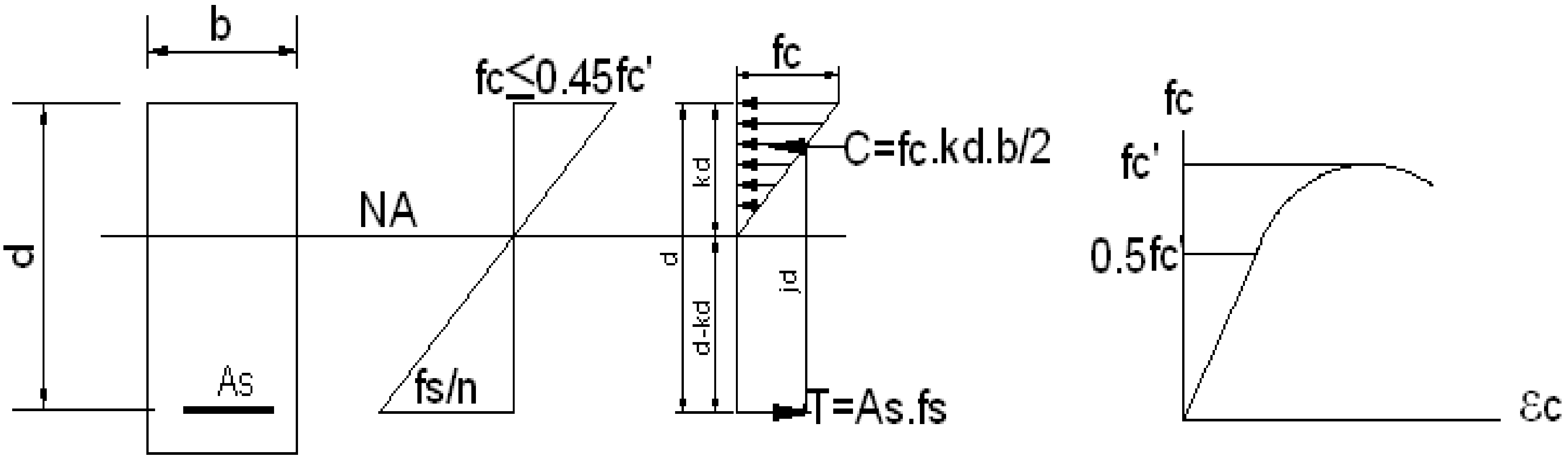
Working Stress Design Method
Singly Reinforced Concrete beams:



Based on cracked-elastic section:

$$[\ fct \geq fr = 0.62\sqrt{fc'}\ ,\quad fc_{comp} \leq 0.5fc'\ ]$$

$$fc_{comp} \leq 0.45fc'$$

$$fs \leq 0.5fy \nless \begin{cases} 140MPa \quad for\ fy = 300MPa \\ 170MPa\ for\ fy = 400MPa \end{cases}$$

$$n = \frac{Es}{Ec}$$

$$\sum M_{NA}$$

$$b.kd.\frac{kd}{2} = n.As.(d - kd), \qquad \rho = \frac{As}{bd}$$

$$b.\frac{(kd)^2}{2} = n.\rho bd.(d - kd)$$

$$b.\frac{(kd)^2}{2} = n.\rho bd^2.(1 - k)$$

$$\frac{(k)^2}{2} = n\rho - n\rho k$$

$$(k)^2 + 2n\rho k - 2n\rho = 0 \rightarrow k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho$$

$$C = \frac{fckdb}{2}$$

$$T = As\,fs$$

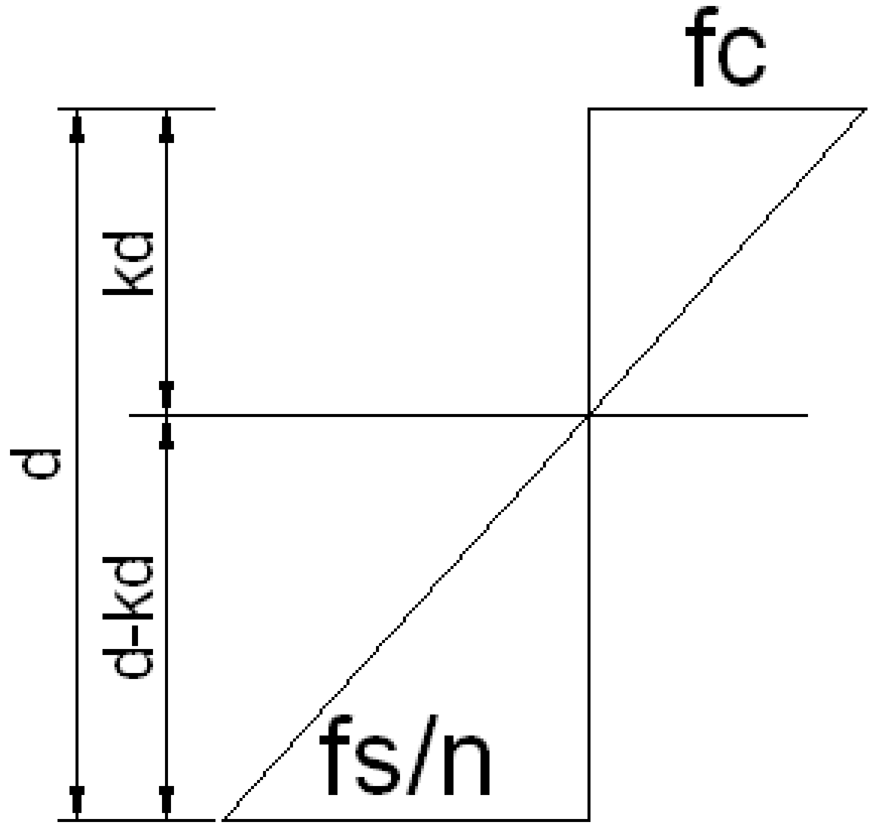
$$Mc = C.jd = \frac{fckdb}{2}.jd$$

$$Mc = C.jd = \frac{fc.kj.b.d^2}{2}$$

$$Ms = T.jd = As.fs.jd$$

$$jd = d - \frac{jd}{3} \rightarrow \quad j = 1 - \frac{k}{3}$$

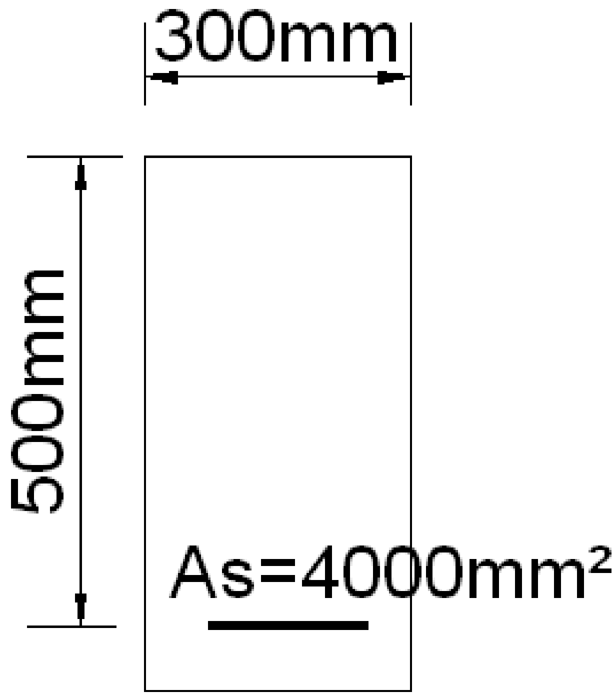
$$\frac{fc}{kd} = \frac{fc + \frac{fs}{n}}{d} \rightarrow k = \frac{1}{1 + \frac{fs}{nfc}}$$



Ex1:

If M=150kN.m, n=8, As=4000mm2, d=500mm, b=300mm, find stresses in steel and concrete.

Solution:



$$\rho = \frac{4000}{300 * 500} = 0.0267$$

$$k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho$$

$$k = \sqrt{(8 * 0.0267)^2 + 2 * 8 * 0.0267}$$

$$- 8 * 0.0267 = 0.474$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.474}{3} = 0.842$$

$$M = \frac{fc.kj.b.d^2}{2} \rightarrow fc = \frac{2M}{kj.b.d^2}$$

$$= \frac{2 * 150 * 10^{-3}}{0.474 * 0.842 * 0.3 * 0.5^2} = 10MPa$$

$$M = As.fs.jd \rightarrow fs = \frac{M}{As.jd} = \frac{150 * 10^{-3}}{4000 * 10^{-6} * 0.842 * 0.5}$$

$$= 89MPa$$

Ex2:

For data given in Ex1, if fc'=30MPa, $fs_{allowable}$=140MPa, find moment capacity of the beam.

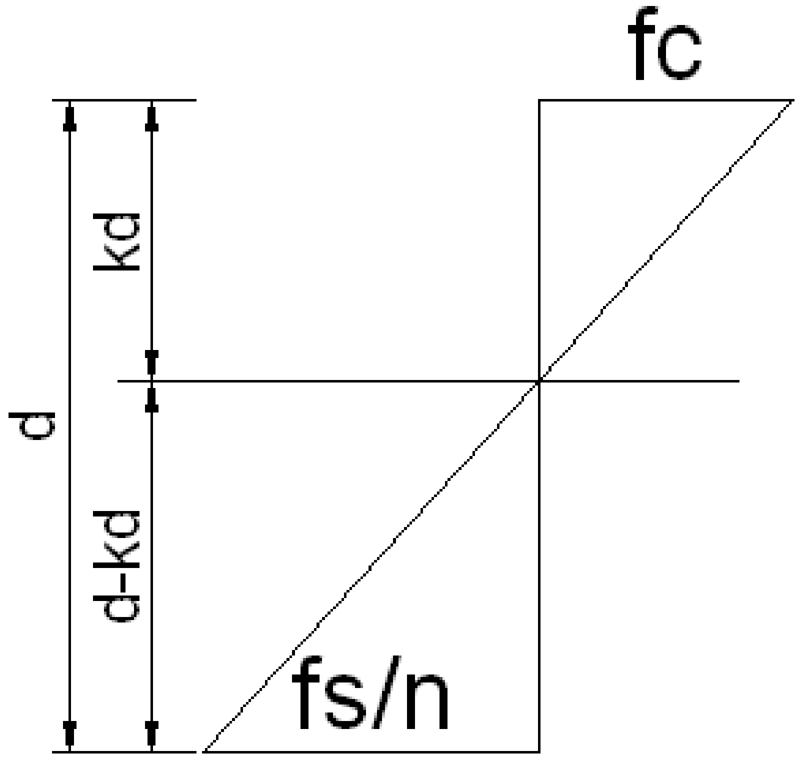
Solution:

$fc_{allowable}$=0.45fc'=0.45*30=13.5MPa

assume steel failure fs=140MPa

$$\frac{fc}{kd}=\frac{\frac{fs}{n}}{d-kd}\rightarrow\frac{fc}{0.474*500}$$

$$=\frac{\frac{140}{8}}{500-0.474*500}$$



$$fc = 15.8MPa > fc_{allow} = 13.5MPa \rightarrow$$

$$\therefore concrete\ failure, fc = 13.5Mpa$$

$$\frac{fc}{kd} = \frac{\frac{fs}{n}}{d - kd} \rightarrow \frac{13.5}{0.474 * 500} = \frac{\frac{fs}{8}}{500 - 0.474 * 500}$$

$$fs = 120MPa < fs_{allow} = 140MPa$$

$$Mc = C.jd = \frac{fc.kj.b.d^2}{2} = \frac{13.5 * 0.474 * 0.842 * 0.3 * 0.5^2}{2}$$

$$= 0.202MN.m$$

OR $Ms = T.jd = As.fs.jd = 4000 * 10^{-6} ** 120 * 0.842 *$

$0.5 = 0.202MN.m$

Ex3: Design of concrete beam.
If M=300kN.m, fc'=30MPa, fs=140MPa, n=10, fy=300MPa.
Solution:

$fc_{allowable}$=0.45fc'=0.45*30=13.5MPa

$$k = \frac{1}{1+\frac{fs}{nfc}} = \frac{1}{1+\frac{140}{10*13.5}} = 0.49 \qquad balanced\ failure$$

$$j = 1-\frac{k}{3} = 1-\frac{0.49}{3} = 0.84$$

$$M = \frac{fc.kj.b.d^2}{2} \rightarrow 0.3 = \frac{13.5*0.49*0.84*bd^2}{2} \rightarrow bd^2$$

$$= 108*10^{-3}m^3$$

Assume d=2b→ b=d/2

$$\frac{d^3}{2} = 108 * 10^{-3} \rightarrow d = 600mm, \quad b = 300mm$$

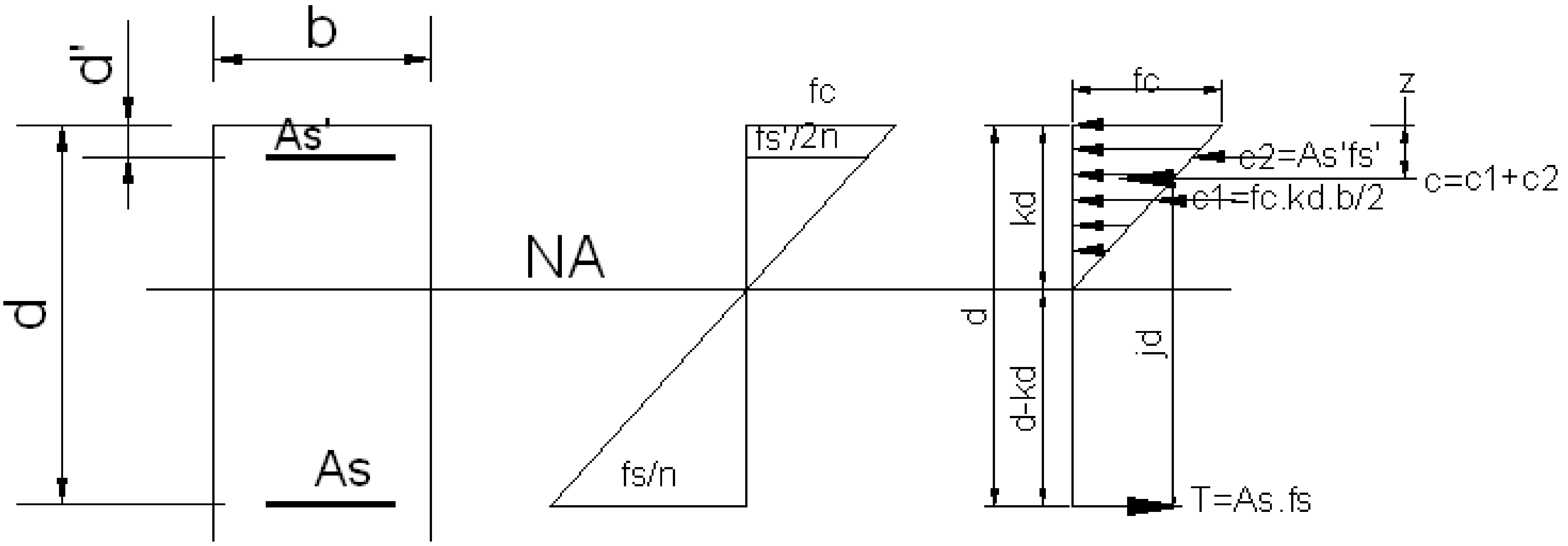
$$M = As.fs.jd \rightarrow 0.3 = As * 140 * 0.84 * 0.5 \rightarrow As$$

$$= 4251mm^2$$

$$use\ 7\emptyset 28 = 4312mm^2$$

$$\rho = \frac{4251}{300 * 600} = 0.0226 > \rho_{min} = \frac{1.4}{300} = 0.0046\ O.K$$

Doubly Reinforced Beams:



$$As' \xrightarrow[transformed\ \ to\ concrete]{} (2n-1)As',$$

$$A \xrightarrow[transformed\ \ to\ \ concrete]{} nAs$$

$$\sum M_{NA}$$

$$b * kd * \frac{kd}{2} + (2n - 1)As' * (kd - d') = nAs(d - kd)$$

$$\rho = \frac{As}{bd}, \quad \rho' = \frac{As'}{bd}$$

$$\frac{b(kd)^2}{2} + (2\mathrm{n} - 1)\mathrm{\rho' bd(kd - d')} = \mathrm{n\rho bd(d - kd)}$$

$$k = \sqrt{2(2n-1)\rho'\frac{d'}{d} + 2n\rho + n^2\left(2\rho' + \rho - \frac{\rho'}{n}\right)^2} - n\left(2\rho'\right.$$

$$\left. + \rho - \frac{\rho'}{n}\right)$$

$$If\ As' = 2n\rho' bd$$

$$k = \sqrt{2n\left(\rho'\frac{d'}{d} + \rho\right) + n^2(\rho' + \rho)^2} - n(\rho' + \rho)$$

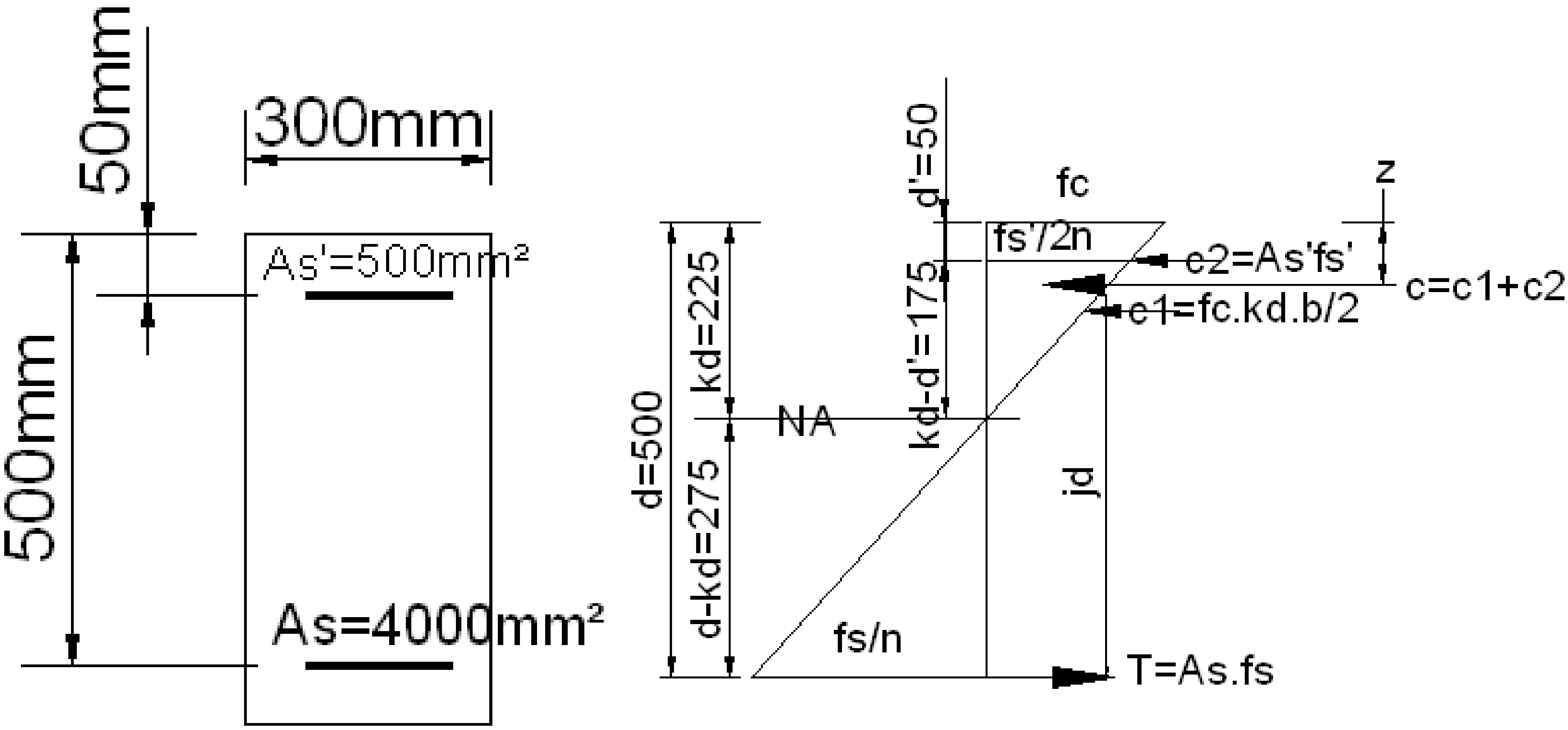
jd=d-z →j=1-z/d

Ms=As fs jd

$$\mathrm{Mc} = (\mathrm{c1} + \mathrm{c2})(\mathrm{d} - \mathrm{z}) = (\frac{fckdb}{2} + As'fs')(d - z)$$

$$z = \frac{\frac{d}{6}\,k^2 + \rho'(2n - 1)(1 - \frac{d'}{kd})d'}{\frac{k}{2} + \rho'(2n - 1)(1 - \frac{d'}{kd})}$$

Ex1:
If M=150kN.m, n=8, find fs',fs,fc



Solution:

$$\rho = \frac{As}{bd} = \frac{4000}{300 * 500} = 0.027, \quad \rho' = \frac{As'}{bd} = \frac{500}{300 * 500}$$

$$= 0.0033$$

$$k = \sqrt{2(2n-1)\rho'\frac{d'}{d} + 2n\rho + n^2(2\rho' + \rho - \frac{\rho'}{n})^2} - n(2\rho'$$

$$+ \rho - \frac{\rho'}{n})$$

$$k = \left[2(2*8-1)*0.0033*\frac{50}{500} + 2*8*0.027 + 8^2(2*0.0033+0.027-\frac{0.0033}{8})^2\right]^{\frac{1}{2}} - 8\left(2*0.0033 + 0.027-\frac{0.0033}{8}\right) = 0.45$$

kd=0.45*500=225mm

$$\frac{fc}{kd} = \frac{\frac{fs'}{2n}}{kd-d'} \rightarrow \frac{fc}{225} = \frac{\frac{fs'}{2*8}}{175} \rightarrow fs' = 12.44fc$$

$$c1 = \frac{fc.kd.b}{2} = \frac{fc*0.225*0.3}{2} = 0.03375fc$$

$$c2 = As'fs' = 500*10^{-6}*12.44fc = 6.22*10^{-3}fc$$

$$c = c1 + c2 = 0.03375fc + 6.22 * 10^{-3} fc = 0.03997fc$$

$$\sum M_{top\ fiber} = 0$$

$$c1 * \frac{kd}{3} + c2 * d' = c * z$$

$$z = \frac{c1 * \frac{kd}{3} + c2 * d'}{c}$$

$$= \frac{0.03375fc * \frac{0.225}{3} + 6.22 * 10^{-3} fc * 0.05}{0.03997fc} = 0.0711m$$

OR

$$z=\frac{\frac{d}{6}\ k^2+\rho'(2n-1)(1-\frac{d'}{kd})d'}{\frac{k}{2}\ +\rho'(2n-1)(1-\frac{d'}{kd})}$$

=71.1mm

jd=d-z=500-71.1=428.9mm

$$M=Asfsjd\rightarrow fs=\frac{150*10^{-3}}{4000*10^{-6}*0.4289}=87.43MPa$$

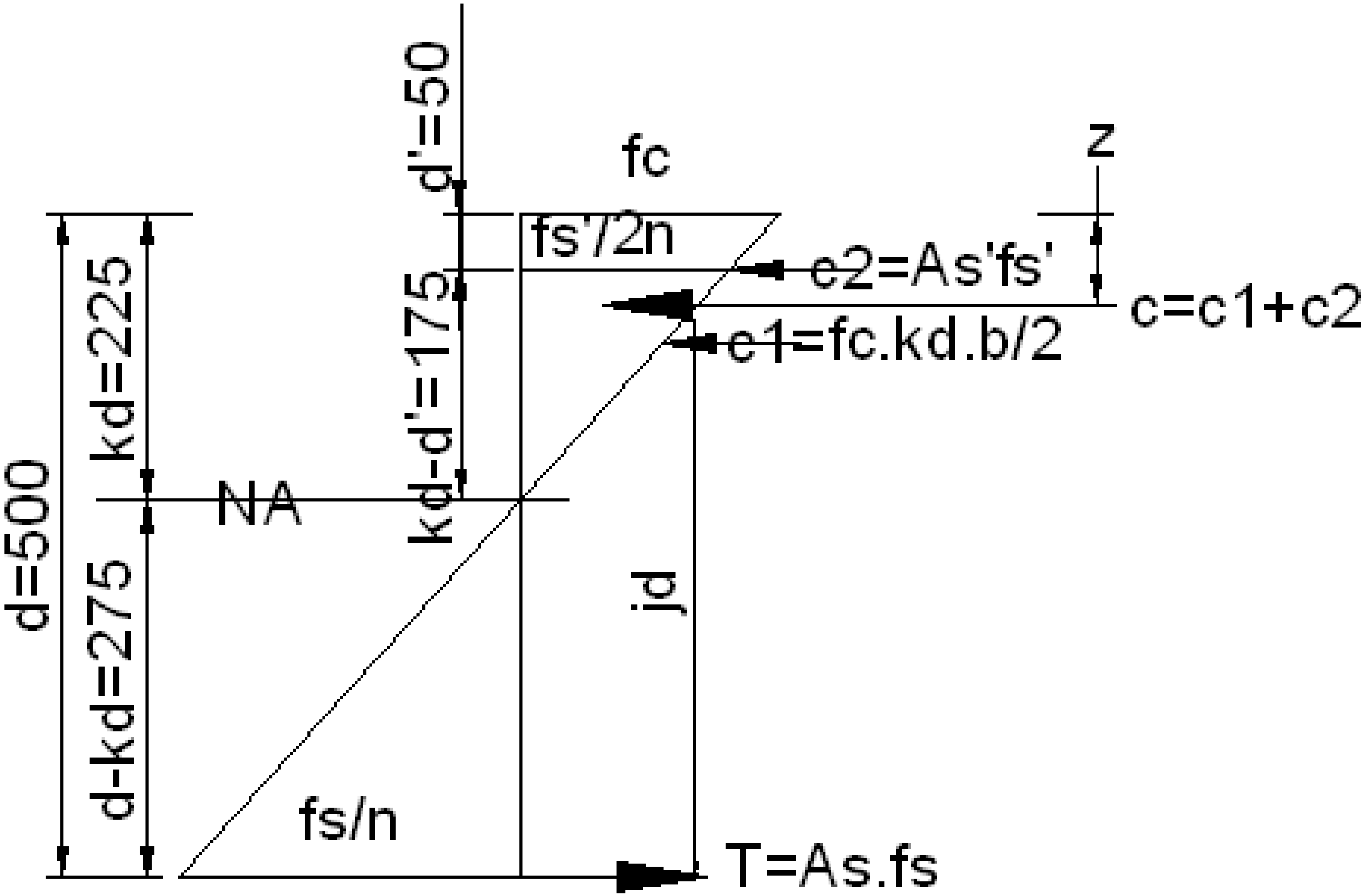
$$\mathrm{Mc}=\mathrm{c.jd}\rightarrow 150*10^{-3}=0.03997fc*0.4289\rightarrow fc$$

$$=8.75MPa$$

$$fs'=12.44fc=12.44*8.75=108.85MPa$$

EX2:

For data given in ex1,if fs=140MPa, fc'=25MPa, n=8, find moment capacity.



Solution:

Assume steel failure, fs=140MPa

$$\frac{fc}{kd}=\frac{\frac{fs}{n}}{d-kd}\rightarrow\frac{fc}{225}=\frac{\frac{140}{8}}{275}\rightarrow fc=14.3MPa>fc_{all}$$

$$=0.45*25=11.25MPa\rightarrow\therefore concrete\ failure$$

$$\frac{fc}{kd}=\frac{\frac{fs}{n}}{d-kd}\rightarrow\frac{11.25}{225}=\frac{\frac{fs}{8}}{275}\rightarrow fs=110MPa>fs_{all}$$

$$=140MPa$$

$$\frac{fc}{kd} = \frac{\frac{fs'}{2n}}{d - d'} \rightarrow \frac{11.25}{225} = \frac{\frac{fs'}{2*8}}{175} \rightarrow fs' = 140MPa = fs_{all}$$

$$= 140MPa$$

$$M = Asfsjd = 4000 * 10^{-6} * 110 * 10^{3} * 0.4289$$

$$= 189\ kN.m$$

OR

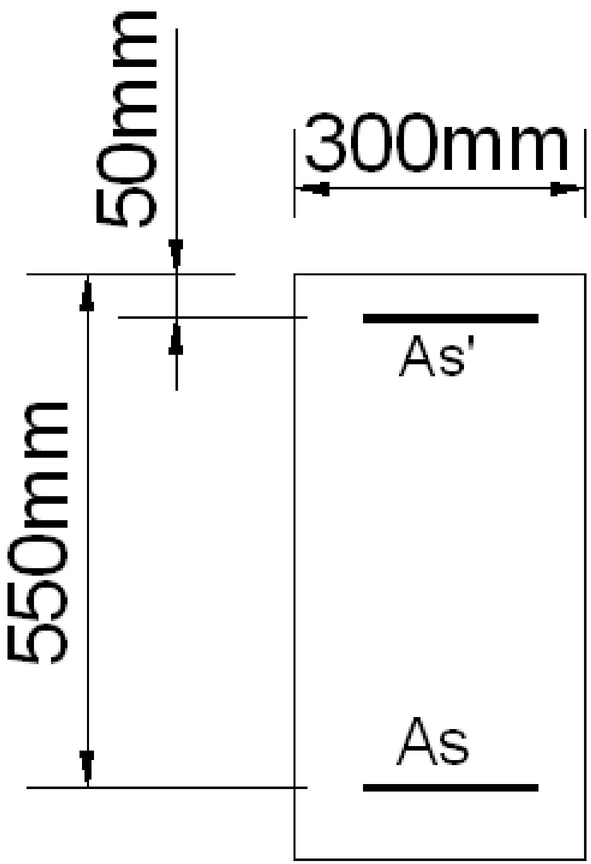
$$M = cjd = 0.03997fc.jd = 0.03997 * 11.25 * 10^{3} * 0.4289$$

$$= 189kN.m$$

Ex3:

If M=300kN.m, fc'=30MPa, fs=140MPa, n=8, design beam for flexure.

Solution:



Assume singly RC beam and balanced failure

$fc_{allowable}$=0.45*30=13.5MPa

$$k = \frac{1}{1+\frac{fs}{nfc}} = \frac{1}{1+\frac{140}{8*13.5}} = 0.44$$

kd=0.44*550=242mm

j=1-k/3=1-0.44/3=0.85

jd=0.85*550=467mm

$$M = \frac{fc.kd.b.jd}{2} \rightarrow 0.3 = \frac{fc * 0.242 * 0.3 * 0.467}{2} \rightarrow fc$$

$$= 17.6MPa > fc_{all} = 13.5MPa \rightarrow$$

$$\therefore compression\ reinforcement\ is\ required$$

M=Asfsjd

$$0.3 = As * 140 * 0.467 \rightarrow \quad As = 4.584 * 10^{-3} m^2$$

$$= 4584mm^2$$

$$Mc = \frac{fc.kd.b.jd}{2} = \frac{13.5 * 10^3 * 0.242 * 0.3 * 0.467}{2}$$

$$= 229kN.m$$

M'=Mt-Mc=300-229=71kN.m

$$M' = As'fs'(d - d') \rightarrow As' = \frac{71}{140 * 10^3 * (0.55 - 0.05)}$$

$$= 1.014 * 10^{-3}m^2 = 1014mm^2$$

$$use\ 9\text{Ø}25 + 1\text{Ø}20 = 4733mm^2\ tension\ reinf.$$

$$use\ 4\text{Ø}20 = 1256mm^2\ compression\ reinf.$$

$$\rho = \frac{As}{bd} = \frac{4733}{300 * 550} = 0.0287, \quad \rho' = \frac{As'}{bd} = \frac{1256}{300 * 550}$$

$$= 0.0076$$

$$k = \sqrt{2(2n-1)\rho'\frac{d'}{d} + 2n\rho + n^2(2\rho' + \rho - \frac{\rho'}{n})^2} - n(2\rho'$$

$$+ \rho - \frac{\rho'}{n})$$

$$k = \Big[2(2 * 8 - 1) * 0.0076 * \frac{50}{500} + 2 * 8 * 0.0287$$

$$+ 8^2(2 * 0.0076 + 0.0287 - \frac{0.0076}{8})^2\Big]^{\frac{1}{2}}$$

$$- 8\left(2 * 0.0076 + 0.0287 - \frac{0.0076}{8}\right) = 0.429$$

kd=0.429*550=236mm

$$z = \frac{\frac{d}{6}\ k^2 + \rho'(2n-1)(1-\frac{d'}{kd})d'}{\frac{k}{2}\ + \rho'(2n-1)(1-\frac{d'}{kd})}$$

$$z=\frac{\frac{550}{6}*\ 0.429^2+0.0076(2*8-1)\left(1-\frac{50}{236}\right)*50}{\frac{0.429}{2}\ +0.0076(2*8-1)(1-\frac{50}{236})}$$

=70.2mm

jd=d-z=550-70.2=480mm

$$M=Asfsjd\ \rightarrow fs=\frac{M}{Asjd}=\frac{0.3}{4733*10^{-6}*0.48}=132MPa$$

$$<fs_{all}=140MPa\ O.K$$

$$\frac{\frac{fs}{n}}{d-kd}=\frac{fc}{kd}\rightarrow\frac{\frac{132}{8}}{550-236}=\frac{fc}{236}\rightarrow fc=12.4MPa<fc_{all}$$

$$=13.5MPa\ O.K$$

$$\frac{\frac{fs\prime}{2n}}{kd-d'}=\frac{fc}{kd}\rightarrow\frac{\frac{fs\prime}{2*8}}{236-50}=\frac{fc}{236}\rightarrow fs'=156.4MPa$$

$$>fs_{all}=140MPa\ N.G$$

Increase As' gradually, till $fs' \leq fs_{all}$

$use\ 2\text{Ø}25+2\text{Ø}22=1740mm^2\ compression\ reinf.$

$$\rho = \frac{As}{bd} = \frac{4733}{300 * 550} = 0.0287, \quad \rho' = \frac{As'}{bd} = \frac{1740}{300 * 550}$$

$$= 0.0105$$

$$k = \sqrt{2(2n-1)\rho'\frac{d'}{d} + 2n\rho + n^2(2\rho' + \rho - \frac{\rho'}{n})^2} - n(2\rho'$$

$$+ \rho - \frac{\rho'}{n})$$

$$k = \Big[2(2*8-1)*0.0105*\frac{50}{500} + 2*8*0.0287 + 8^2(2*0.0105 + 0.0287 - \frac{0.0105}{8})^2\Big]^{\frac{1}{2}} - 8\left(2*0.0105 + 0.0287 - \frac{0.0105}{8}\right) = 0.411$$

kd=0.411*550=226mm

$$z = \frac{\frac{d}{6}\ k^2 + \rho'(2n-1)(1-\frac{d'}{kd})d'}{\frac{k}{2}\ + \rho'(2n-1)(1-\frac{d'}{kd})}$$

$$z=\frac{\frac{550}{6}*\ 0.411^2+0.0105(2*8-1)\left(1-\frac{50}{226}\right)*50}{\frac{0.411}{2}+0.0105(2*8-1)(1-\frac{50}{226})}$$

=65.8mm

jd=d-z=550-65.8=484mm

$$M=Asfsjd\ \rightarrow fs=\frac{M}{Asjd}=\frac{0.3}{4733*10^{-6}*0.484}$$

$$=131MPa<fs_{all}=140MPa\ O.K$$

$$\frac{\frac{fs}{n}}{d-kd}=\frac{fc}{kd}\rightarrow\frac{\frac{131}{8}}{550-226}=\frac{fc}{226}\rightarrow fc=11.42MPa$$

$$<fc_{all}=13.5MPa\ O.K$$

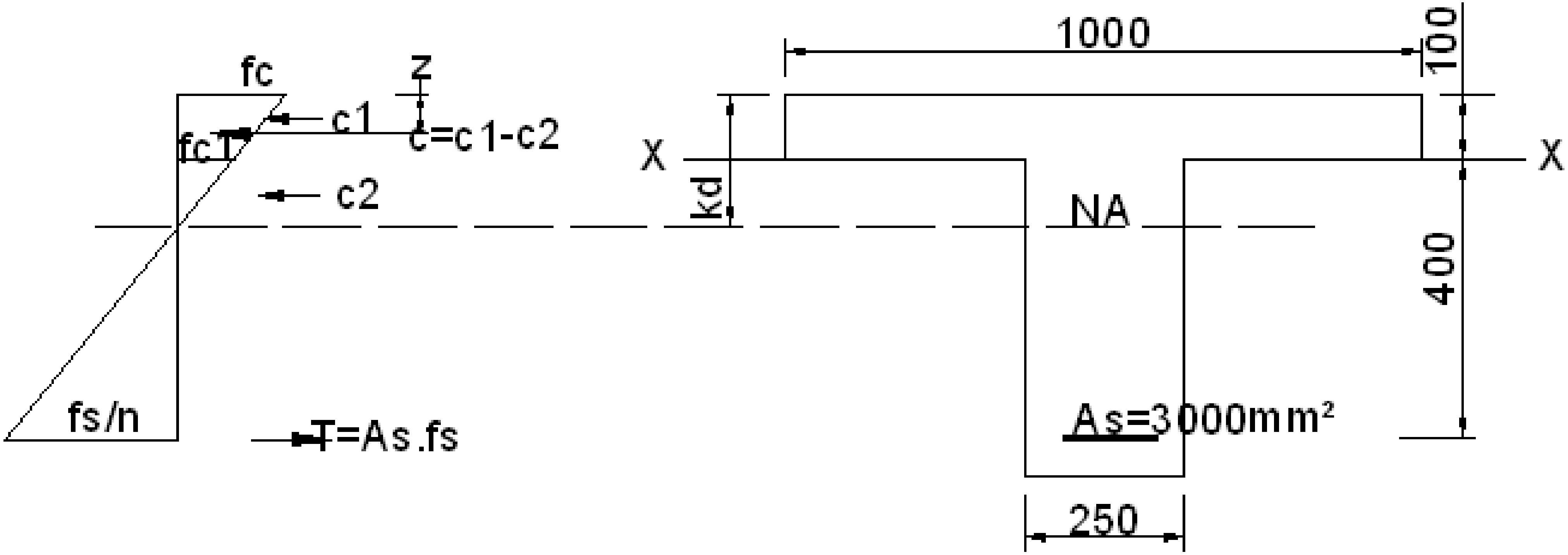
$$\frac{\frac{fs'}{2n}}{kd-d'}=\frac{fc}{kd}\rightarrow\frac{\frac{fs'}{2*8}}{226-50}=\frac{fc}{226}\rightarrow fs'=140MPa\leq fs_{all}$$

$$=140MPa\ O.K$$

T-Beam Section:

EX1:

If M=200kN.m, $n = \frac{Es}{Ec} = 12$, find fc and fs.



Solution:

Assume N.A at x-axis

$$\sum M_{X-X}$$

$$moment\ of\ comp.area = 1000 * 100 * \frac{100}{2} = 5 * 10^6 mm^3$$

$$moment\ of\ ten.area = 12 * 3000 * 400 = 14.4 * 10^6 mm^3$$

Since $moment\ of\ ten.area > moment\ of\ comp.area \rightarrow$

$NA\ within\ web$

$$\sum M_{NA} = 0$$

$$1000 * kd * \frac{kd}{2} - (kd - 100)(1000 - 250)(\frac{kd - 100}{2})$$

$$= nAs(d - kd)$$

$$1000 * \frac{kd^2}{2} - (kd - 100)^2 * 375 = 12 * 3000(500 - kd)$$

$$\rightarrow kd = 165mm > hf$$

$$c1 = \frac{fc}{2} * kd * b = \frac{fc}{2} * 165 * 1000 = 82500fc$$

$$\frac{fc}{kd} = \frac{fc1}{kd - hf} \rightarrow \frac{fc}{165} = \frac{fc1}{165 - 100} \rightarrow fc1 = 0.39fc$$

$$c2 = \frac{0.39fc}{2} * (kd - hf)(b - bw)$$

$$= \frac{0.39fc}{2} * (165 - 100)(1000 - 250) = 9506fc$$

$$c = c1 - c2 = 82500fc - 9506fc = 72994fc$$

$$\sum M_{top\ fiber} = 0$$

$$c1 * \frac{kd}{3} - c2 * \left(hf + \frac{kd - hf}{3}\right) = c * z$$

$$82500fc * \frac{165}{3} - 9506fc * \left(100 + \frac{165 - 100}{3}\right)$$

$$= 72994fc * z \rightarrow z = 46.3mm$$

jd=d-z=500-46.3=453.7mm

$$M = Asfsjd \rightarrow fs = \frac{0.2}{3000 * 10^{-6} * 0.4537} = 147MPa$$

$$M = cjd \rightarrow 0.2 = 72994fc * 0.4537 \rightarrow fc = 6.04MPa$$

EX2:

For data given in ex1,if fs=140MPa, fc'=30MPa, find moment capacity.

Solution:

$$fc = 6.04MPa < fc_{all} = 0.45fc' = 0.45 * 30 = 13.5MPa$$

$$fs = 147MPa > fs_{all} = 140MPa \rightarrow \therefore steel\ failure$$

$$M = Asfsjd = 3000 * 10^{-6} * 140 * 10^{3} * 0.4537$$

$$= 190kN.m$$

OR

$$\frac{fc}{kd} = \frac{\frac{fs}{n}}{d - kd} \rightarrow \frac{fc}{165} = \frac{\frac{140}{12}}{500 - 165} \rightarrow fc = 5.74MPa$$

$$M = cjd = 72994 * 5.74 * 0.4537 * 10^{-3} = 190kN.m$$