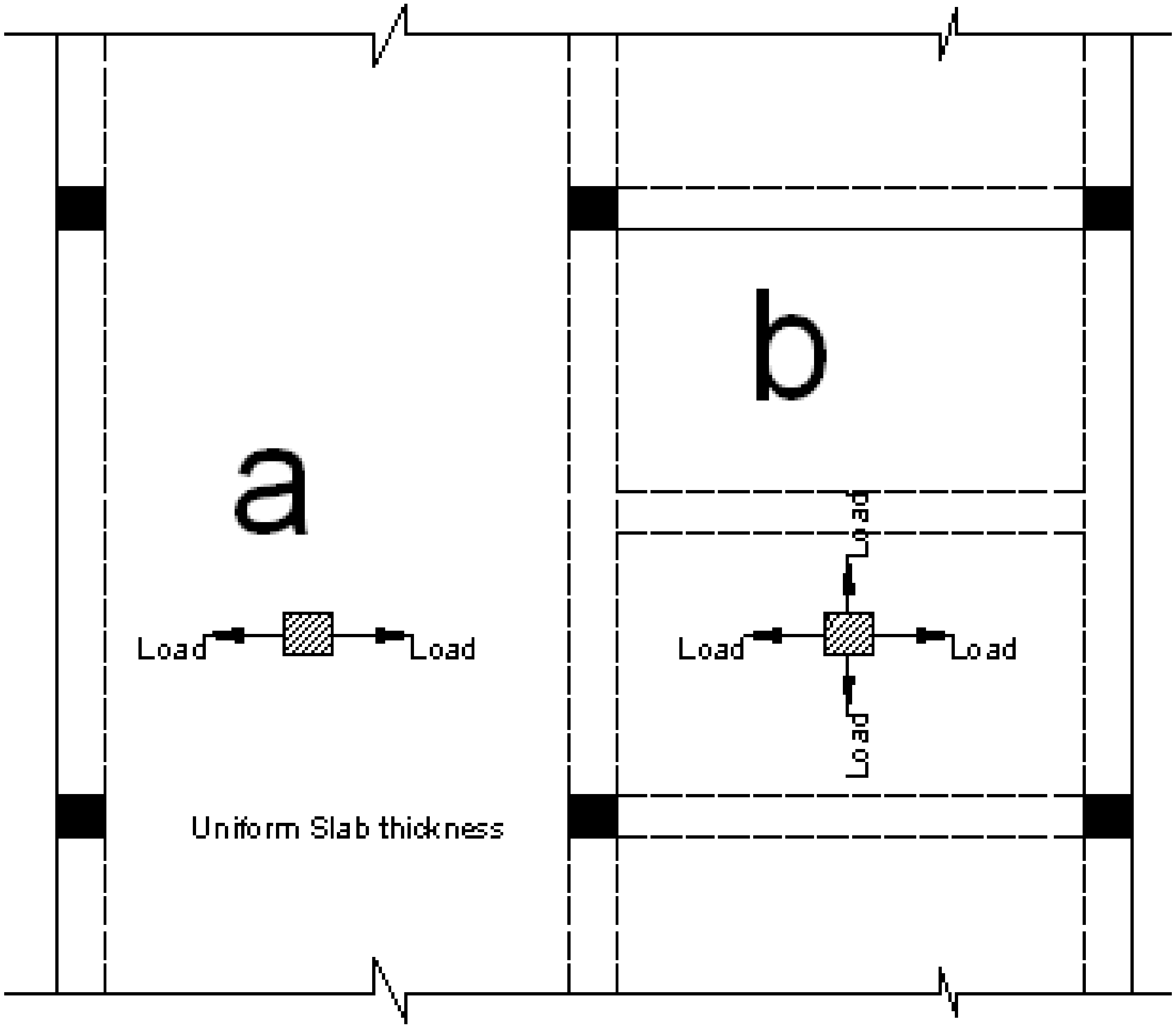
Reinforced Concrete Slabs

Type Of slabs

a. One-way solid slab
b. Two way solid slab
c. One-way ribbed slab (Joist slab)
d. Two way ribbed slab (Waffle or grid slab)
e. Flat plates
f. Flat slab
g. Ground slab



(a) $\frac{Long\ S}{S\quad ort\ S} \geq 2.0$ One way slab

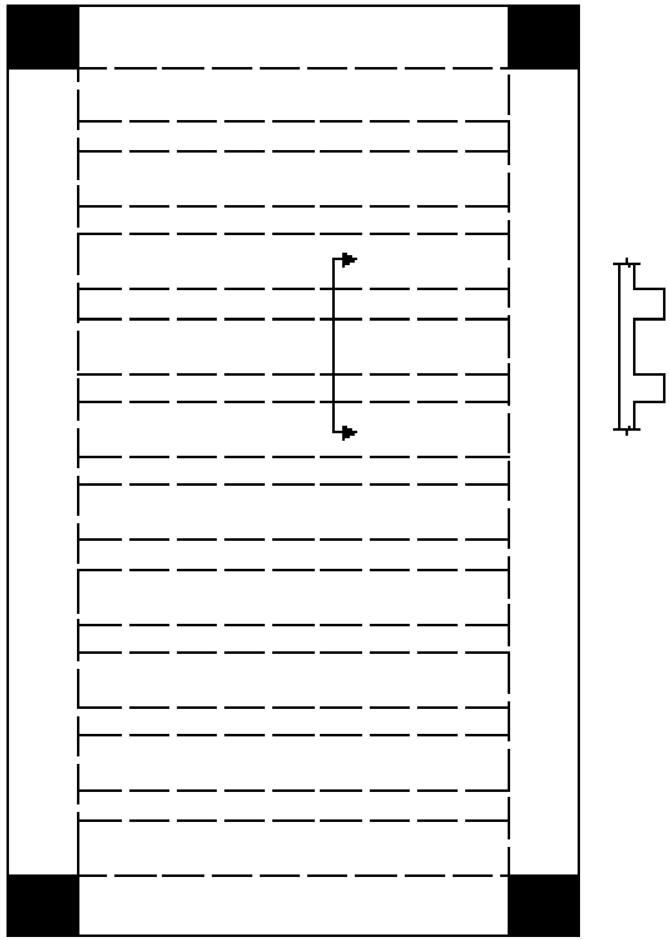
(b) $\frac{Long\ S\qquad n}{S\quad ort\ S\qquad n} < 2.0$ Two way slab

One way solid slab:

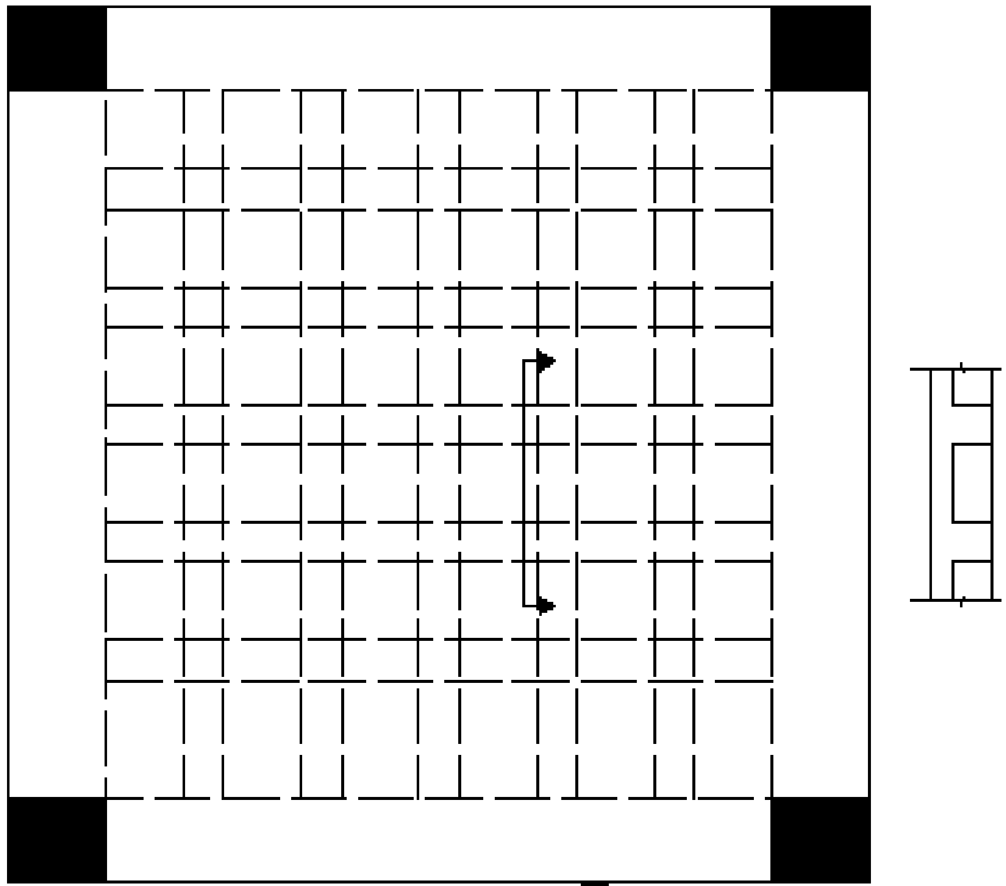
- loads transfer to supports in one direction
- uniform slab thickness

Two way solid slab:

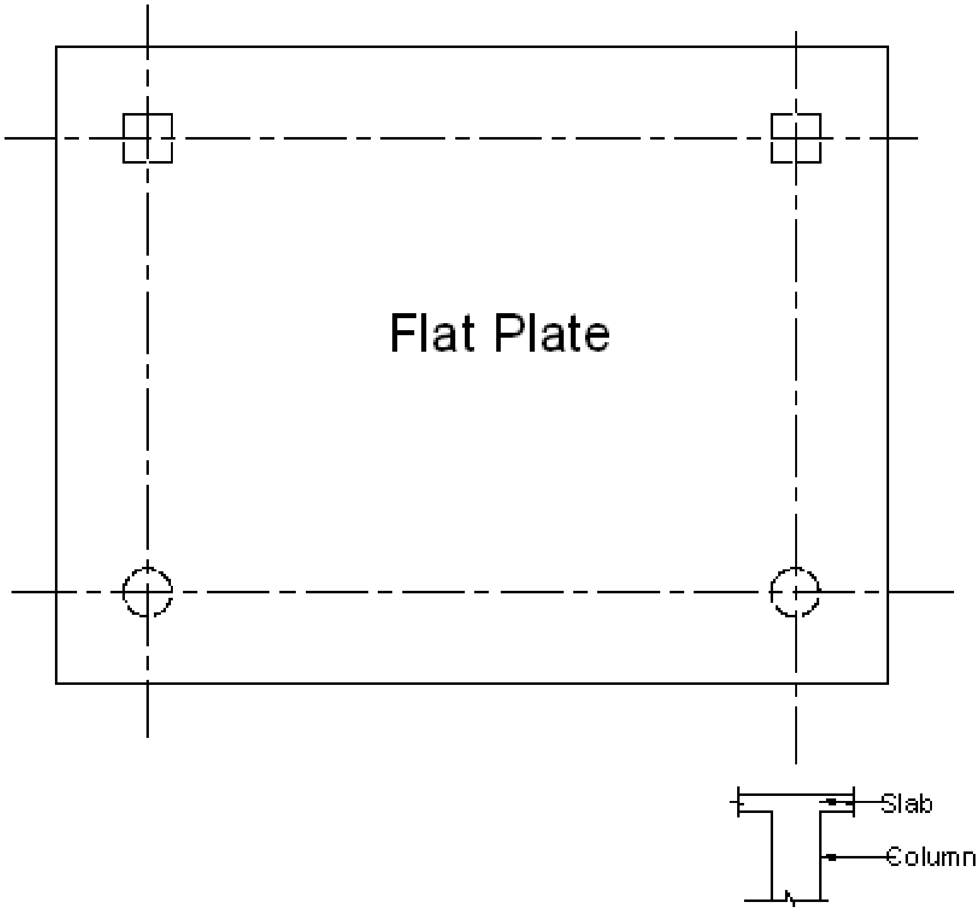
- loads transfer to supports in two perpendicular directions
- uniform slab thickness



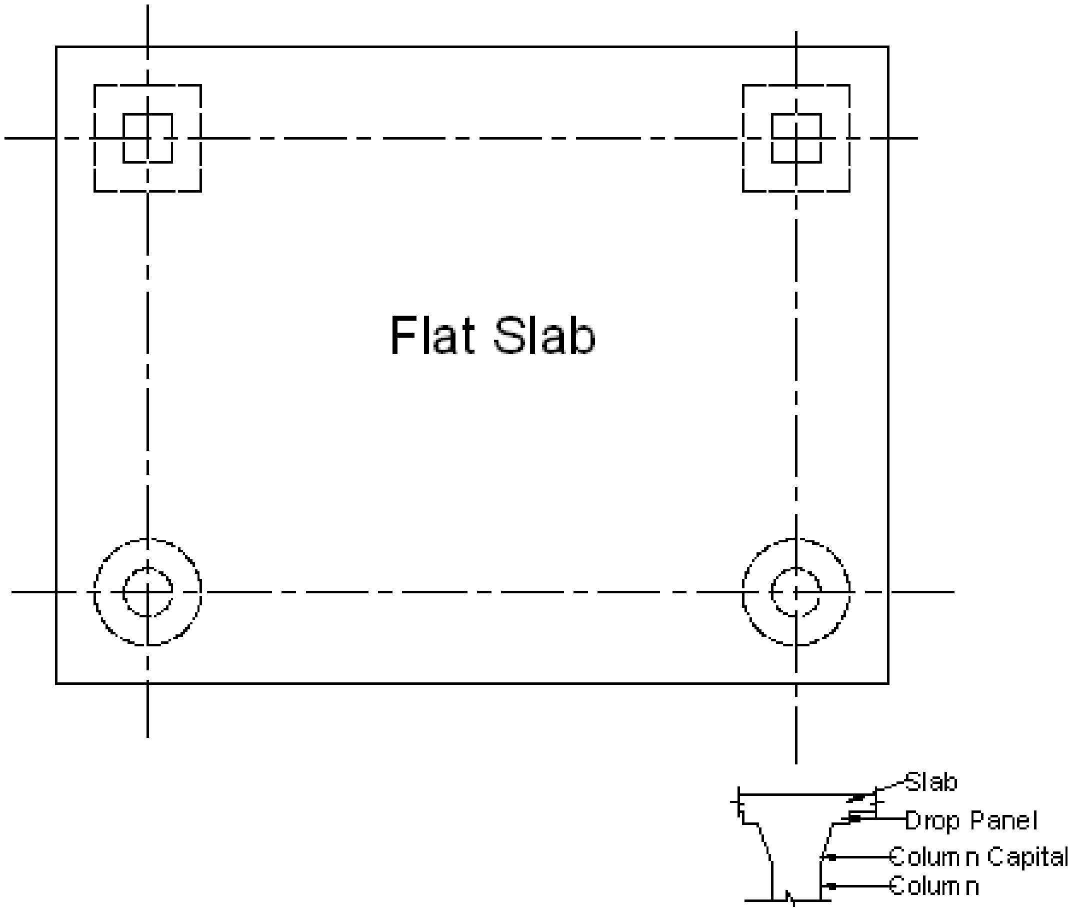
c



d



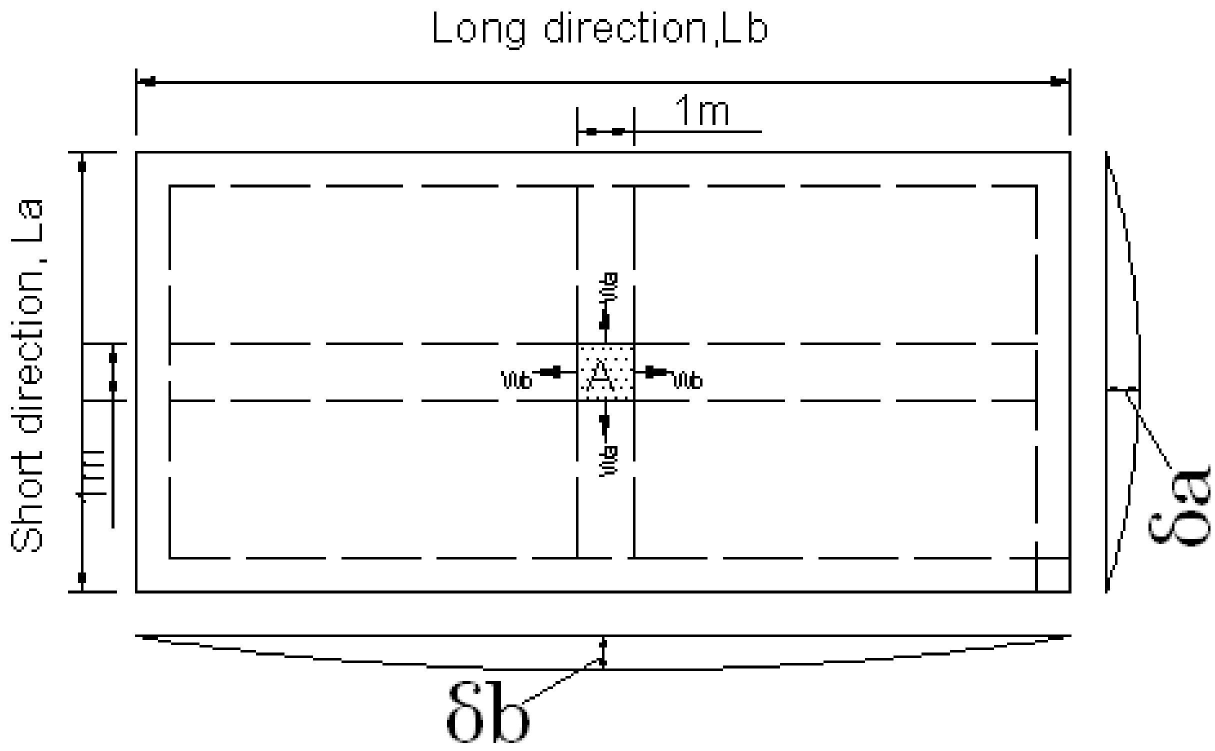
Flat Plate
Slab
Column



Flat Slab
Slab
Drop Panel
Column Capital
Column

One way solid slabs :-

W:uniformly distributed load on slab



$$\delta_A = \frac{5W_a l_a^4}{384EI}$$

$$\delta_B = \frac{5W_b l_b^4}{384EI}$$

$$\delta_A = \delta_b \rightarrow \frac{5W_a l_a^4}{384EI}$$

$$= \frac{5W_b l_b^4}{384EI} \rightarrow \frac{W_a}{W_b} = \frac{l_b^4}{l_a^4}$$

$$W_a = W_b \left(\frac{l_b}{l_a}\right)^4$$

Let $\frac{long}{short}\frac{l_b}{l_a} = 2.0$ (aspect ratio of slab)

$$W_a + W_b = W$$

$$W_a + W_a \left(\frac{l_a}{l_b}\right)^4 = W$$

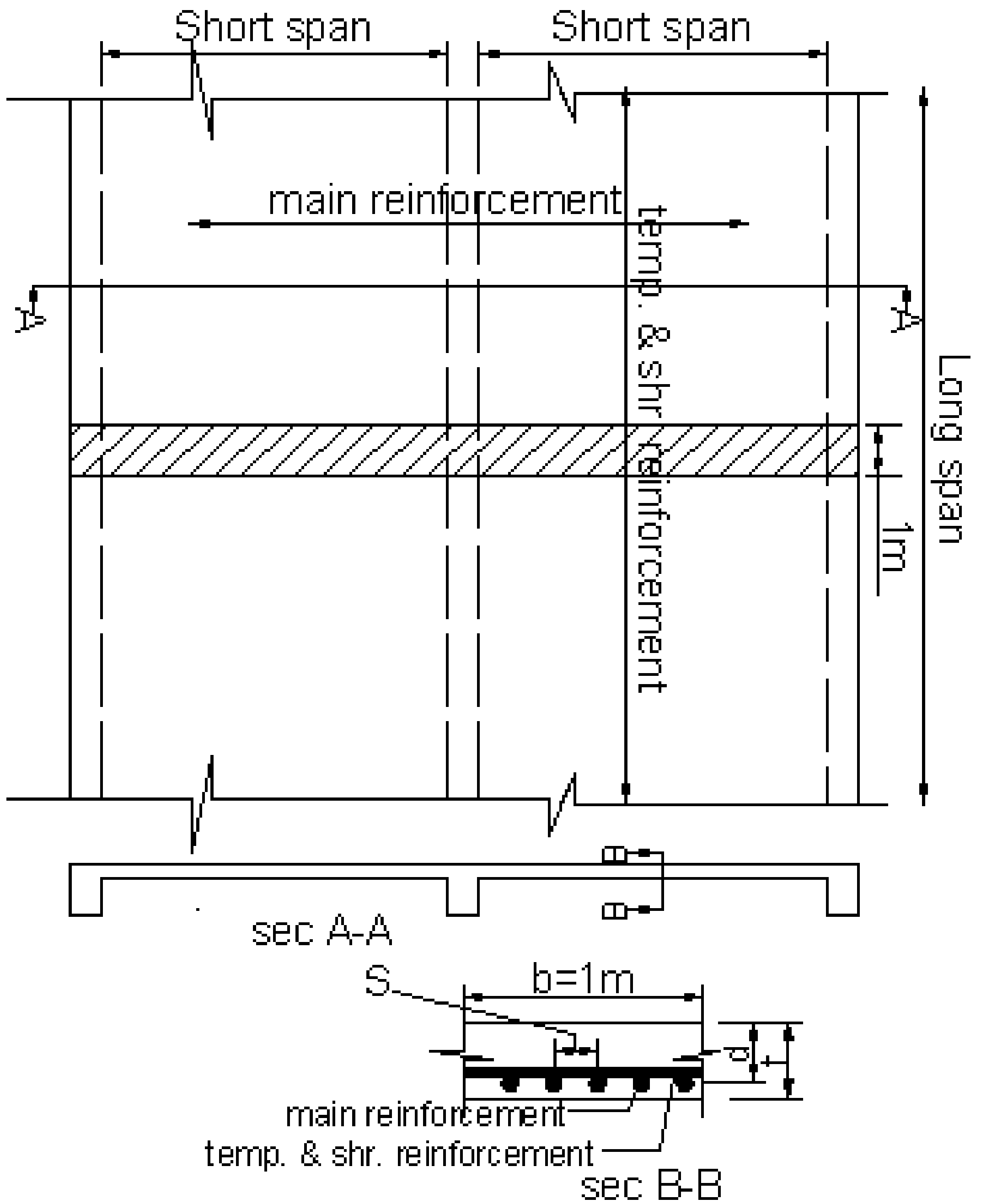
$$W_a + W_a \left(\frac{1}{2}\right)^4 = W$$

$$W_a + \frac{1}{16} W_a = W \rightarrow \boldsymbol{w_a = 0.94\, w,}$$

$$W_b = \mathbf{0.06}\, \boldsymbol{w}$$

if $\frac{l_b}{l_a} \geq \mathbf{2.0}$ **one way slab**

if $\frac{l_b}{l_a} < 2.0$ **two way slab**


Short span
Short span
main reinforcement
temp. & shr. reinforcement
A
A
Long span
1m
B
B
sec A-A
S
b=1m
d
t
main reinforcement
temp. & shr. reinforcement
sec B-B

Min thickness of one way solid slabs Table (9.5.a ACI code)

We can use ACI code method for shear and moment coefficients after satisfy its conditions.

Unless slab thickness should be checked

Support condition	Simply supported	One end continuous	Both ends continuous	Cantilever	For normal concrete(γc=24 kN/m^3),fy=400 MPa
Min. thickness	L/20	L/24	L/28	L/10	

$$d = t - cover - \frac{1}{2}\, db$$

Min. concrete cover for slab=20mm

$d_b = 10, 12$ mm …. $< 16mm$ *used for ribbed slab*

$let\, d_b = 10 \rightarrow \quad d = t - 20 - \frac{1}{2} * 10 \quad = t - 25$

$$Mu = \emptyset\, \rho\, b\, d^2 fy \left(1\text{-}0.59\, \rho \frac{fy}{fc'}\right) \rightarrow solve\ get\ \rho$$

$As = \rho\, b\, d = \rho * 1000 * d$ $\quad mm^2/m$ width

$$\rho \leq \rho_{max} = 0.85\, \beta_1 \frac{fc'}{fy} \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.004}$$

$if\ \rho > \rho_{max} \rightarrow$ *increas slab thickness*

- in slab, use tensile reinforcement only.
- In practice ρ economy between (0.004-0.008).

Spacing of bars (c. to .c)

$$S = \frac{1000}{no.of\ bars} \quad , \qquad no.of\ bars = \frac{As}{As_b}$$

$$\therefore S = 1000 * \frac{As_{bar}}{As}$$

$$\begin{pmatrix} S_{min}=40mm \text{ for practice} \\ S_{max}=\min(3t, 500mm) \text{ACI 7.6.5} \end{pmatrix} \text{for main reinforcement,}$$

In practice $S_{economy} \leq 1.5t.$

Shrinkage and temperature reinforcement As_{min}

ACI(7.12.2)

$$\rho_{min} = 0.0018\ for\ \text{fy} = 420\ MPa$$

$$\rho_{min} = 0.002\ for\ \text{fy} = 300\ ,350\ MPa$$

$$\rho_{min} = 0.0018 * \frac{420}{\text{fy}} \geq 0.0014 for\ \text{fy}\ \geq 420\ MPa$$

$$S_{max} = \min(5t, 500\ mm) \rightarrow for\ secondary\ reinforcement$$

$$As > (As_{min} = \rho_{min} * b * t\ )$$

$$for\ both\ main\ and\ secondary\ reiorcement$$

Moment equation:

$$Mu=\emptyset\ \rho\ b\ d^2 fy\left(1\text{-}0.59\ \rho\frac{fy}{fc}\right)\ \div\ \emptyset b d^2 f c'$$

$$\frac{Mu}{\emptyset b d^2 f c'}=\rho\frac{fy}{fc'}\left(1-0.59\ \rho\frac{fy}{fc'}\right)$$

$$R=\frac{Mu}{\emptyset b d^2 fc'}\quad,\ \omega=\ \rho\frac{fy}{fc'}$$

$$R\rightarrow chart\rightarrow\omega=\qquad Desgin\ Problem$$

$$\omega\rightarrow chart\rightarrow R=\qquad Analysis\ Problem$$

$$R=W(1-0.59\omega)$$

$$\rho_{max}=0.85\ \beta_1\ \frac{fc'}{fy}*\frac{0.003}{0.003+0.004}$$

$$\omega_{max}=\rho_{max}.\frac{fy}{fc'}=0.85\ \beta_1*\frac{0.003}{0.003+0.004}*\frac{fc'}{fy}*\frac{fy}{fc'}$$

$$\therefore\ \omega_{max} = 0.85\,\beta_1 * \frac{0.003}{0.003 + 0.004} = 0.364\,\beta_1$$

$$fc' \le 28\ MPa \to \beta_1 = 0.85 \to \omega_{max} = 0.309$$

Other method:

$$\text{Mu} = \text{Ø}\ \rho\ \text{b}\ \text{d}^2\text{fy}\left(1\text{-}0.59\ \rho\frac{\text{fy}}{\text{fc}}\right) \div\ Øbd^2fc'$$

$$\frac{Mu}{Øbd^2} = \rho\text{fy}\left(1 - \frac{\rho\text{fy}}{2*0.85\text{fc}'}\right)$$

$$R = \frac{Mu}{Øbd^2} \quad ,\ m = \frac{\text{fy}}{0.85\text{fc}'}$$

$$2R = 2\rho fy - \rho^2 fym$$

$$\rho = \frac{1}{m}(1 - \sqrt{1 - \frac{2mR}{fy}}$$

MOMENT STRENGTH ($M_u/\phi f_c' bd^2$) OF RECTANGULAR SECTION WITH TENSION REINFORCEMENT ONLY ....

ω	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009
.00	0	.0010	.0020	.0030	.0040	.0050	.0060	.0070	.0080	.0090
.01	.0099	.0109	.0119	.0129	.0139	.0149	.0159	.0168	.0178	.0188
.02	.0197	.0207	.0217	.0226	.0236	.0246	.0256	.0266	.0275	.0285
.03	.0295	.0304	.0314	.0324	.0333	.0343	.0352	.0362	.0372	.0381
.04	.0391	.0400	.0410	.0420	.0429	.0438	.0448	.0457	.0467	.0476
.05	.0485	.0495	.0504	.0513	.0523	.0532	.0541	.0551	.0560	.0569
.06	.0579	.0588	.0597	.0607	.0616	.0625	.0634	.0643	.0653	.0662
.07	.0671	.0680	.0689	.0699	.0708	.0717	.0726	.0735	.0744	.0753
.08	.0762	.0771	.0780	.0789	.0798	.0807	.0816	.0825	.0834	.0843
.09	.0852	.0861	.0870	.0879	.0888	.0897	.0906	.0915	.0923	.0932
.10	.0941	.0950	.0959	.0967	.0976	.0985	.0994	.1002	.1011	.1020
.11	.1029	.1037	.1046	.1055	.1063	.1072	.1081	.1089	.1098	.1106
.12	.1115	.1124	.1133	.1141	.1149	.1158	.1166	.1175	.1183	.1192
.13	.1200	.1209	.1217	.1226	.1234	.1243	.1251	.1259	.1268	.1276
.14	.1284	.1293	.1301	.1309	.1318	.1326	.1334	.1342	.1351	.1357
.15	.1367	.1375	.1384	.1392	.1400	.1408	.1416	.1425	.1433	.1441
.16	.1449	.1457	.1465	.1473	.1481	.1489	.1497	.1506	.1514	.1522
.17	.1529	.1537	.1545	.1553	.1561	.1569	.1577	.1585	.1593	.1601
.18	.1609	.1617	.1624	.1632	.1640	.1648	.1656	.1664	.1671	.1679
.19	.1687	.1695	.1703	.1710	.1718	.1726	.1733	.1741	.1749	.1756
.20	.1764	.1772	.1779	.1787	.1794	.1802	.1810	.1817	.1825	.1832
.21	.1840	.1847	.1855	.1862	.1870	.1877	.1885	.1892	.1900	.1907
.22	.1914	.1922	.1929	.1937	.1944	.1951	.1959	.1966	.1973	.1981
.23	.1988	.1995	.2002	.2010	.2017	.2024	.2031	.2039	.2046	.2053
.24	.2060	.2067	.2075	.2082	.2089	.2096	.2103	.2110	.2117	.2124
.25	.2131	.2138	.2145	.2152	.2159	.2166	.2173	.2180	.2187	.2194
.26	.2201	.2208	.2215	.2222	.2229	.2236	.2243	.2249	.2256	.2263
.27	.2270	.2277	.2284	.2290	.2297	.2304	.2311	.2317	.2324	.2331
.28	.2337	.2344	.2351	.2357	.2364	.2371	.2377	.2384	.2391	.2397
.29	.2404	.2410	.2417	.2423	.2430	.2437	.2443	.2450	.2456	.2463
.30	.2469	.2475	.2482	.2488	.2495	.2501	.2508	.2514	.2520	.2527
.31	.2533	.2539	.2546	.2552	.2558	.2565	.2571	.2577	.2583	.2590
.32	.2596	.2602	.2608	.2614	.2621	.2627	.2633	.2639	.2645	.2651
.33	.2657	.2664	.2670	.2676	.2682	.2688	.2694	.2700	.2706	.2712
.34	.2718	.2724	.2730	.2736	.2742	.2748	.2754	.2760	.2766	.2771
.35	.2777	.2783	.2789	.2795	.2801	.2807	.2812	.2818	.2824	.2830
.36	.2835	.2841	.2847	.2853	.2858	.2864	.2870	.2875	.2881	.2887
.37	.2892	.2898	.2904	.2909	.2915	.2920	.2926	.2931	.2937	.2943
.38	.2948	.2954	.2959	.2965	.2970	.2975	.2981	.2986	.2992	.2997
.39	.3003	.3008	.3013	.3019	.3024	.3029	.3035	.3040	.3045	.3051

Area Of Group Of Bars (mm^2/ Number Of Bars):

Bar No. / Dia (MM)	1	2	3	4	5	6	7	8	9	10	11	12	13	14
10	79	157	236	314	393	471	550	628	707	785	864	942	1021	1099
12	113	226	340	453	566	679	792	905	1018	1131	1244	1357	1471	1584
16	201	402	604	805	1006	1207	1408	1609	1810	2011	2212	2413	2615	2816
18	255	509	764	1018	1273	1527	1782	2036	2291	2545	2800	3054	3309	3563
22	380	760	1140	1520	1901	2281	2661	3041	3421	3801	4181	4561	4941	5321
25	491	982	1473	1964	2455	2946	3437	3928	4419	4910	5401	5892	6383	6874
28	616	1232	1848	2464	3080	3696	4312	4928	5544	6160	6776	7392	8008	8624
32	805	1609	2413	3217	4021	4826	5630	6434	7238	8043	8847	9651	10455	11260
36	1018	2036	3054	4072	5090	6108	7126	8144	9162	10180	11198	12216	13234	14252
38	1134	2268	3402	4536	5671	6805	7939	9073	10207	11341	12475	13609	14743	15877
40	1257	2514	3771	5028	6285	7542	8799	10056	11313	12570	13827	15084	16341	17598

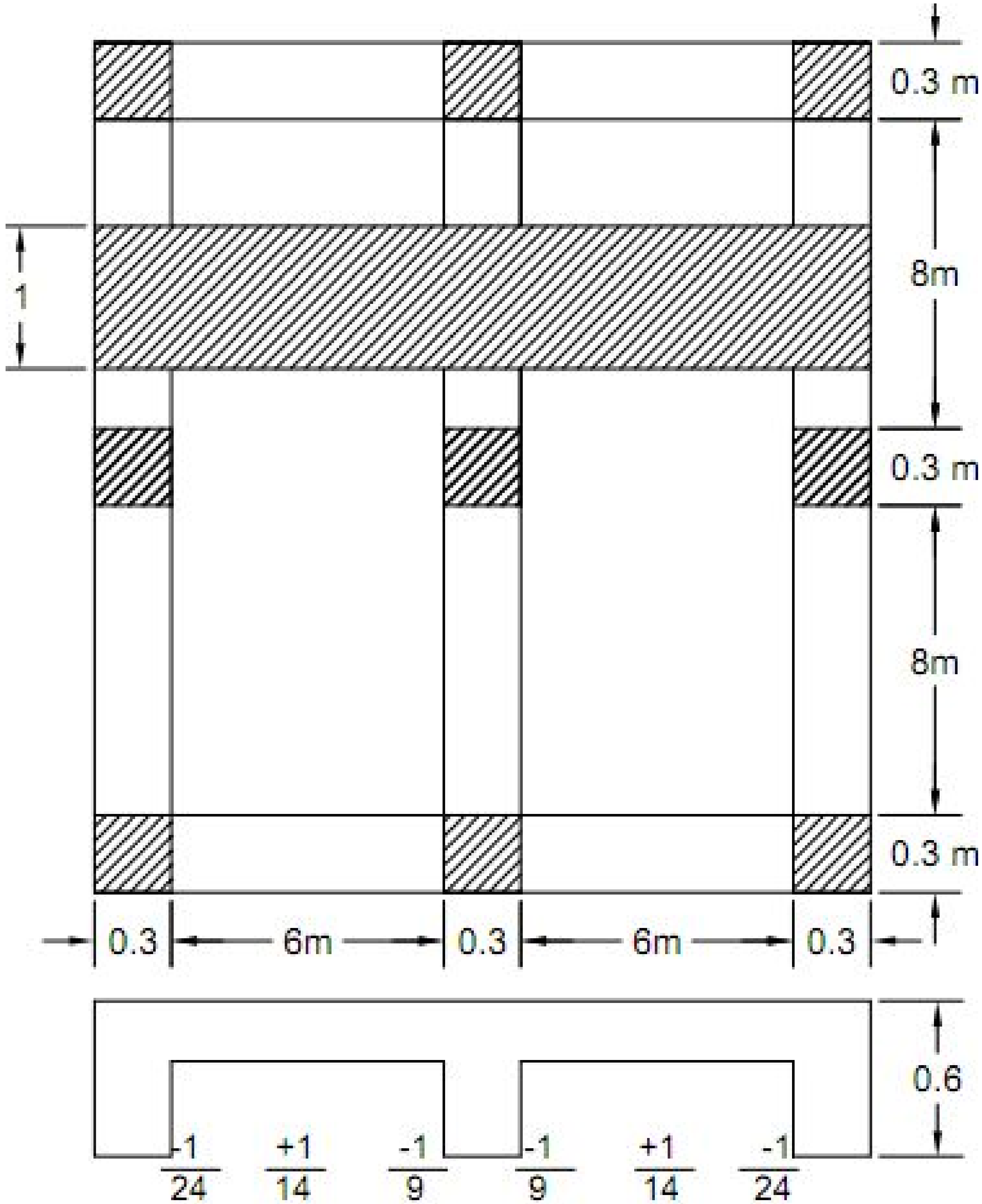
Area Of Group Of Bars (Cm^2/ M. Lenght) :

SPACING (CM) / DIA (mm)	7.5	10	12.5	15	17.5	20	22.5	25	27.5	30	35	40	45	50
10	10.47	7.85	6.28	5.24	4.49	3.93	3.49	3.14	2.86	2.62	2.23	1.95	1.75	1.57
12	15.08	11.31	9.05	7.54	6.46	5.65	5.03	4.52	4.11	3.77	3.23	2.83	2.52	2.26
16	26.81	20.17	16.08	13.4	11.49	10.05	8.94	8.04	7.31	6.7	5.74	5.03	4.47	4.02
18	33.87	25.4	20.32	16.93	14.51	12.7	11.29	10.16	9.24	8.47	7.26	6.35	5.65	5.08
22	50.67	38.0	30.4	25.33	21.71	19.0	16.89	15.2	13.82	12.67	10.86	9.5	8.45	7.60
25	65.45	49.09	38.27	32.72	28.05	24.54	21.82	19.63	17.85	16.36	14.03	12.28	10.91	9.82
28	82.13	61.60	44.26	41.07	35.20	30.80	27.38	24.64	22.40	20.54	17.6	15.4	13.69	12.32
32	107.24	80.43	64.34	53.62	45.96	40.21	35.75	32.17	29.25	26.81	22.98	20.11	17.29	16.09
36	135.74	101.80	81.44	67.87	58.17	50.90	45.25	40.72	37.02	33.44	29.09	25.45	22.63	20.36
38	151.22	113.41	90.73	75.61	64.81	56.71	50.41	45.73	41.24	37.81	32.41	28.36	25.21	22.69
40	167.60	125.70	100.56	83.78	71.83	62.85	55.87	50.28	45.71	41.90	35.92	31.43	27.94	25.14

Example 1:-

Design the two panel one way slab system shown to carry service live load of 5 kN/m^2 all supports are reinforced concrete beam, $\frac{S}{C} = \frac{400}{30}$,beam dimension (h=0.6 m, bw=0.3 m), Column dimension (0.3 , 0.3)m

Table 1.1 page 11 ,Winter→ min. uniformly distributed Live Loads


0.3 m
8m
0.3 m
8m
0.3 m
1
0.3
6m
0.3
6m
0.3
0.6
-1/24
+1/14
-1/9
-1/9
+1/14
-1/24

Solution:-

$$\frac{clear\ long\ span}{clear\ short\ span} = \frac{8 + 0.3 + 8}{6} = 2.7 > 2 \therefore one - way$$

Min thickness for deflection (Table 9.5 a ACI code) both end continuous

$$t = \frac{l}{28} = \frac{6}{28} = 0.214m = 0.22m = 220mm$$

$$t \leq 150mm \rightarrow t \cong 5mm$$

$$t > 150mm \rightarrow t \cong 10mm$$

$$use\ t = 220mm$$

$$d = t - cover - \frac{1}{2} d_b = (t - 25)mm$$

$$= 220 - 25 = 195\ mm$$

Dead load calculation

$Slab\ weight = t * \gamma_{concrete} = 0.22 * 24.5 = 5.39 kN/m^2$

$Tiling\ with\ mortar = 0.04 * 24.5 = 0.98\ kN/m^2$

$\sum D.L = 6.37 kN/m^2$

$W_u = 1.2 * W_{D.L} + 1.6\ W_{L.L} = 1.2 * 6.37 + 1.6 * 5 = 15.65 kN/m^2$

$W_u = 15.65 kN/m, \qquad sice\ strip\ width = 1m$

B.M coff.	$Mu = W_u * l_n$ *coeff.	$R = \frac{Mu}{Øfc' bd^2}$	ω from table	$\rho = \omega * \frac{fc'}{fy}$	$As = \rho bd$ $\rho * 1000 * 195$
-1/9	62.6	0.0610	0.0635	0.00476	929
+1/14	40.2	0.0391	0.0400	0.00300	585
-1/24	23.5	0.0229	0.0235	0.00172	335→396
			All ω $< \omega_{max}$		

$$Mu = Wu * ln^2 * coeff = 15.6 * 6^2 * \frac{1}{9} = 62.4kN.m$$

Assume Ø=0.9 to be check later

$$R = \frac{Mu}{\text{Øfc}' bd^2} = \frac{0.0624}{0.9 * 30 * 1 * 0.195^2} = 0.061$$

$$\omega(from\ table) = 0.0635$$

$$\rho = \omega * \frac{\text{fc}'}{\text{fy}} = 0.0635 * \frac{30}{400} = 0.00476$$

$$As = \rho bd = 0.00476 * 1000 * 195 = \frac{929mm^2}{m\ width}$$

$$\omega_{max} = 0.364 * \beta_1 = 0.364 * 0.85 = 0.3094$$

$All\ \ \omega < \omega_{max}$ else increase slab thickness

$since\ fy = 400MPa,$

$$As_{min} = 0.0018 * b * t = 0.0018 * 1000 * 220$$

$$= 396 \frac{mm^2}{m\ width}$$

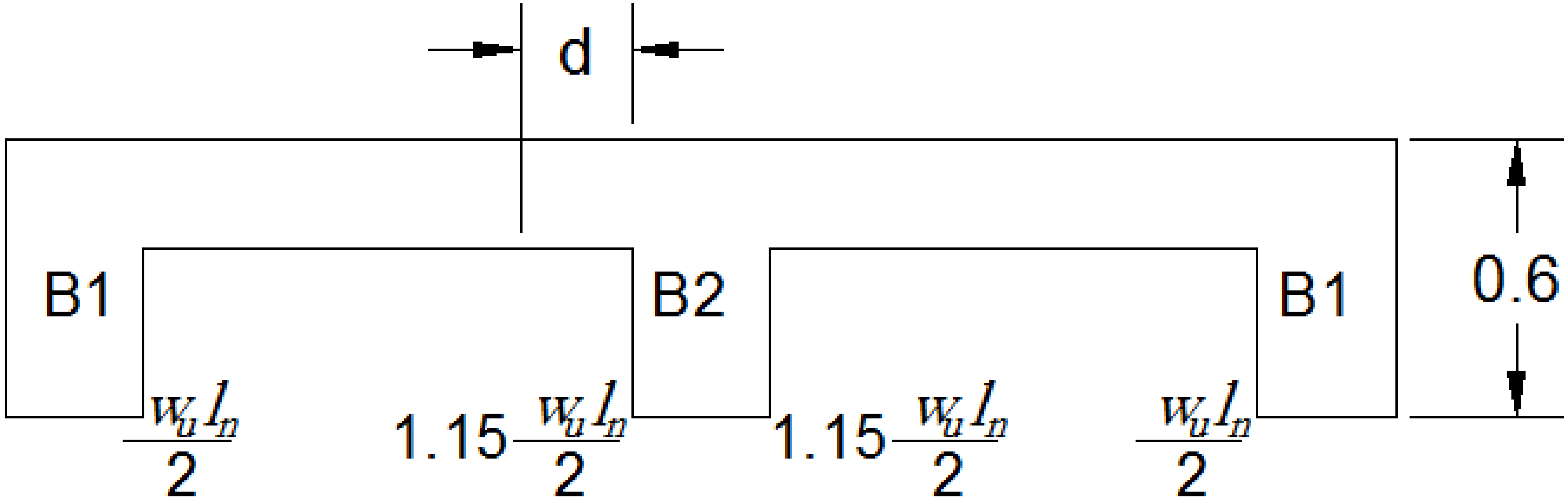
***Check redaction factor* $\emptyset$**

$$\rho_t = 0.85 * \beta_1 * \frac{\mathrm{fc}'}{\mathrm{fy}} * \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.005}$$

$$= 0.85 * 0.85 * \frac{30}{400} * \frac{0.003}{0.003 + 0.005} = 0.02$$

$$all\ \rho < \rho_t \ \rightarrow \emptyset = 0.9 \therefore o.k$$

Check for shear



Max shear

$$V_u = 1.15 * \frac{W_u * l_n}{2} = 1.15 * \frac{15.65 * 6}{2} = 54\ kN$$

$$\mathrm{V_{ud} = V_u - w_u d = 54 - 15.65 * 0.195 = 51\ kN}$$

$$V_c = 0.17 * \sqrt{fc'} b_w d = 0.17\sqrt{30} * 1 * 0.195 * 10^3$$

$$= 181.6 \text{ kN}$$

$$\emptyset V_c = 0.75 * 181.6 = 136 \text{ kN}$$

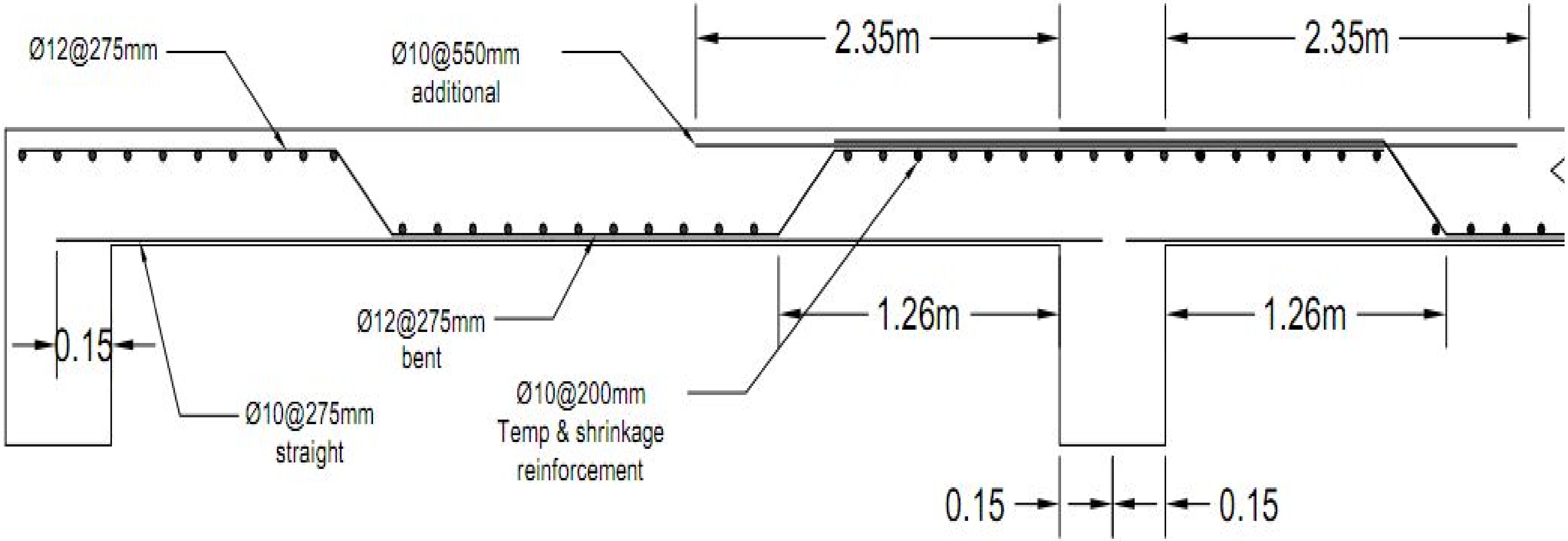
$$V_{ud} = 51 \text{ kN} < \emptyset V_c = 136 \text{ kN} \therefore \text{o.k}$$

$$\text{if } V_{ud} > \emptyset V_c \rightarrow \text{increuss (t)ACI (11.5.5.1)}$$

Arrangement of reinforcement

$$S_{max} = min.(3t, 500mm) = 500mm$$

$$S_{min} = 40\ mm$$



- ***Exterior negative reinforcement 396 mm^2/m:***

$$\emptyset 12mm\ @275mm \rightarrow As = 411\frac{mm^2}{m} > 396\frac{mm^2}{m}$$

- ***Positive reinforcement 585 mm^2/m***

$$\begin{pmatrix} \emptyset 12mm@275mm(bent) = 411\frac{mm^2}{m} \\ \emptyset 10mm@275mm\ (stright) = 286\frac{mm^2}{m} \end{pmatrix} \sum As$$

$$= 286 + 411 = 697\frac{mm^2}{m} > 585\frac{mm^2}{m} \ \therefore o.k$$

- ***interior negative reinforcement 929 mm^2/m***

$$\begin{pmatrix} \emptyset 12mm@275\ bent\ (left) = 411\frac{mm}{m} \\ \emptyset 12mm@275mm\ bent\ (right) = 411\frac{mm}{m} \\ \sum As = 411 + 411 = 822\frac{mm}{m} < 929\frac{mm}{m} \\ \emptyset 10\ \mathrm{mm}\ @\ 550mm\ additional\ reinforcment\ 193\frac{mm}{m} \end{pmatrix}$$

$$\sum As = 411 + 411 + 193 = 1015\frac{mm^2}{m} > 929\frac{mm^2}{m}\ O.K$$

Bent and Cut off point

Positive reinforcement(+1/14), % bent up=411/697=59%

$$\text{bent point} = 0.21 * l_n$$

$$= 1.26\ \text{m}$$

Negative reinforcement(-1/9) % cut off =100%

$$\text{cut off point} = 0.33 * l_n + \max.\left(12d_b, d, \frac{l_n}{16}\right) = 2.35\text{m}$$

Temperature and shrinkage reinforcement

$$As = 396\,\frac{\text{mm}^2}{\text{m width}} \rightarrow \text{use } \emptyset 10\ @200\text{mm} = 393\,\frac{\text{mm}^2}{\text{m}}, \text{diff.}$$

$$< 10\ mm^2\ 0.\text{k}$$

Load transferred to beam

B1

$$W_d = \frac{WL_n}{2} = \frac{6.37 * 6}{2} = 19.11 \frac{kN}{m} + *$$

$$W_l = \frac{W_l * L_n}{2} = \frac{5 * 6}{2} = 15 \frac{kN}{m}$$

B2

$$W_d = \frac{1.15 * W_d L_n}{2} * 2 = \frac{1.15 * 6.37 * 6}{2} * 2 = 43.95 \frac{kN}{m} + *$$

$$W_l = \frac{1.15W_l * L_n}{2} * 2 = \frac{1.15 * 5 * 6}{2} * 2 = 34.5 \frac{kN}{m}$$

*and any additional dead load carried directly by beam