \tan x \sec x$

  $f(x) = \sec 5x \Rightarrow f'(x) = \sec 5x \tan 5x \cdot 5 = 5 \sec 5x \tan 5x$

- $f(x) = \csc x \Rightarrow f'(x) = -\cot x \csc x$

  $f(x) = \csc 3x^2 \Rightarrow f'(x) = -\cot(3x^2) \csc(3x^2) \cdot 6x$

  $= -6x \cot(3x^2) \csc(3x^2)$

Partial derivatives of functions with two variables :-

Ex : Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ if $f(x,y) = 2 - x^2 - 3y^4 - \frac{x^3}{y}$

Sol:

$$\frac{\partial f}{\partial x} = 0 - 2x - 0 - \frac{3x^2}{y}$$

$$= -2x - \frac{3x^2}{y}$$

$$\frac{\partial f}{\partial y} = 0 - 0 - 12y^3 - \frac{x^3}{y^2}$$

Ex : if $z = e^{2x} \ln(x^2 + 2y^2 - x \sin y + 1)$ find $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$?

Sol:

$$\frac{\partial z}{\partial x} = e^{2x} * \frac{2x + 0 - [x * 0 + \sin y * 1] + 0}{(x^2 + 2y^2 - x \sin y + 1)} + \ln(x^2 + 2y^2 - x \sin y + 1) * 2e^{2x}$$

$$= e^{2x} \frac{2x - \sin y}{(x^2 + 2y^2 - x \sin y + 1)} + \ln(x^2 + 2y^2 - x \sin y + 1) + 2e^{2x}$$

$$\frac{\partial z}{\partial y} = e^{2x} * \frac{0 + 4y - [x \cdot \cos y\,(1) + \sin y \cdot 0] + 0}{(x^2 + 2y^2 - x \sin y + 1)} + \ln(x^2 + 2y^2 - x \sin y + 1) * 0$$

$$= e^{2x} * \frac{4y - x \cos y}{(x^2 + 2y^2 - x \sin y + 1)}$$

partial derivatives of functions with more than two variables :-

$$W = f(X_1, X_2, X_3, ---, X_n)$$

$$f_x :, \frac{\partial f}{\partial X}, \frac{\partial w}{\partial X}$$

Ex : let $f(x,y,z) = X Z^2 \tan^{-1}\left(\frac{y}{x}\right)$ find f_x, f_y, f_z ?

Sol :

$$f_x = Z^2 \left[X \tan^{-1} \frac{y}{X} \right]$$

$$= Z^2 \left[X \frac{y/x^2}{1+(y/x)^2} + \tan^{-1} \frac{y}{X} * 1 \right]$$

$$f_y = Z^2 \left[x * \frac{1/x}{1+(y/x)^2} + \tan^{-1} \frac{y}{X} * 0 \right]$$

$$= Z^2 * x * \frac{1/x}{1+\left(\frac{y}{X}\right)^2}$$

$$f_z = X \tan^{-1} \frac{y}{X} \, (2Z)$$