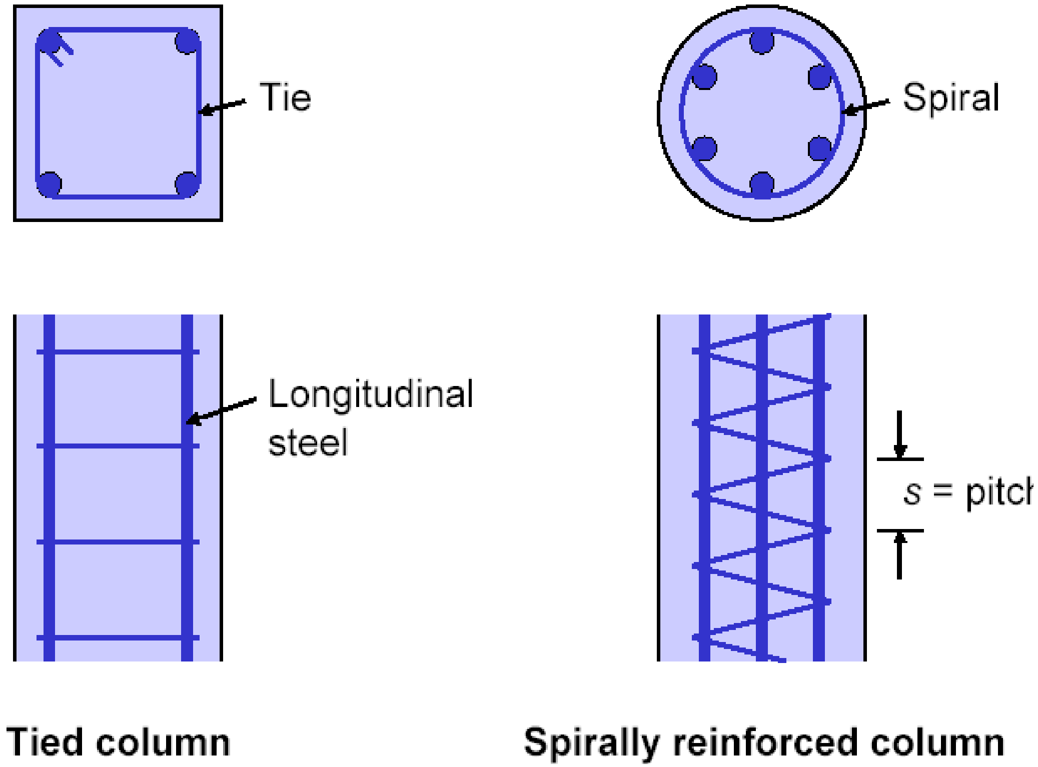
Reinforced Concrete Columns

Member with ratio of height to least lateral dimension exceeding 3 used primarily to support axial compressive load.

Types of columns:

1. Tied columns: which have a rectangular, circular or square cross section with longitudinal bars and lateral ties.
2. Spirally reinforced columns: which have a circular or square cross section with longitudinal bars circular shape arrangement and closely spaced spirals.
3. Composite columns: reinforced longitudinally with structural steel shapes, pipes or tubing with or without additional bars and lateral reinforcement.



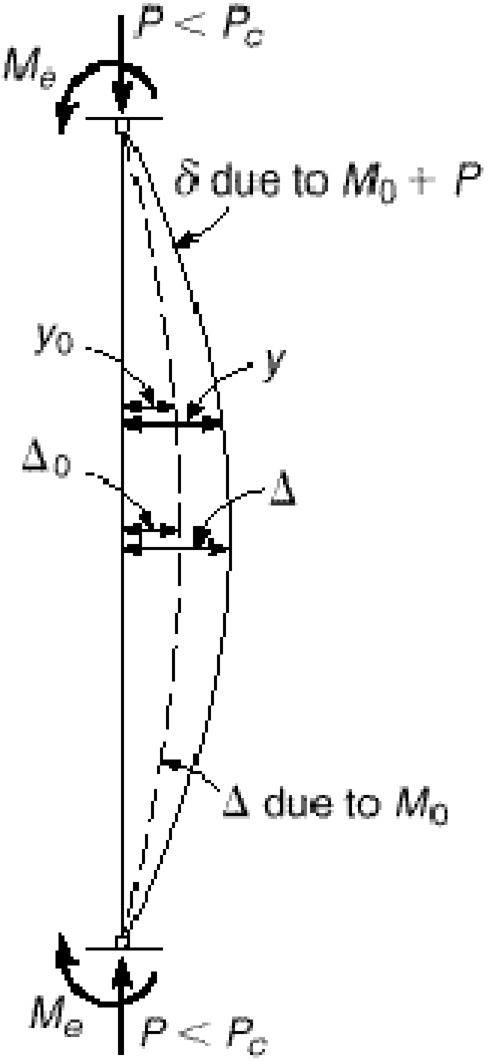
Columns may be divided into :

1. Short columns:

which fail due to initial material failure, depending on the dimensions and strength of materials used.

2. Long columns(slender columns):

Columns which may be failed by buckling due to applied moments on columns, columns deflects laterally causing additional moment=P.Δ, called secondary moment or P-Δ moment.



Behavior of axially loaded columns:

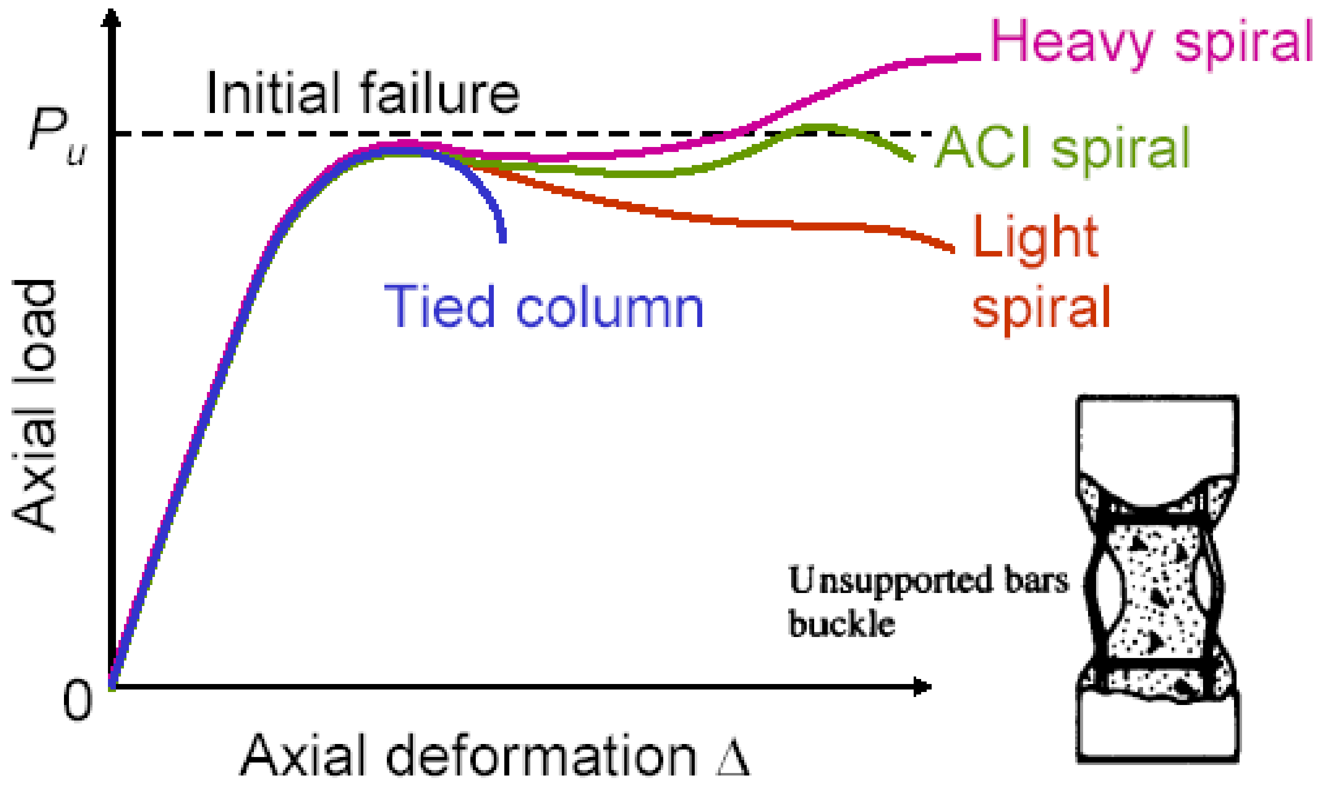
Tied columns:

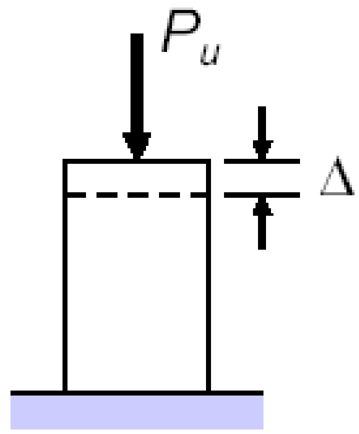
At this load(failure load, Pn), the concrete fails by crushing and shearing outward along inclined planes, and the longitudinal steel by buckling outward between ties fig.(a) below.

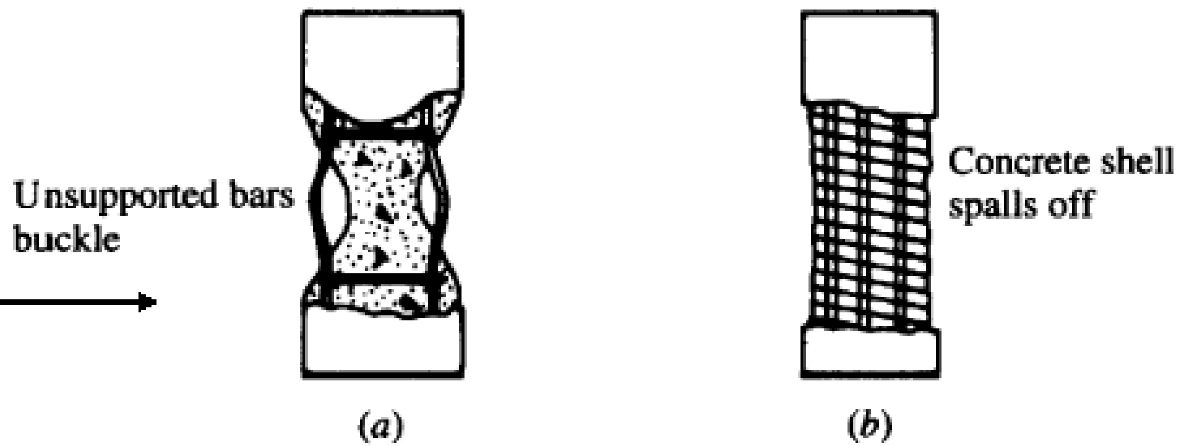
Spirally reinforced columns:

When the same load(Pn above) is reached, the longitudinal steel and the concrete within the core are prevented from moving outward by the spiral. The concrete in the outer shell, however, not being so confined, does fail, i.e the outer shell spalls off when the load Pn is reached fig.(b) below.

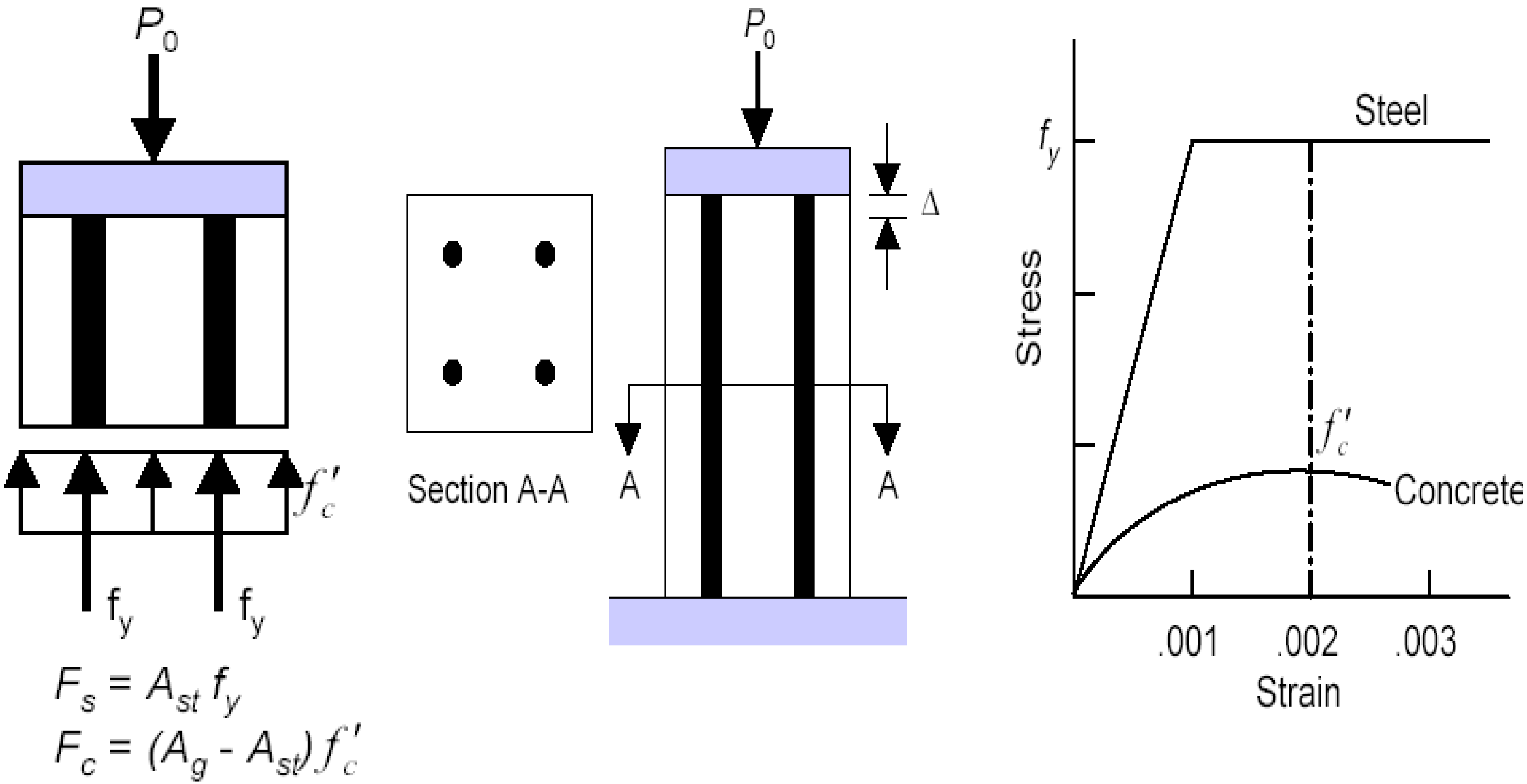
Any excess capacity beyond the spalling load of the shell is wasted because the member, although not actually failed, would no longer be considered serviceable.







Strength of short axially loaded columns:



$$\sum Force = 0$$

$$Po = fc\ Ac + fs\ As \quad elastic\ stage$$

$$Po = fc\ Ac + n\ fc\ As \quad , \quad n = \frac{ES}{Ec}$$

$$Po = fc(\ Ac + n\ \ As) = fc(\underbrace{Ac + As}_{Ag} + (n-1)As)$$

$$Po = fc(Ag + (n-1)As)$$

The design axial load strength of an axially loaded columns can be evaluated by:

$$\emptyset Pn_{,max} = 0.85\emptyset[0.85fc'(Ag - Ast) + Astfy] \quad , \emptyset = 0.7$$

$$\ldots \ldots ACI\ eq. 10 - 1 for\ spiral\ columns, sec\ 10.3.6.1$$

$$\emptyset Pn_{,max} = 0.80\emptyset[0.85fc'(Ag - Ast) + Astfy] \ , \emptyset = 0.65$$

$$\ldots ACI\ eq. 10 - 2 for\ tied\ columns, \text{sec } 10.3.6.1$$

Ac: concrete area, mm^2.

Ast: total area of longitudinal reinforcement, (bars, or steel shapes).

Ag: gross area of section, mm^2.

Pn: nominal strength of column.

The present code does not specify min. eccentricity of load(as in previous code), but satisfy the same objective by multiplying the nominal strength(axial load) by a factor of 0.85 for spiral reinforced column and 0.80 for tied column.

ACI Code requirements for columns:

1. $0.01 \leq (\rho_g = \frac{Ast}{Ag}) \leq 0.08$ ACI 10.9.1 usually $\rho_g \leq 0.05$

   Lower limit=0.01 :to prevent failure mode plain concrete

   Upper limit=0.08 :to maintain proper clearances between bars

2. Min. no. of longitudinal bars(ACI 10.9.2) ≥ 4 for tied columns

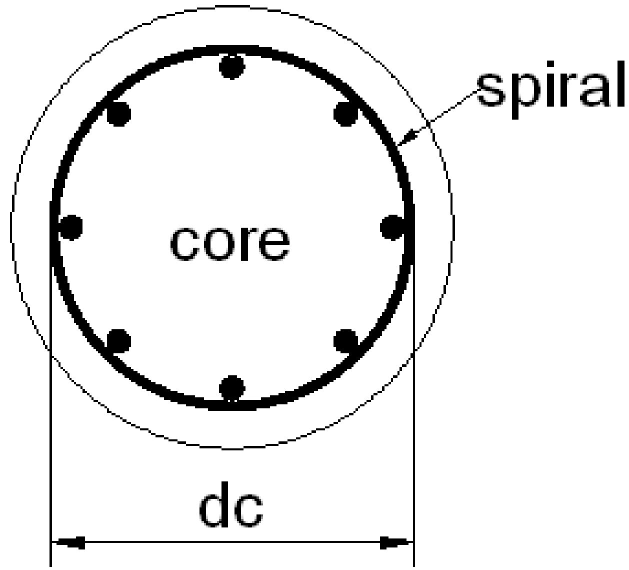
   ≥ 6 for spiral columns

3. $\rho_s \geq 0.45\left(\frac{Ag}{Ac}-1\right)\frac{fc'}{fy}$,

$fy \leq 420MPa\ yield\ strength\ of\ spiral$ ACI 10.9.3

ρ_s : the ratio of volume of spiral reinforcement to the total volume of core(out-to-out of spiral) of spirally columns.

$$\rho_s = \frac{\pi\, d_c Asp}{\frac{\pi (d_c)^2}{4} s} = \frac{4Asp}{d_c.s}$$



4. $\emptyset_{tie} \geq$

$$\begin{cases} 10mm\ for\ long.bars\ diamster \leq 32m \\ 12mm\ for\ long.bars\ diamster > 32m \end{cases}$$

5. $spacing\ between\ ties \leq$

$$\begin{cases} 16d_b \\ 48d_{tie} \\ least\ dimension\ of\ column\ cross\ section \end{cases} \quad ACI 7.10.5.2$$

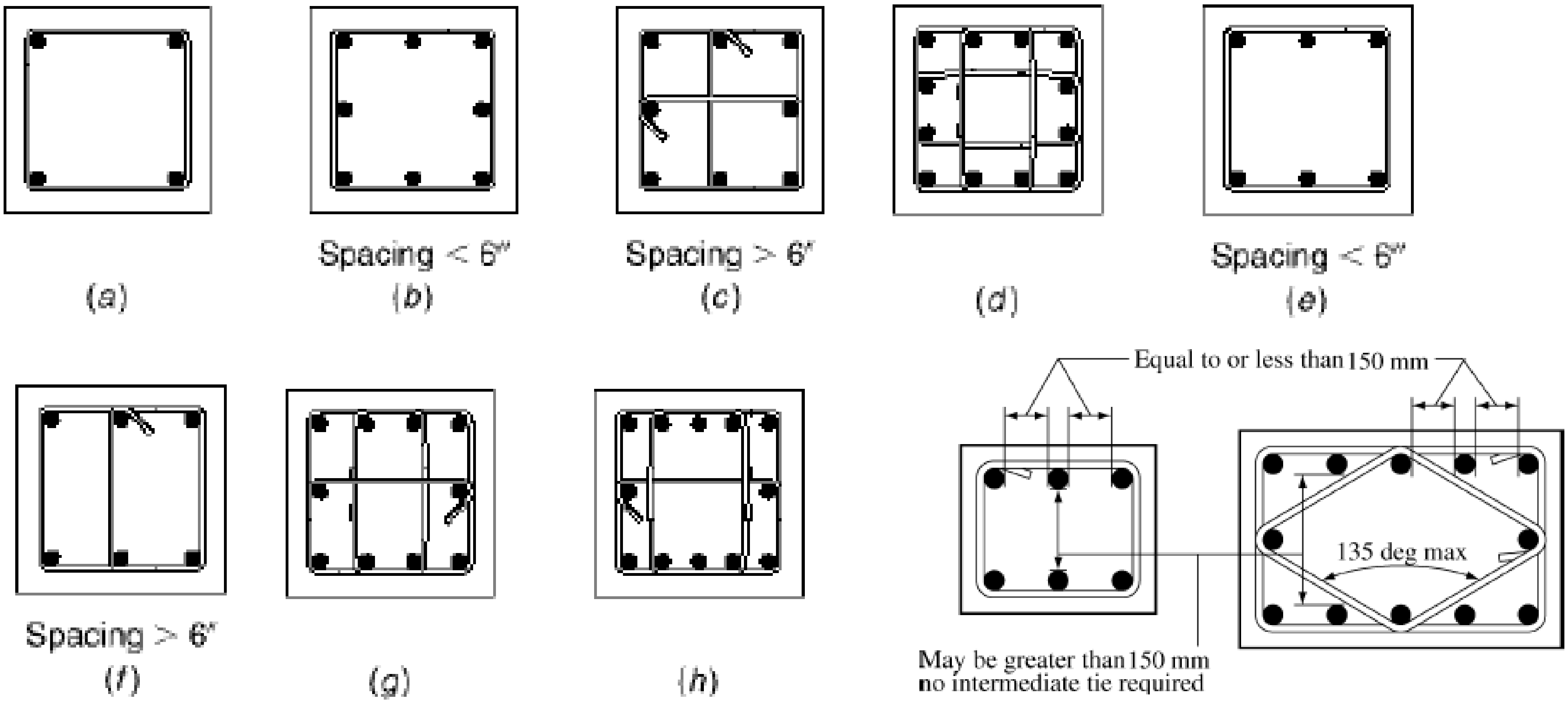
6. For spirally reinf. Columns $25mm \leq pitch(s) \leq 75mm$,

$lap\ splice = \max(48d_{spiral}, 300mm)$ ACI 7.10.4

7. $Clear\ spacing\ between\ long.bars \geq$

$$\begin{cases} 1.5d_b \\ 40mm \\ \frac{4}{3} max.of\ aggregate \end{cases}$$ ACI 7.6.3

8. Ties shall be arranged such that every corner and alternate long. bar shall have lateral support provided by the corner of a tie with an included angle not more than 135^o and no bar shall be further than 150mm clear spacing on each side along the tie from such a lateral supported. Where long. bars are located around the perimeter of a circle , a complete circular ***ties*** shall permitted. ACI 7.10.5.3


(a)
Spacing < 6"
(b)
Spacing > 6"
(c)
(d)
Spacing < 6"
(e)
Spacing > 6"
(f)
(g)
(h)
Equal to or less than 150 mm
135 deg max
May be greater than 150 mm
no intermediate tie required

6"=150mm

Ex: Design of axially loaded columns

a. Design of tied square column to support an axial service load(Pd=1400kN, Pl=1600 kN). Given: fy=400 MPa, fc'=30MPa, steel ratio=0.03.

Solution:

Pu=1.2DL+1.6LL=1.2*1400+1.6+1600=4240kN

$$Pu = \emptyset Pn_{,max} = 0.80\emptyset[0.85fc'(Ag - Ast) + Astfy]$$

$$4240000 = 0.80 * 0.65[0.85 * 30(Ag - 0.03Ag) + 0.03Ag * 400]$$

Ag=221964 mm^2=h^2→h=471mm

Use 480*480mm tie column

$$4240000 = 0.80 * 0.65[0.85 * 30(480^2 - Ast) + Ast * 400]$$

As=6085 mm^2

Use 12Ø28mm long. bars (Ast=7390 mm^2

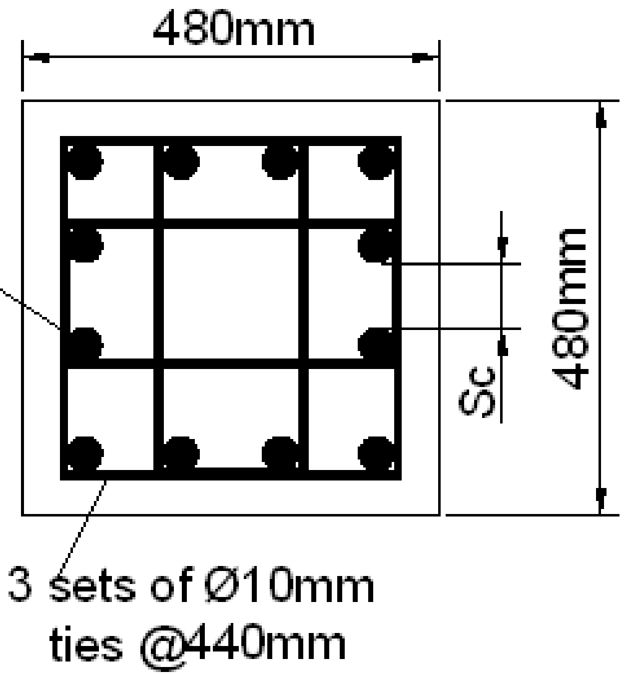
$$for\ db = 28mm < 32mm, use\ tie\ \emptyset 10mm\ @$$

$$min\begin{cases} 16d_b = 16 * 28 = 448mm\ \ control \\ 48d_{tie} = 48 * 10 = 480mm \\ least\ dimension\ of\ column\ cross\ section = 480mm \end{cases}$$

$$use\ tie\ \emptyset 10mm\ @440mm\ c/c$$

$$s_c = \frac{480-2*40-4*28-2*10}{4-1} = 89mm \geq$$

$$max\begin{cases} 1.5db = 1.5*28 = 42mm \\ 40mm \end{cases} \quad O.K$$



b. Design of spirally circular column to support an axial service load(Pd=1400kN, Pl=1600 kN). Given: fy=400 MPa, fc'=30MPa, steel ratio=0.03.

Solution:

Pu=1.2DL+1.6LL=1.2*1400+1.6+1600=4240kN

$$Pu = \emptyset Pn_{,max} = 0.85\emptyset[0.85fc'(Ag - Ast) + Astfy]$$

$$4240000 = 0.85 * 0.7[0.85 * 30(Ag - 0.03Ag) + 0.03Ag * 400]$$

Ag=193985 $mm^2 = \frac{\pi D^2}{4} \rightarrow D = 497mm$

Use D=500mm → $Ag = \frac{\pi * 500^2}{4} = 196350\ mm^2$

4240000

$= 0.85 * 0.7[0.85 * 30(196350 - Ast) + Ast * 400]$

As=5659 mm^2

Use 10Ø28mm long. bars (Ast=6160 mm^2)

dc=500-2*40=420mm

Ac=$\frac{\pi(420)^2}{4} = 138544\text{mm}^2$

$$\rho_{s,min} \geq 0.45\left(\frac{Ag}{Ac} - 1\right)\frac{fc'}{fy} = 0.45\left(\frac{196350}{138544} - 1\right)\frac{30}{400}$$

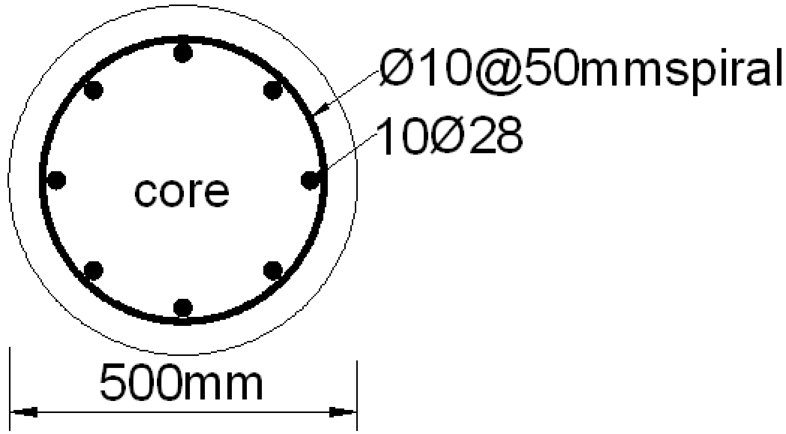
$$= 0.0141$$

$$\rho_s = \frac{4Asp}{d_c.s} = \frac{4 * 79}{420.s} = 0.0141 \rightarrow s = 53mm \begin{cases} < 75mm \\ > 25mm \end{cases} O.K$$

$$use\ spiral\ \emptyset 10mm\ @50mm\ pitch$$

$$s_c = \frac{\pi(420 - 2 * 10 - 2 * \frac{28}{2})}{10} = 93mm$$

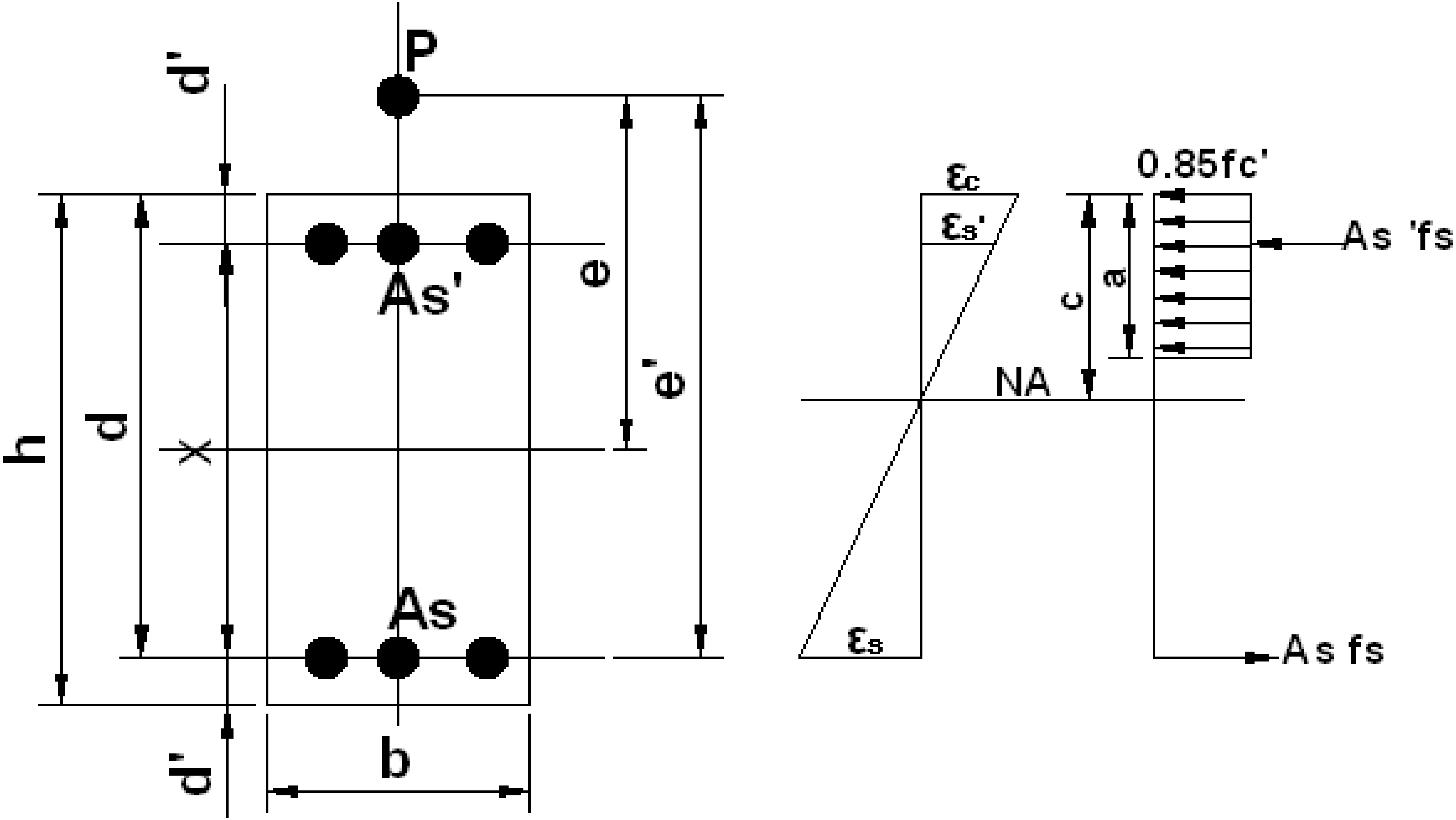
$$\geq max \begin{cases} 1.5db = 1.5 * 28 = 42mm \\ 40mm \end{cases} O.K$$


Ø10@50mmspiral
10Ø28
core
500mm

Axial Compression Plus Bending About One Axis

Eccentric load: load applied at point of plastic centroid of section

Concentric load: load applied out of the point of plastic centroid of section


P
d'
As'
e
e'
h
d
x
As
b
d'
εc
εs'
0.85fc'
As 'fs
c
a
NA
εs
As fs

Assume:

- As=As',
- $\varepsilon s' \geq \varepsilon y \rightarrow fs'=fy$

$$\sum force = 0$$

$$Pu' = 0.85fc'ba + As'fs' - Asfs$$

$$\sum M_{AS} = 0$$

$$Pu'e' = 0.85fc'ba\left(d - \frac{a}{2}\right) + As'fs'(d - d')$$

All possible failures are allowed in columns.

1. Balance failure

$$\varepsilon s = \varepsilon y, \quad \varepsilon c = \varepsilon cu$$

Pu'=P'$_b$, e=e$_b$, e'=e'$_b$

$$, \sum force = 0 \; P_b{}' = 0.85 fc' b a_b + As' fs' - As fs$$

εs' ≥ εy⟶ fs'=fy, As=As'

$$P_b{}' = 0.85 fc' b a_b$$

From strain diagram $a_b = \beta 1 a_b = \frac{600\beta 1}{600+fy}\, d$

$$P_b{}' = 0.85 fc' b \frac{600\beta 1}{600 + fy}\, d$$

$$\sum M_{AS} = 0$$

$$P_b{}'e_b{}' = 0.85fc'ba_b\left(d - \frac{a_b}{2}\right) + As'fy(d - d')$$

$$e_b = e_b{}' - (\frac{h}{2} - d')$$

$$M_b{}'$$

$$= P_b{}'e_b \ \ bending\ moment\ capacity\ at\ balanced\ failure$$

2. If $e > e_b$

$\varepsilon s = \varepsilon y, \quad \varepsilon c < \varepsilon cu$ tensile steel yielding

$\varepsilon s > \varepsilon y, \quad \varepsilon c = \varepsilon cu$ secondary compression failure

$$\sum force = 0, \quad P_u' = 0.85 f c' ba \ldots \ldots \ldots \ldots \ldots 1$$

$$\sum M_{AS} = 0, P_u' e' = 0.85 f c' ba \left(d - \frac{a}{2}\right) + As' fy(d - d') \ldots 2$$

Solution of eq1&2, gives a second degree polynomial in P_u'

$$P_u' = 0.85 f c' bd[-\left(\frac{e'}{d} - 1\right) + \sqrt{\left(\frac{e'}{d} - 1\right)^2 + 2\rho\mu(1 - \frac{d'}{d})}$$

$$\rho = \frac{As}{bd} = \frac{As'}{bd}$$

$$\mu = \frac{fy}{0.85fc'}$$

$$P_u' < P_b'$$

3. If e<eb

$\varepsilon_u = \varepsilon_{cu}$ $\quad$ $\varepsilon_s < \varepsilon_y,$ $\quad$ primary compression failure

$$\sum force = 0,$$

$$P_u' = 0.85fc'ba + As'fy - Asfs \ldots\ldots\ldots\ldots\ldots\ldots\ldots..1$$

$$\sum M_{AS} = 0, P_u'e' = 0.85fc'ba\left(d - \frac{a}{2}\right) + As'fy(d - d') \ldots.2$$

From strain diagram:

$$\frac{\varepsilon_s}{d-c} = \frac{\varepsilon_{cu}}{c}$$

$$fs = Es\varepsilon_s = 600\left(\frac{\beta 1d}{a} - 1\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3$$

Solve eq.1,2,&3 for the three unknowns a, fs and Pu'

Note: cubic polynomial in one of the unknowns

Newton-Raphson method for finding roots of polynomial.

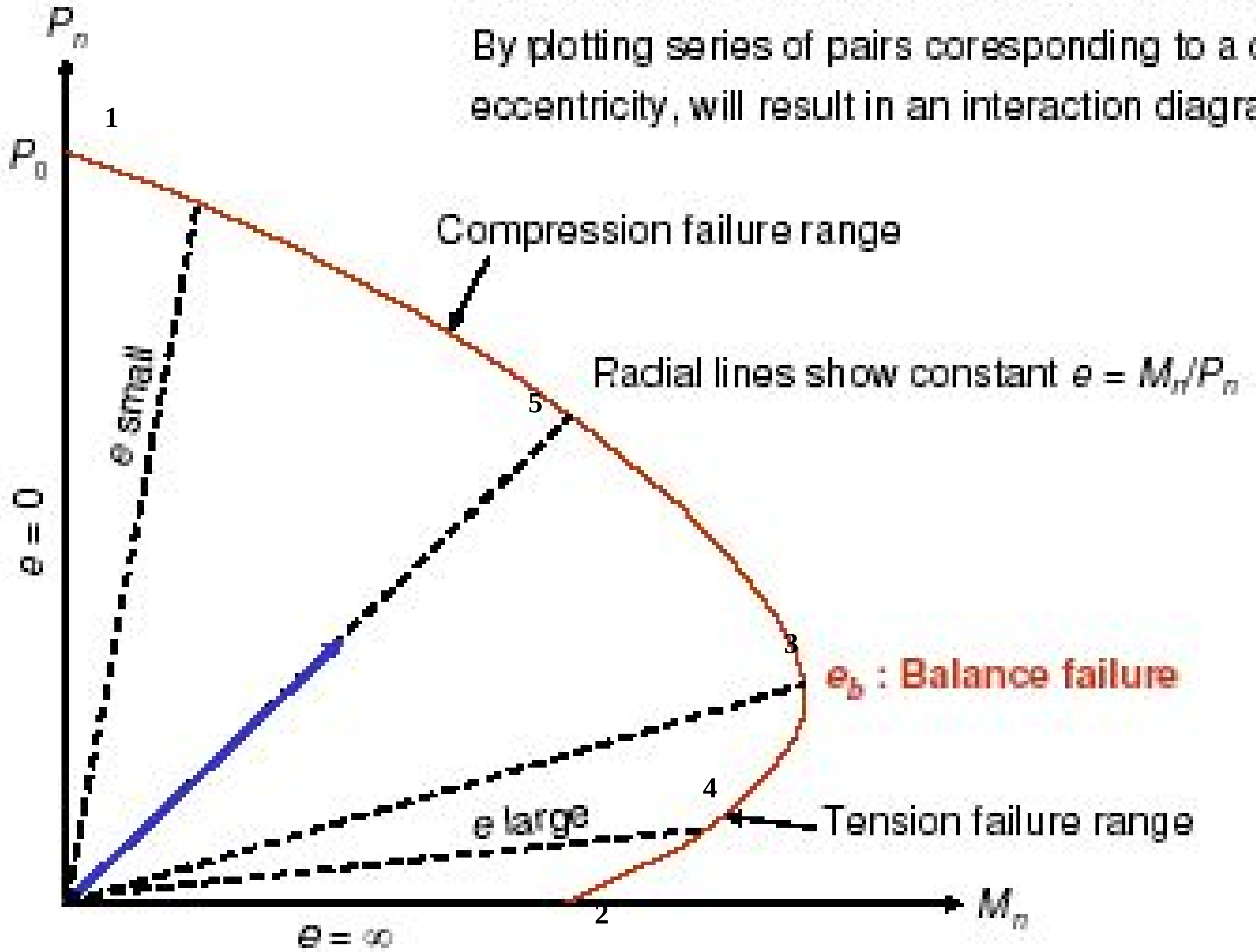
Summary:

1. $e=0$, $Pu' = Po' = 0.85fc'bh + As'fy + Asfy$
2. $e \to \infty$ $Mu' = Mo'$ = Pure bending moment capacity of doubly reinforced section.
3. $e = e_b$ $Pu' = P_b'$, $Mu' = M_b'$ balanced failure
4. $e > e_b$, $Pu' < P'_b$, $Mu' = Pu'.e$ tensile failure
5. $e < e_b$, $Pu' > P'_b$, $Mu' = Pu'.e$ compressive failure

Interaction Diagram for Combined Bending and Axial Load

For any eccentricity, there is a unique pair of P_n and M_n.

By plotting series of pairs coresponding to a different eccentricity, will result in an interaction diagram.



path 1-5-3	compression failure
point 3	balanced failure
path 3-4-2	tensile failure

Example:

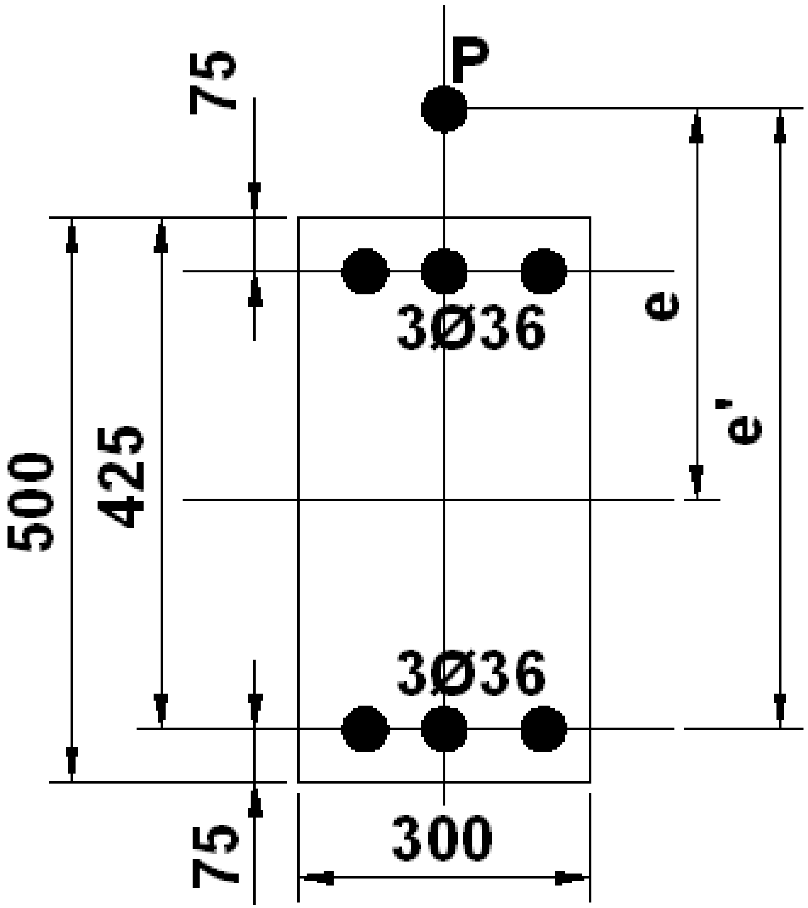
fy=414MPa, fc'=28MPa, d'=75mm,

Find: P'_o, M'_o, e_b, P'_b, M'_b

If e=700mm > e_b, find P'u

If e=180mm < e_b, find P'u

Solution:



1. balanced failure condition:

$\varepsilon s = \varepsilon y, \quad \varepsilon c = \varepsilon cu$

From strain diagram $a_b = \beta 1 a_b = \frac{600 * .85}{600+414} * 0.425 = 0.214m$

$\sum force = 0 \,, P_b' = 0.85 fc' b a_b$

$$= 0.85 * 28 * 0.3 * 0.214 = 1.528MN$$

$$\sum M_{AS} = 0, P_b{}'e_b{}' = 0.85fc'ba_b\left(d - \frac{a_b}{2}\right) + As'fy(d - d')$$

$$1.528 * e_b{}' = 0.85 * 28 * 0.3 * 0.214\left(0.425 - \frac{0.214}{2}\right) +$$

$$3053 * 10^{-6} * 414(0.425 - 0.075)$$

$$e_b{}' = 0.607m$$

$$e_b = 607 - \left(\frac{500}{2} - 75\right) = 432mm$$

$M'_b=P'_b*e_b=1.528*0.432=0.660MN.m$

2. If e=700m > e_b =432mm tensile failure

$\varepsilon s = \varepsilon y, \quad \varepsilon c < \varepsilon cu$ tensile steel yielding

$\varepsilon s > \varepsilon y, \quad \varepsilon c = \varepsilon cu$ secondary compression failure

$$\sum force = 0\,, P_u' = 0.85fc'ba = 0.85 * 28 * 0.3 * a \quad \ldots\ldots 1$$

$$\mathrm{e}' = \mathrm{e} + \frac{\mathrm{h}}{2} - \mathrm{d}' = 0.7 + \frac{0.5}{2} - 0.075 = 0.875\ \mathrm{m}$$

$$\sum M_{AS} = 0\,, P_u'e' = 0.85fc'ba\left(d - \frac{a}{2}\right) + As'fy(d - d') \ldots 2$$

$$P_u' * 0.875 = 0.85 * 28 * 0.3 * a\left(0.425 - \frac{a}{2}\right) +$$

$$3053 * 10^{-6} * 414(0.425 - 0.075) \ldots\ldots\ldots 2$$

Solution of eq1&2, gives a second degree polynomial in P'_u

$$P_u'^2 + 6.437P_u' - 6.314 = 0$$

$$P_u' = 0.865MN < \mathrm{P}'\mathrm{b} = 1.528MN$$

a=0.121m

$M_u'=P_u'*e=0.865*0.7=0.605MN.m < M_b'=0.660MN.m$

3. If e=180mm < e_b= 432mm primary compression failure

$$\varepsilon_u = \varepsilon_{cu} \qquad \varepsilon_s < \varepsilon_y,$$

$$\sum force = 0,$$

$$P_u' = 0.85fc'ba + As'fy - Asfs \ldots\ldots\ldots\ldots\ldots\ldots\ldots..1$$

$$P_u' = 0.85 * 28 * 0.3 * a + 3053 * 10^{-6} * 414 - 3053 * 10^{-6} * fs\ldots1$$

$$e' = e + \frac{h}{2} - d' = 180 + \frac{500}{2} - 75 = 355mm$$

$$\sum M_{AS} = 0\,, P'_u e' = 0.85 fc' ba\left(d - \frac{a}{2}\right) + As' fy(d - d') \ldots .2$$

$$P'_u * 0.355 = 0.85 * 28 * 0.3 * a\left(0.425 - \frac{a}{2}\right) +$$

$$3053 * 10^{-6} * 414(0.425 - 0.075) \ldots \ldots . . .2$$

From strain diagram:

$$\frac{\varepsilon_s}{d - c} = \frac{\varepsilon_{cu}}{c}$$

$$fs = Es\varepsilon_s = 600\left(\frac{\beta 1 d}{a} - 1\right)$$

$$= 600\left(\frac{0.85 * 0.425}{a} - 1\right) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots 3$$

Solution of eq.1,2 &3 gives

$$3.57a^3 - 0.499a^2 + 0.657a - 0.235 = 0$$

$$f(a) = 3.57a^3 - 0.499a^2 + 0.657a - 0.235$$

$$f'(a) = 10.71a^2 - 0.998a + 0.657$$

$$a_{i+1} = a_i - \frac{f(ai)}{f'(ai)}$$

a	$f(a)$	$f'(a)$	$a_{i+1} = a_i - \frac{f(ai)}{f'(ai)}$
0.25m	-0.0461	1.077	0.293
0.293	0.00461	1.284	0.289
0.289	-0.000633	1.263	0.289

a=0.289m

$$a = \beta 1c \rightarrow c = \frac{0.289}{0.85} = 0.340m$$

$$fs = 600\left(\frac{0.85 * 0.425}{0.289} - 1\right) = 150MPa < fy$$

$$P'_u = 0.85 * 28 * 0.3 * 0.289 + 3053 * 10^{-6} * 414 - 3053 * 10^{-6} * 150 = 2.869MN > P'_b = 1.528MN$$

$M_u'=P_u'*e=2.869*0.18=0.516MN.m < M_b'=0.660MN.m$

4. Axially loaded column, e=0

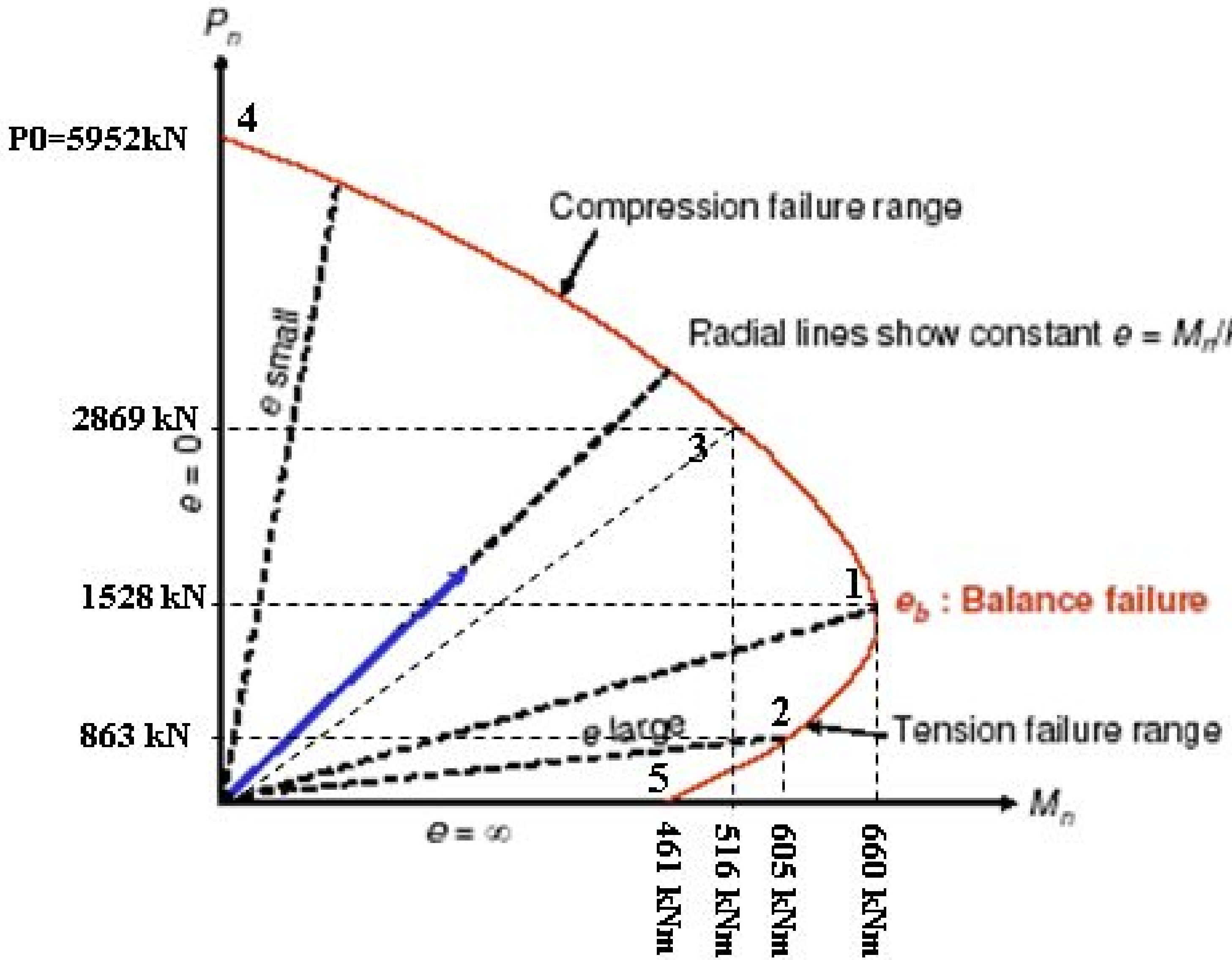
$$P_{\circ}' = [0.85fc'(Ag - Ast) + Astfy]$$

$$P_{\circ}' = 0.85 * 28(0.5 * 0.3 - 2 * 3053 * 10^{-6}) + 2 * 3053 * 10^{-6} * 414 = 5.952MN$$

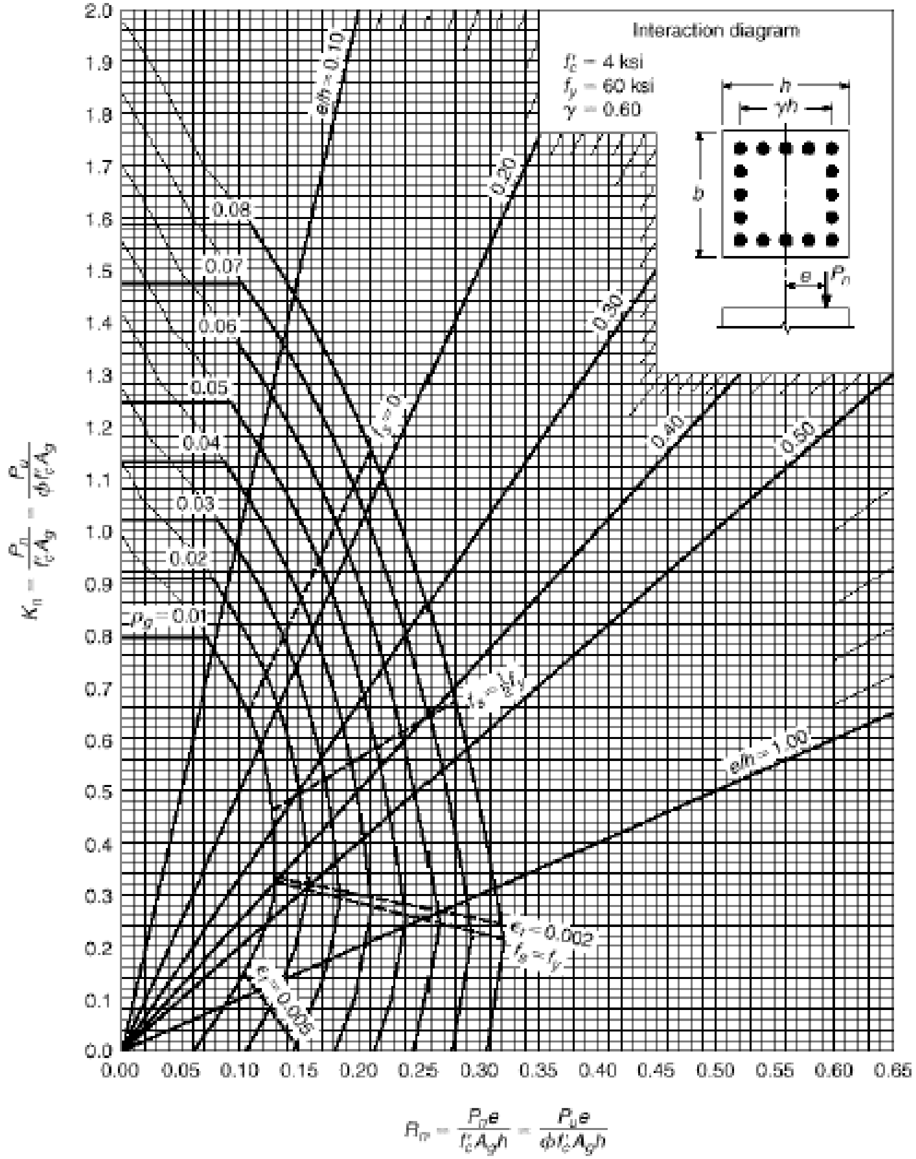
5. Pure bending moment, Mo', e →∞

Mo'=0.461MN.m

Case	**e (m)**	**Pu' (kN)**	**Mu' (kN.m)**	**C (m)**	**Point**
Pure comp.	0	5952	0	-∞	1
Comp. failure	0.180	2869	516	0.340	2
Balanced failure	0.432	1528	660	0.252	3
Tensile failure	0.700	865	605	0.142	4
Pure bending	∞	0	461		5

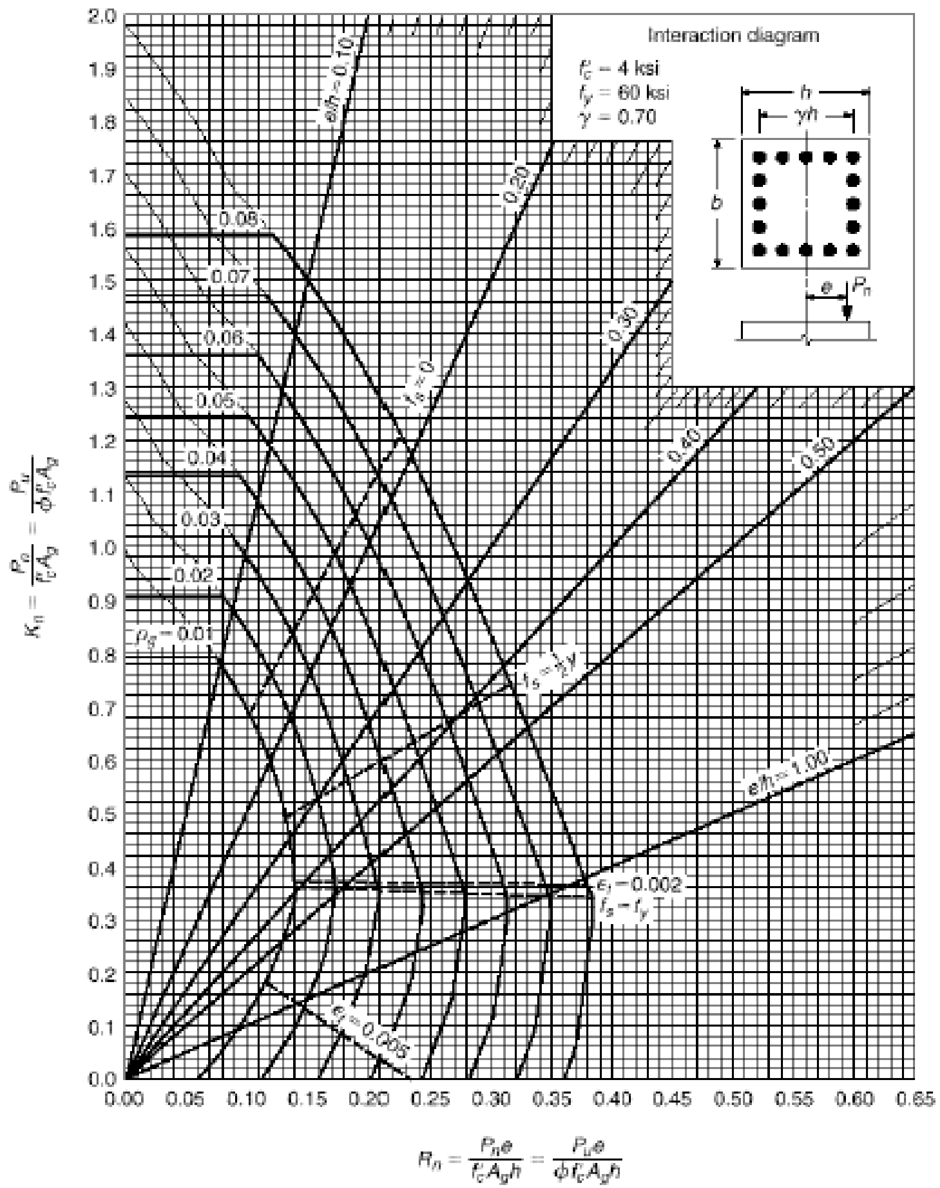


Failure envelope (inter action diagram)



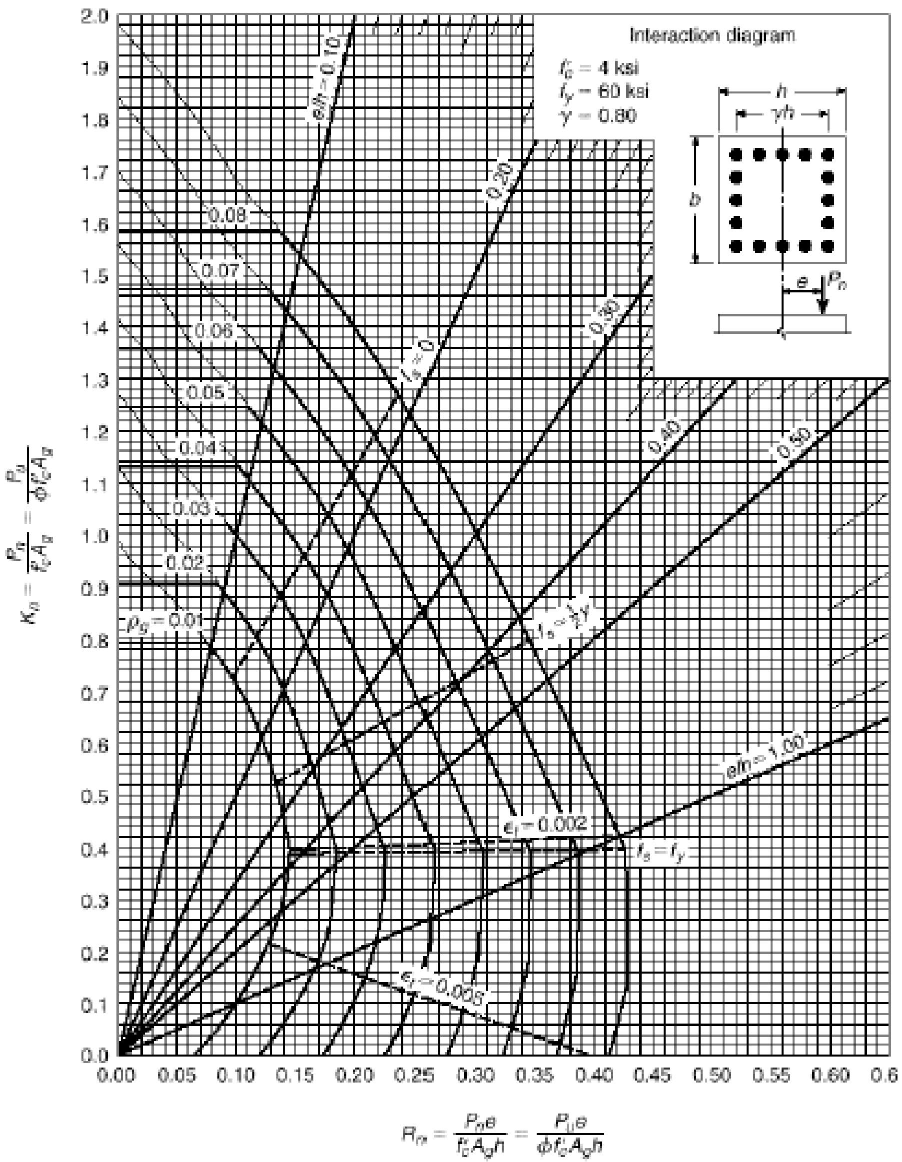
GRAPH A.5

Column strength interaction diagram for rectangular section with bars on four faces and γ = 0.60 (for instructional use only).



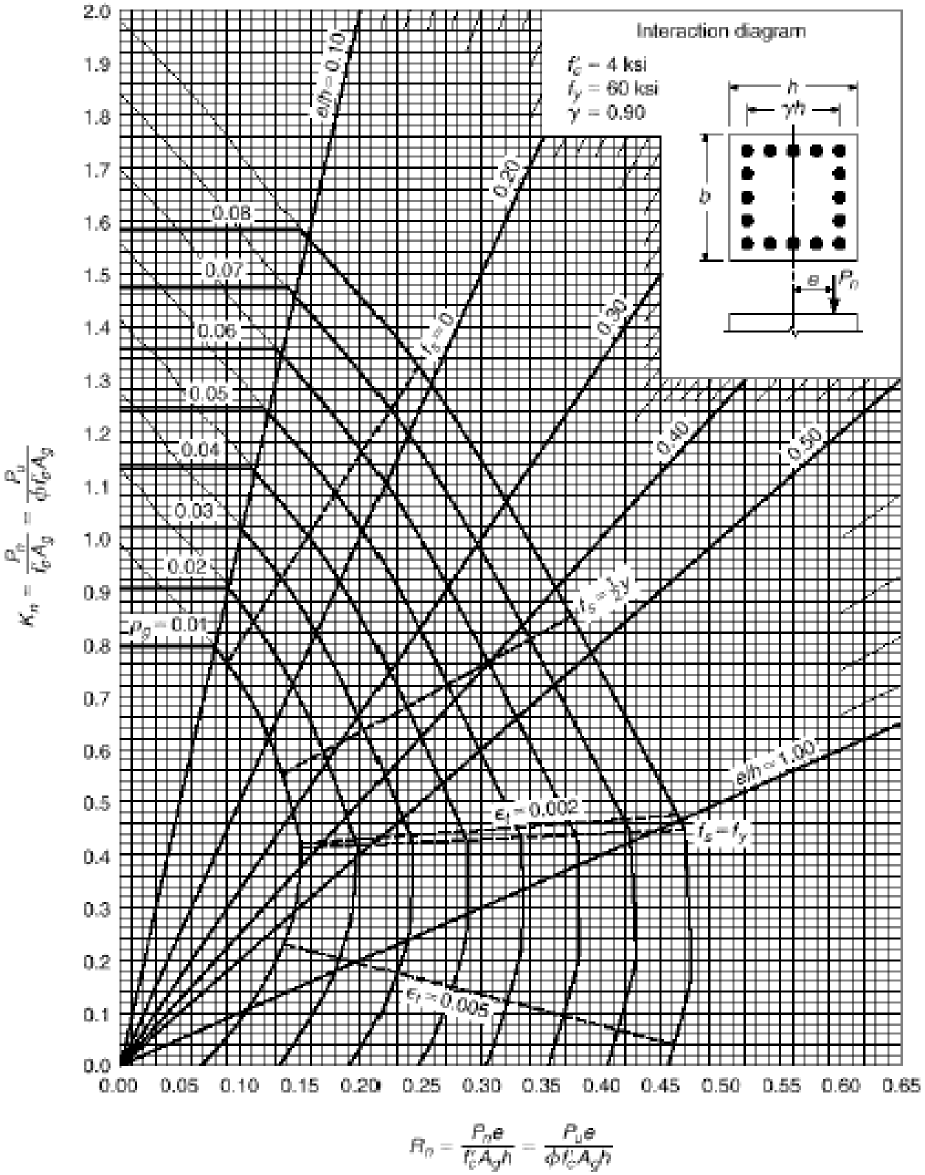
GRAPH A.6

Column strength interaction diagram for rectangular section with bars on four faces and γ = 0.70 (for instructional use only).



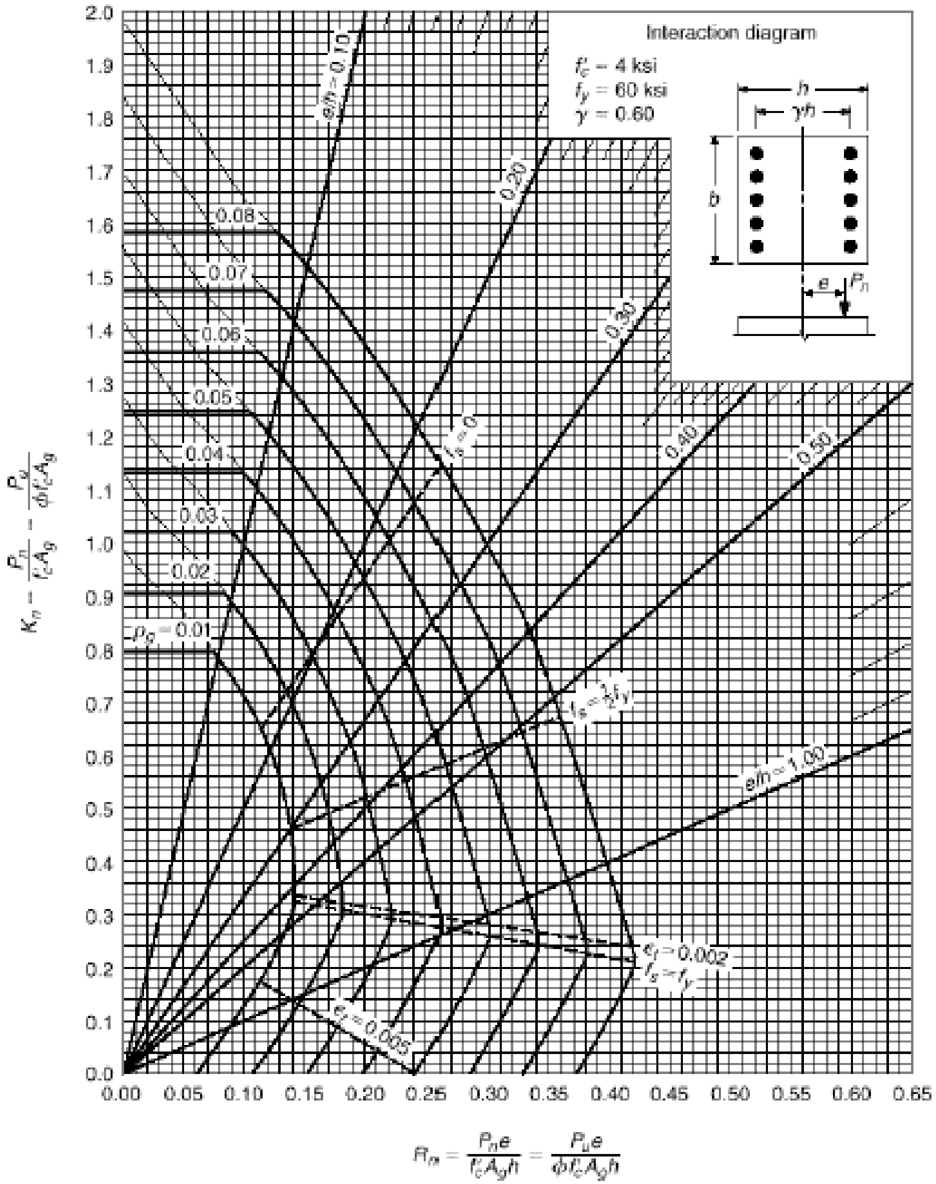
GRAPH A.7

Column strength interaction diagram for rectangular section with bars on four faces and - = 0.80 (for instructional use only).



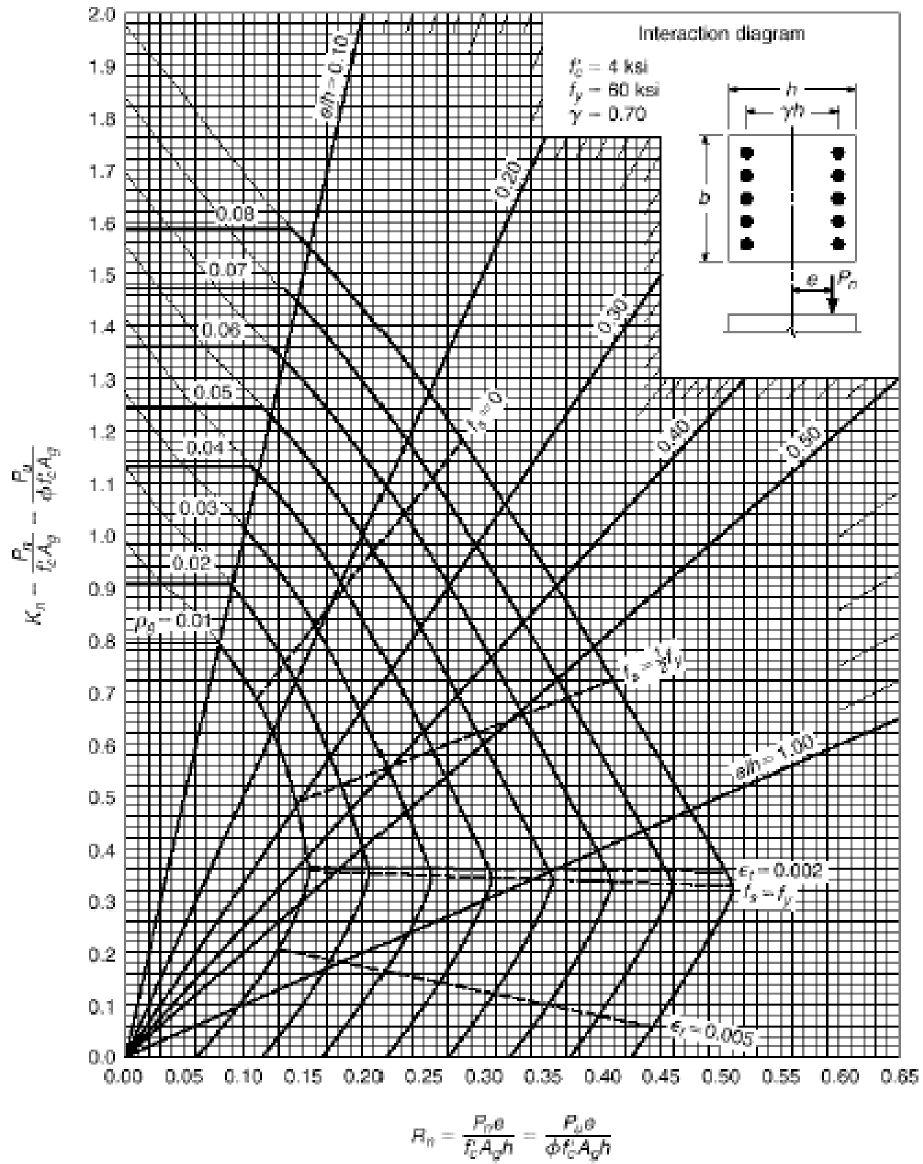
GRAPH A.8

Column strength interaction diagram for rectangular section with bars on four faces and - = 0.90 (for instructional use only).



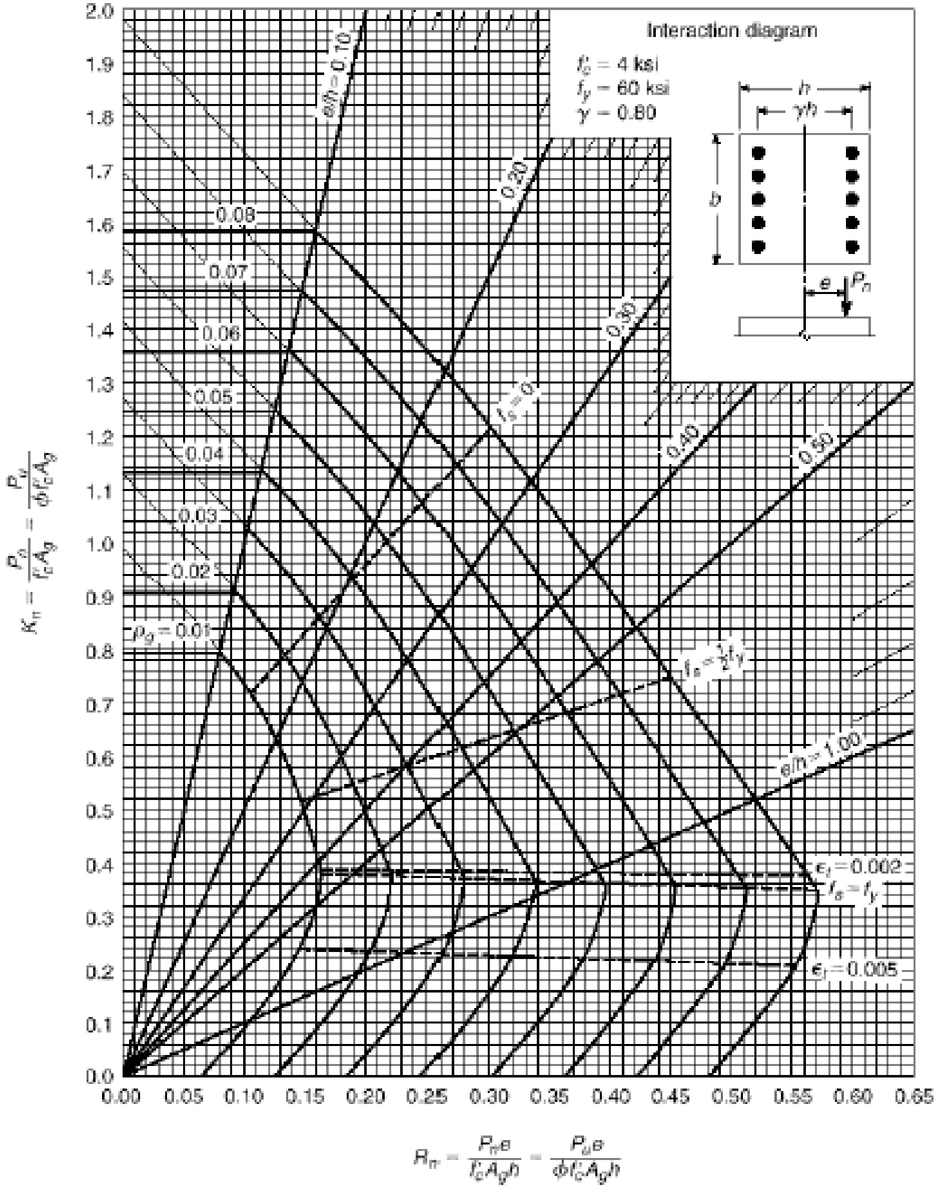
GRAPH A.9

Column strength interaction diagram for rectangular section with bars on end faces and · = 0.60 (for instructional use only).



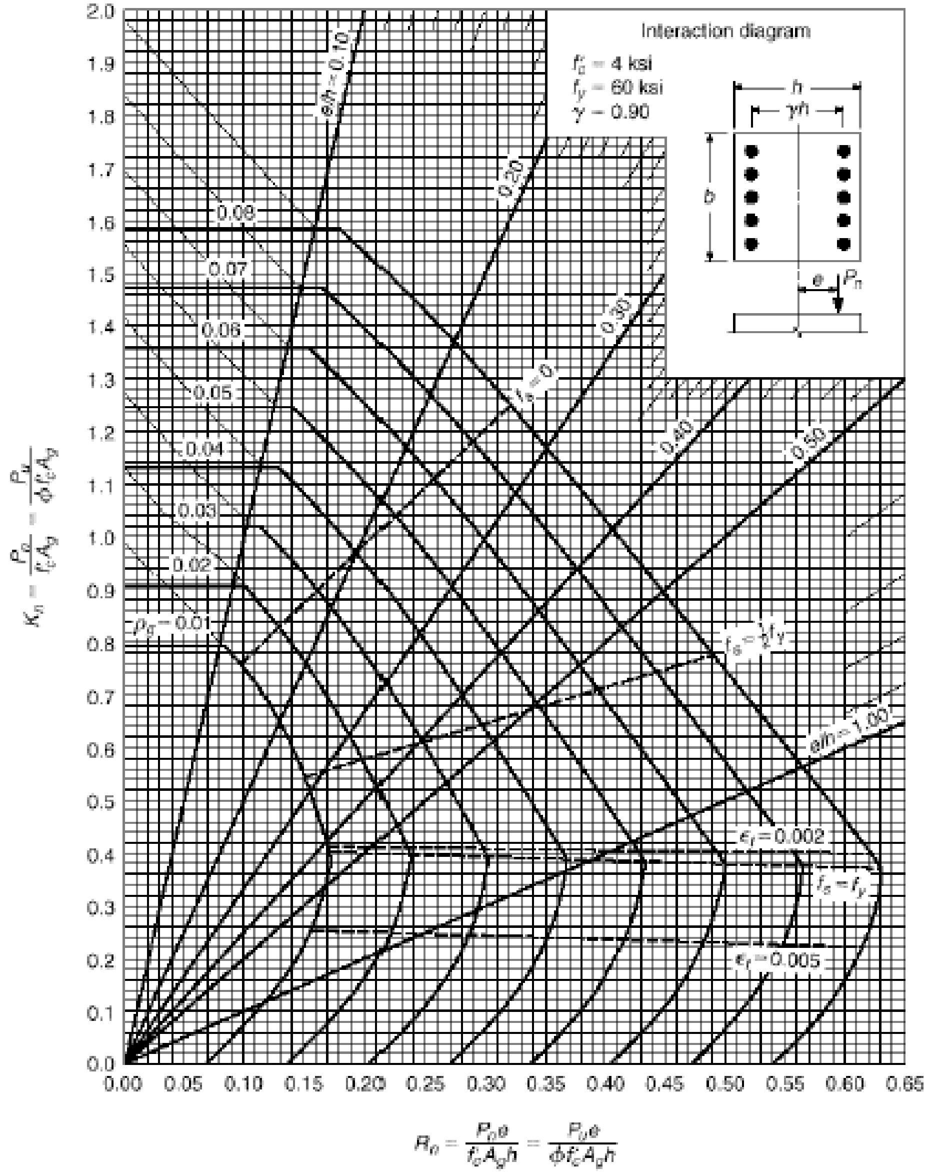
GRAPH A.10

Column strength interaction diagram for rectangular section with bars on end faces and · = 0.70 (for instructional use only).



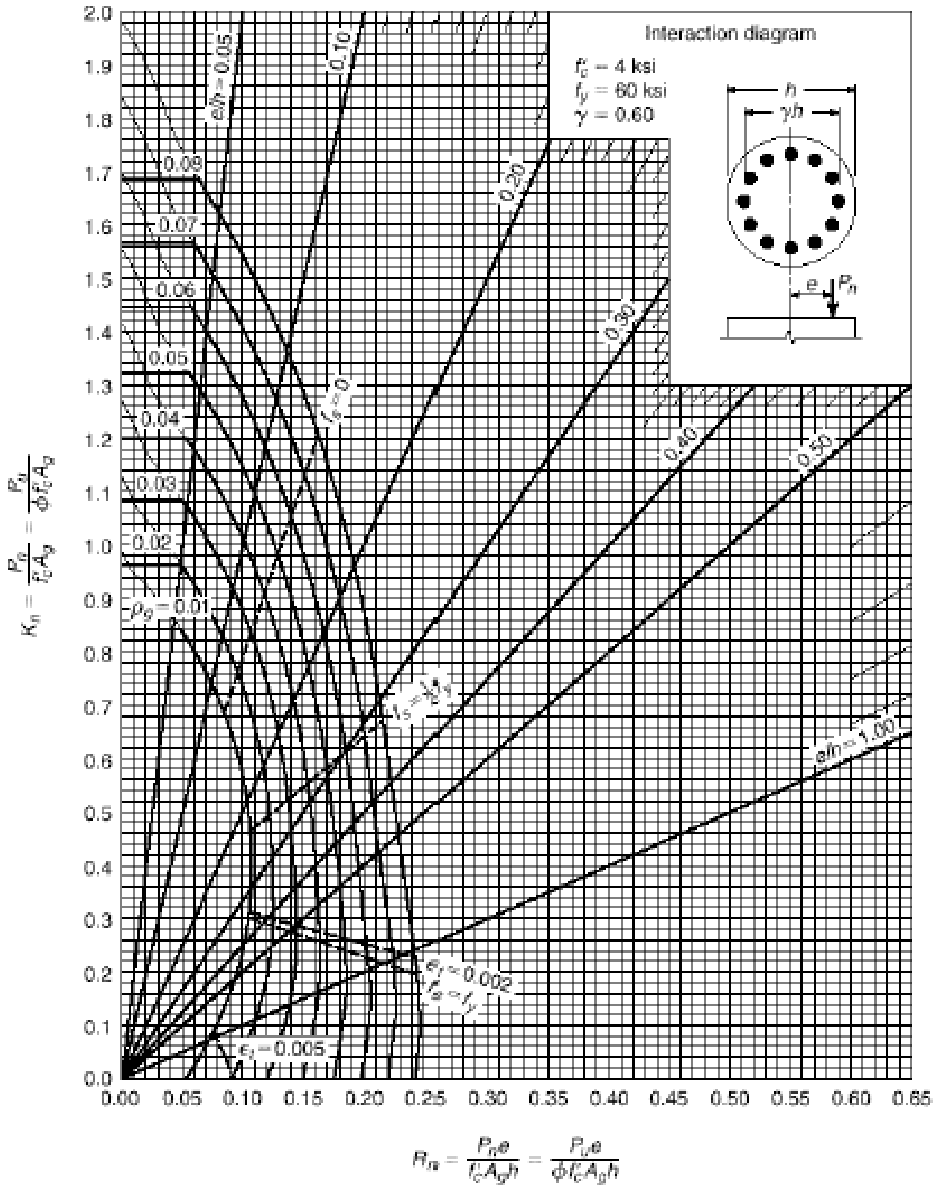
GRAPH A.11

Column strength interaction diagram for rectangular section with bars on end faces and = 0.80 (for instructional use only).



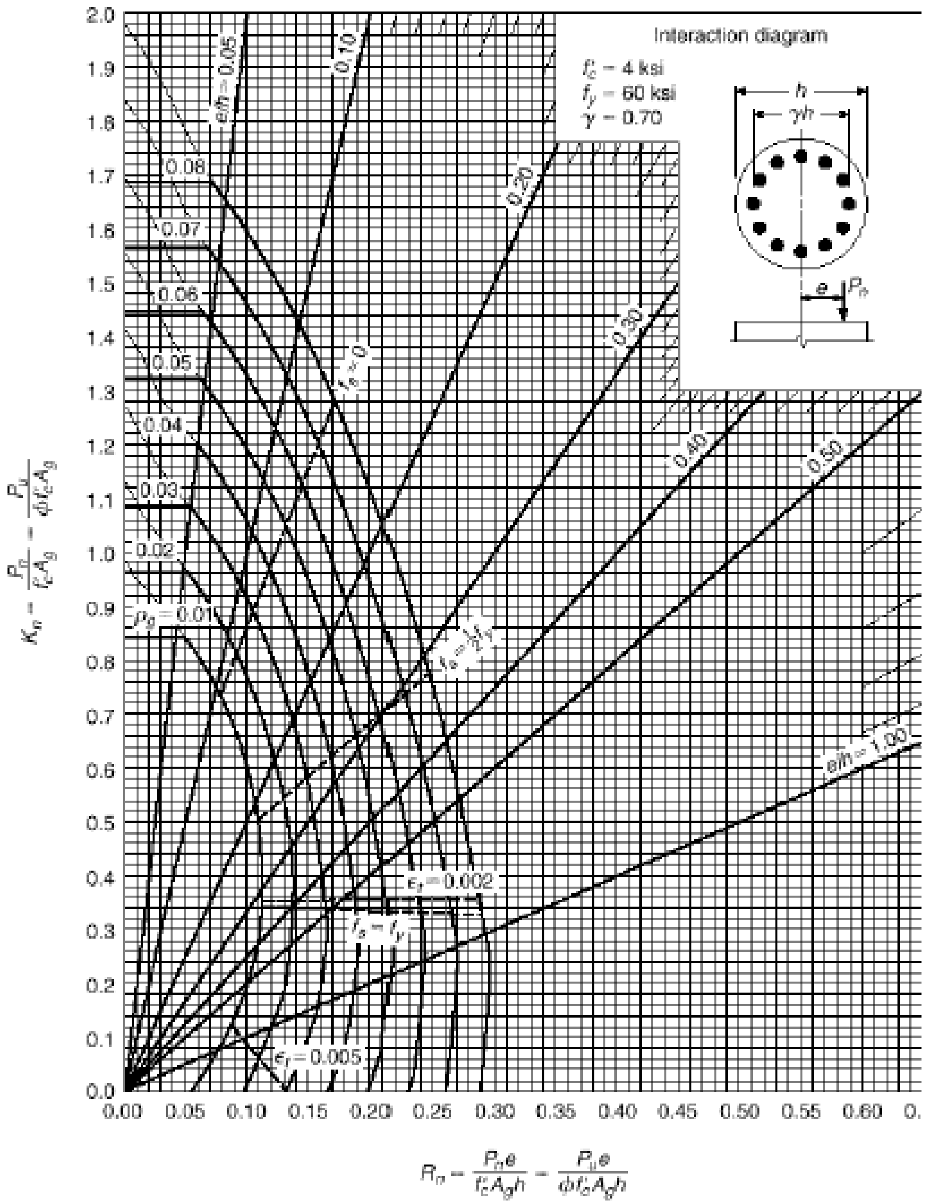
GRAPH A.12

Column strength interaction diagram for rectangular section with bars on end faces and = 0.90 (for instructional use only).



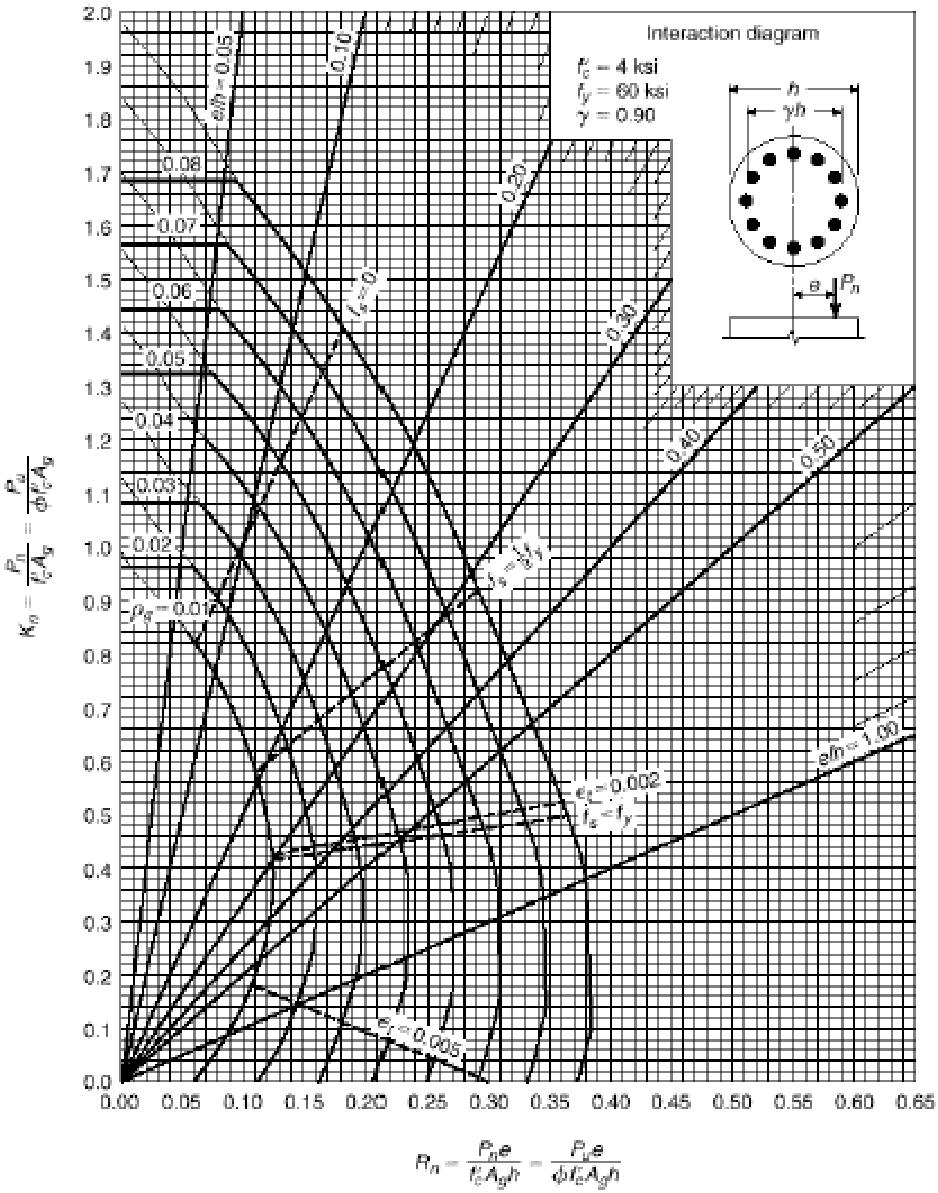
GRAPH A.13

Column strength interaction diagram for circular section with · = 0.60 (for instructional use only).



GRAPH A.14

Column strength interaction diagram for circular section with · = 0.70 (for instructional use only).



GRAPH A.16

Column strength interaction diagram for circular section with · = 0.90 (for instructional use only).

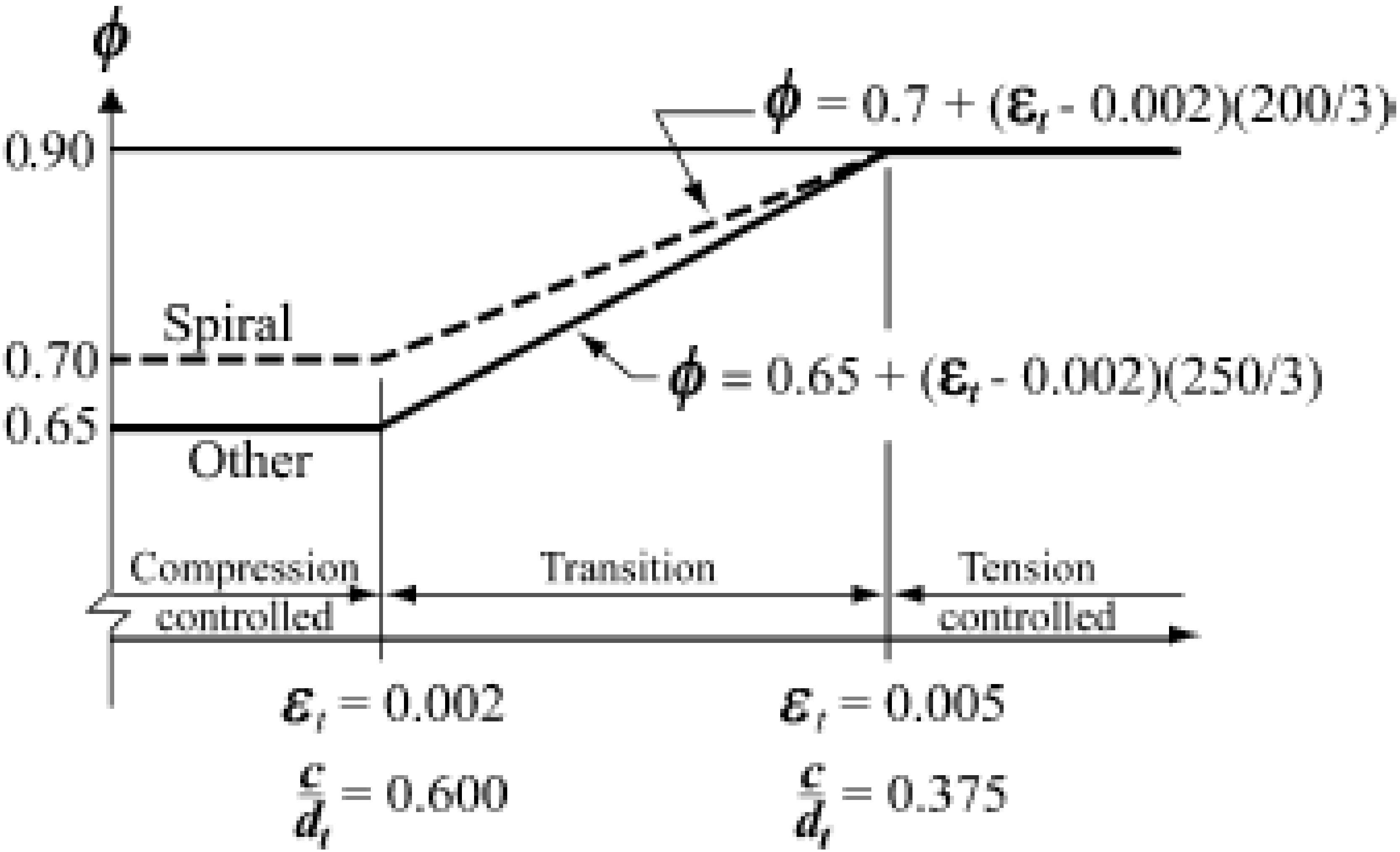
Design of short columns

- Reduction factor Ø for tied columns

Ø=0.65	For $\varepsilon_t \le \varepsilon_y = 0.002$	Compression controlled
$\phi = 0.483 + 83.3\varepsilon_t$	$\varepsilon_y = 0.002 < \varepsilon_t < 0.005$	Transition
Ø=0.9	$\varepsilon_t \ge 0.005$	Tension controlled

- Reduction factor Ø for spiral columns

Ø=0.7	For $\varepsilon_t \le \varepsilon_y = 0.002$	Compression controlled
$\phi = 0.576 + 66.7\varepsilon_t$	$\varepsilon_y = 0.002 < \varepsilon_t < 0.005$	Transition
Ø=0.9	$\varepsilon_t \ge 0.005$	Tension controlled



φ
φ = 0.7 + (εt - 0.002)(200/3)
0.90
Spiral
0.70
0.65
φ = 0.65 + (εt - 0.002)(250/3)
Other
Compression controlled
Transition
Tension controlled
εt = 0.002
c/dt = 0.600
εt = 0.005
c/dt = 0.375

7.8 — Special reinforcement details for columns

7.8.1 — Offset bars

Offset bent longitudinal bars shall conform to the following:

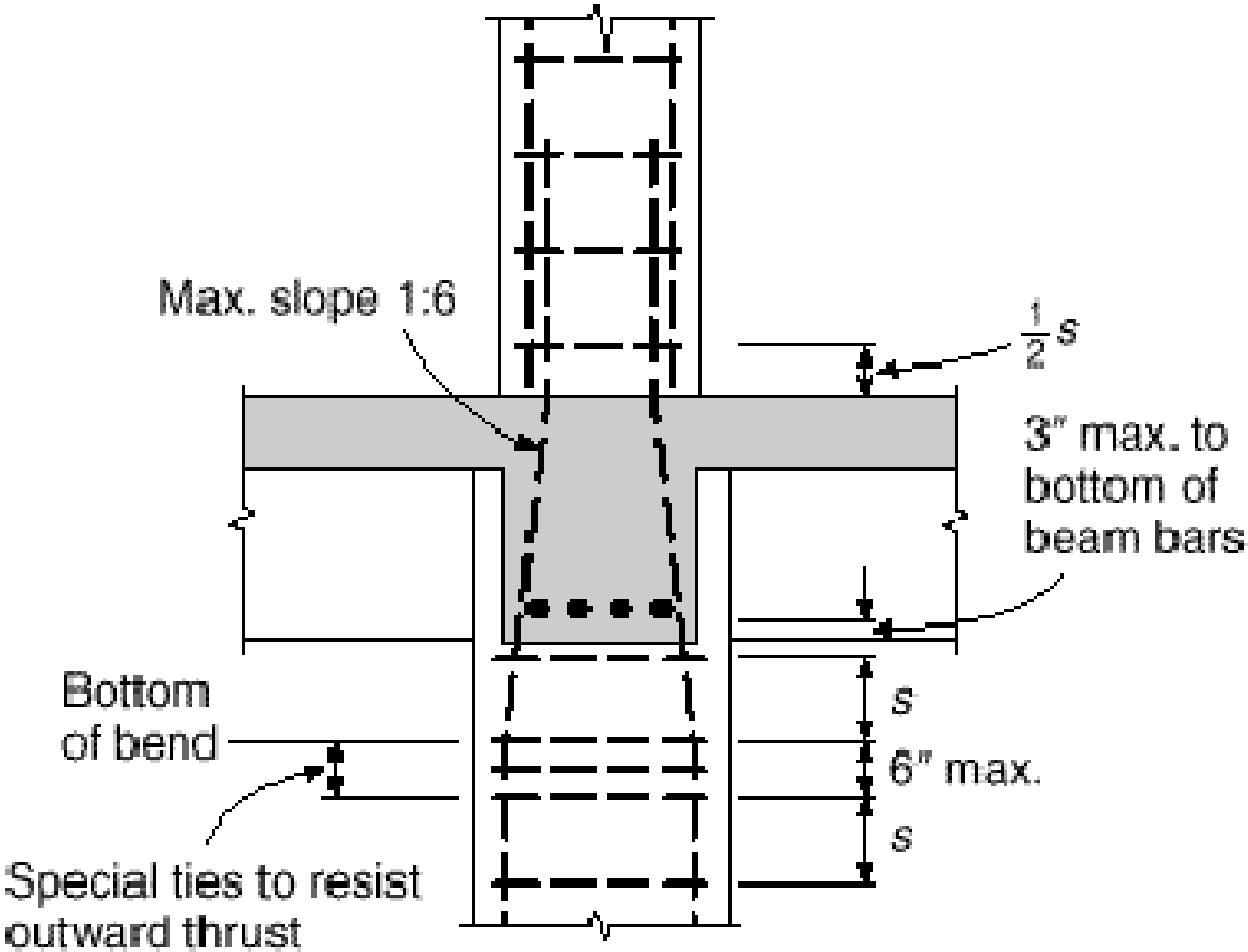
7.8.1.1 — Slope of inclined portion of an offset bar with axis of column shall not exceed 1 in 6.

7.8.1.2 — Portions of bar above and below an offset shall be parallel to axis of column.

7.8.1.3 — Horizontal support at offset bends shall be provided by lateral ties, spirals, or parts of the floor construction. Horizontal support provided shall be designed to resist 1-1/2 times the horizontal component of the computed force in the inclined portion of an offset bar. Lateral ties or spirals, if used, shall be placed not more than 150 mm from points of bend.

7.8.1.4 — Offset bars shall be bent before placement in the forms. See 7.3.

7.8.1.5 — Where a column face is offset 75 mm or greater, longitudinal bars shall not be offset bent. Separate dowels, lap spliced with the longitudinal bars adjacent to the offset column faces, shall be provided. Lap splices shall conform to 12.17.



Max. slope 1:6
$\frac{1}{2}s$
3" max. to bottom of beam bars
Bottom of bend
s
6" max.
s
Special ties to resist outward thrust

Example1:

Short columns (500*600mm), service load(D=990kN, L=1480kN, Md=220kN.m, Ml=300kN.m), d'=65mm, fy=414MPa, fc'=28MPa.

Required: design of short column, if:

- Bending about strong axis
- Bending about weak axis.

Solution:

Pu=1.2*990+1.6*1480=3556kN

Mu=1.2*220+1.6*300=744 kN.m

e=Mu/Pu=744/3556=0.209m

- Bending about strong axis,(h=600mm)

$$\gamma = \frac{h-2d'}{h} = \frac{600-2*65}{600} = 0.78$$

$$Kn = \frac{Pn}{fc'.Ag} = \frac{Pu}{\emptyset fc'.Ag} = \frac{3556}{0.65*28000*0.6*0.5}$$

$$= 0.65$$

$$Rn = \frac{Pn}{fc'.Ag}\frac{e}{h} = \frac{Pu}{\emptyset fc'.Ag}\frac{e}{h} = Kn\frac{e}{h} = 0.65*\frac{0.209}{0.6}$$

$$= 0.226, \qquad h: dimension\ normal\ to\ axis\ of\ bending$$

From graphs:

For γ=0.7 (graph A10)→ρg=0.035

For γ=0.8 (graph A11)→ρg=0.030

For γ=0.78 (use

interpolation)→$\rho g = 0.031 \begin{cases} < \rho_{max} = 0.08 \\ > \rho_{min} = 0.01 \end{cases} \quad O.K$

Since εt<0.002→Ø=0.65

As= ρg*Ag=0.031*600*500=9300mm^2

Use 10Ø36mm(As=10178 mm^2)

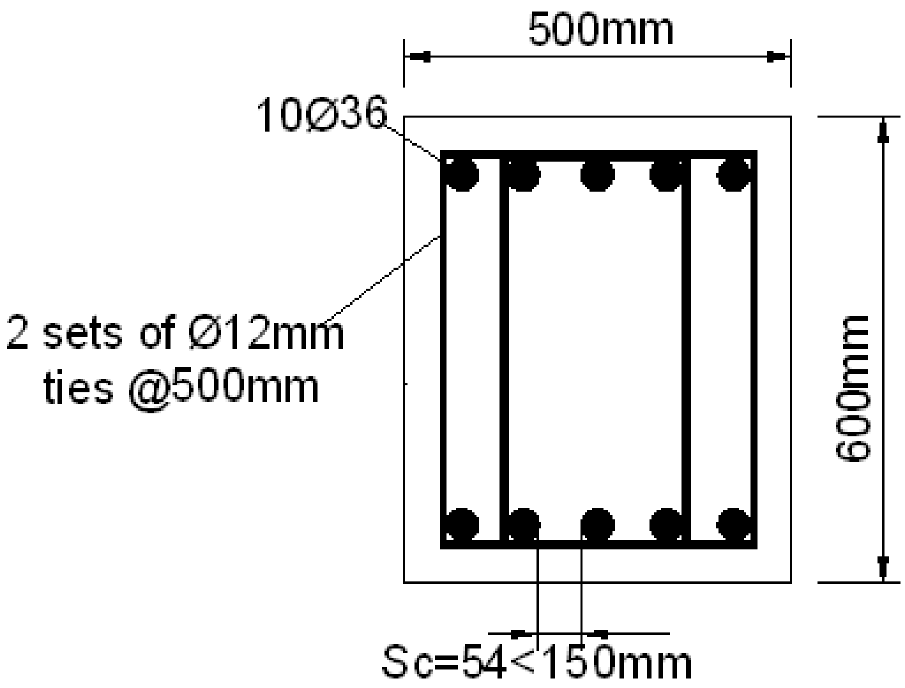
$for\ db = 36mm > 32mm, use\ tie\ \emptyset 12mm\ @$

$$min\begin{cases} 16d_b = 16 * 36 = 576mm \\ 48d_{tie} = 48 * 12 = 576mm \\ least\ dimension\ of\ column\ cross\ section = 500mm \end{cases}$$

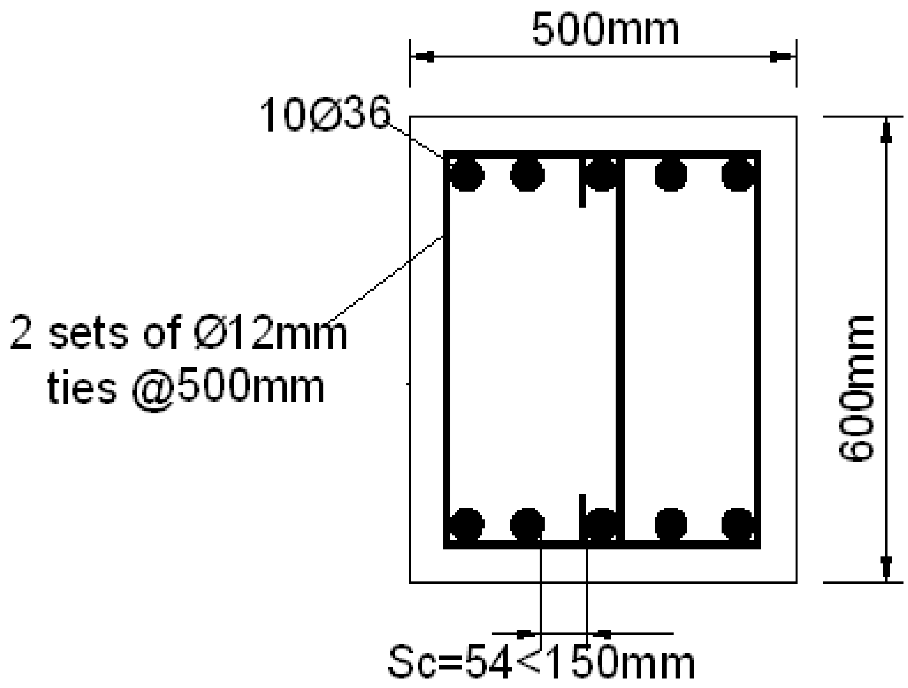
$$use\ tie\ \emptyset 12mm\ @500mm\ c/c$$

$$s_c = \frac{500 - 2 * 40 - 5 * 36 - 2 * 12}{5 - 1} = 54mm$$

$$\geq max\begin{cases} 1.5db = 1.5 * 36 = 54mm \\ 40mm \end{cases} \quad O.K$$


500mm
10Ø36
2 sets of Ø12mm
ties @500mm
600mm
Sc=54<150mm

OR


500mm
10Ø36
2 sets of Ø12mm
ties @500mm
600mm
Sc=54<150mm

- Bending about weak axis,(h=500mm)

$$\gamma = \frac{h - 2d'}{h} = \frac{500 - 2 * 65}{500} = 0.74$$

$$Kn = \frac{Pn}{fc'.Ag} = \frac{Pu}{\emptyset fc'.Ag} = \frac{3556}{0.65 * 28000 * 0.6 * 0.5}$$

$$= 0.65$$

$$Rn = \frac{Pn}{fc'.Ag}\frac{e}{h} = \frac{Pu}{\emptyset fc'.Ag}\frac{e}{h} = Kn\frac{e}{h} = 0.65 * \frac{0.209}{0.5}$$

$$= 0.272, \qquad h: dimension\ normal\ to\ axis\ of\ bending$$

From graphs:

For γ=0.7 (graph A10)→ρg=0.046

For γ=0.8 (graph A11)→ρg=0.040

For γ=0.74 (use interpolation)→$\rho g = 0.0436 \begin{cases} < \rho_{max} = 0.08 \\ > \rho_{min} = 0.01 \end{cases} \quad O.K$

Since $\varepsilon t < 0.002$→Ø=0.65

As= ρg*Ag=0.0436*600*500=13080mm^2

Use 10Ø44mm(As=15205 mm^2)

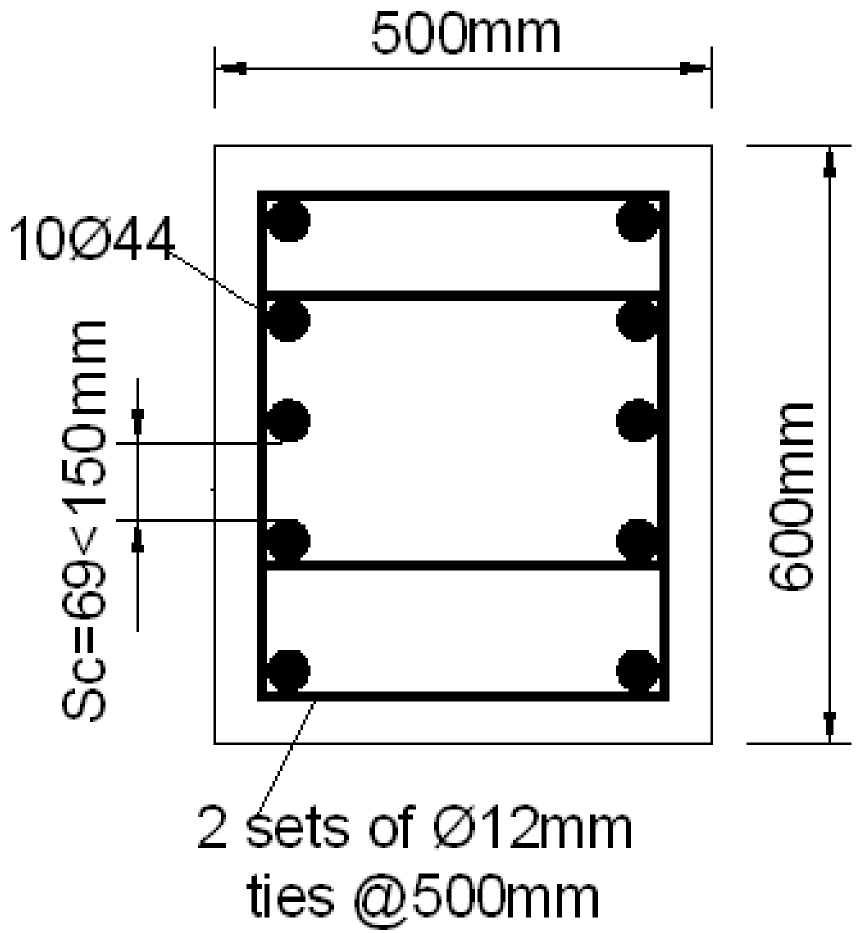
$$for\ db = 44mm > 32mm, use\ tie\ \emptyset 12mm\ @$$

$$min\begin{cases} 16d_b = 16 * 44 = 704mm \\ 48d_{tie} = 48 * 12 = 576mm \\ least\ dimension\ of\ column\ cross\ section = 500mm \end{cases}$$

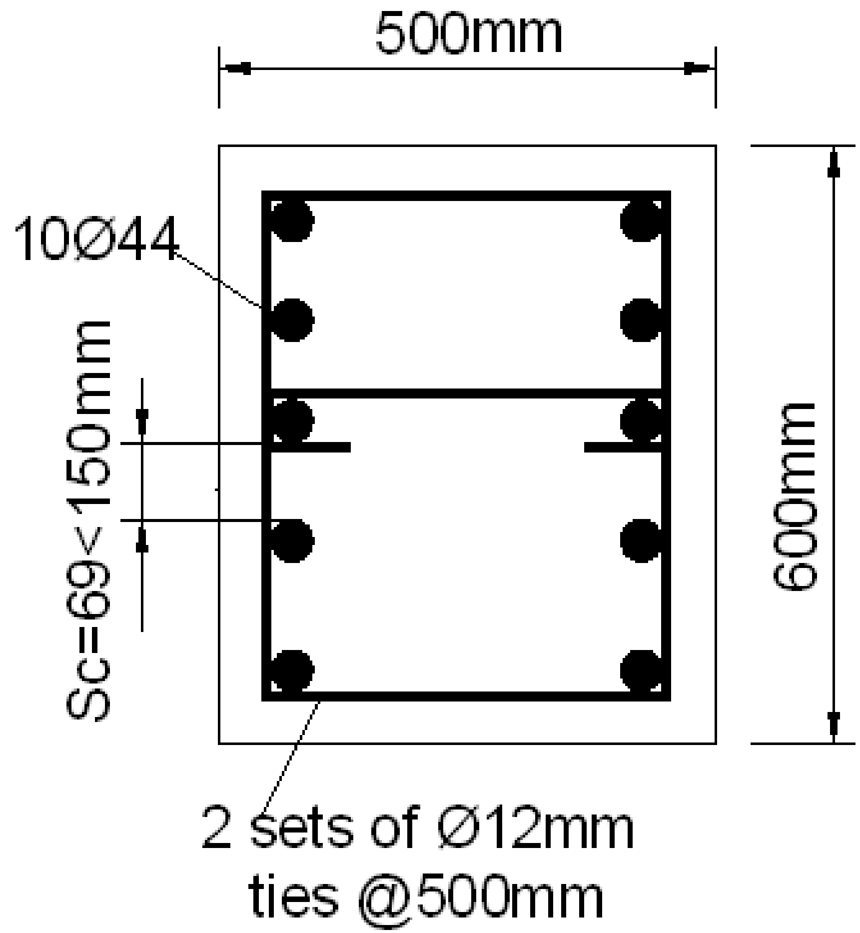
$$use\ tie\ \emptyset 12mm\ @500mm\ c/c$$

$$s_c = \frac{600 - 2 * 40 - 5 * 44 - 2 * 12}{5 - 1} = 69mm$$

$$\geq max\begin{cases} 1.5db = 1.5 * 44 = 66mm \\ 40mm \end{cases} \quad O.K$$


500mm
10Ø44
Sc=69<150mm
600mm
2 sets of Ø12mm
ties @500mm

OR


500mm
10Ø44
Sc=69<150mm
600mm
2 sets of Ø12mm
ties @500mm

Example2:

Short columns, ultimate load(Pu=2140kN, Mu=670kN.m,), d'=60mm, fy=414MPa, fc'=28MPa.

Required: column cross section dimensions, As:

Solution:

assume:

- $\rho g = 0.03 \begin{cases} < \rho_{max} = 0.08 \\ > \rho_{min} = 0.01 \end{cases}$
- h=600mm

e=Mu/Pu=670/2140=0.313m

$$\gamma = \frac{h-2d'}{h} = \frac{600-2*60}{600} = 0.8$$

$$\frac{e}{h} = \frac{0.313}{0.6} = 0.52$$

Graph A11→Kn=0.48

$$Kn = \frac{Pu}{\emptyset fc'.Ag} \rightarrow 0.48 = \frac{2140}{0.65*28000*b*0.6} = 0.65 \rightarrow b = 0.408$$

Use b=410mm

As= ρg*Ag=0.03*600*410=7380mm^2

Use 8Ø36mm(As=8143 mm^2)

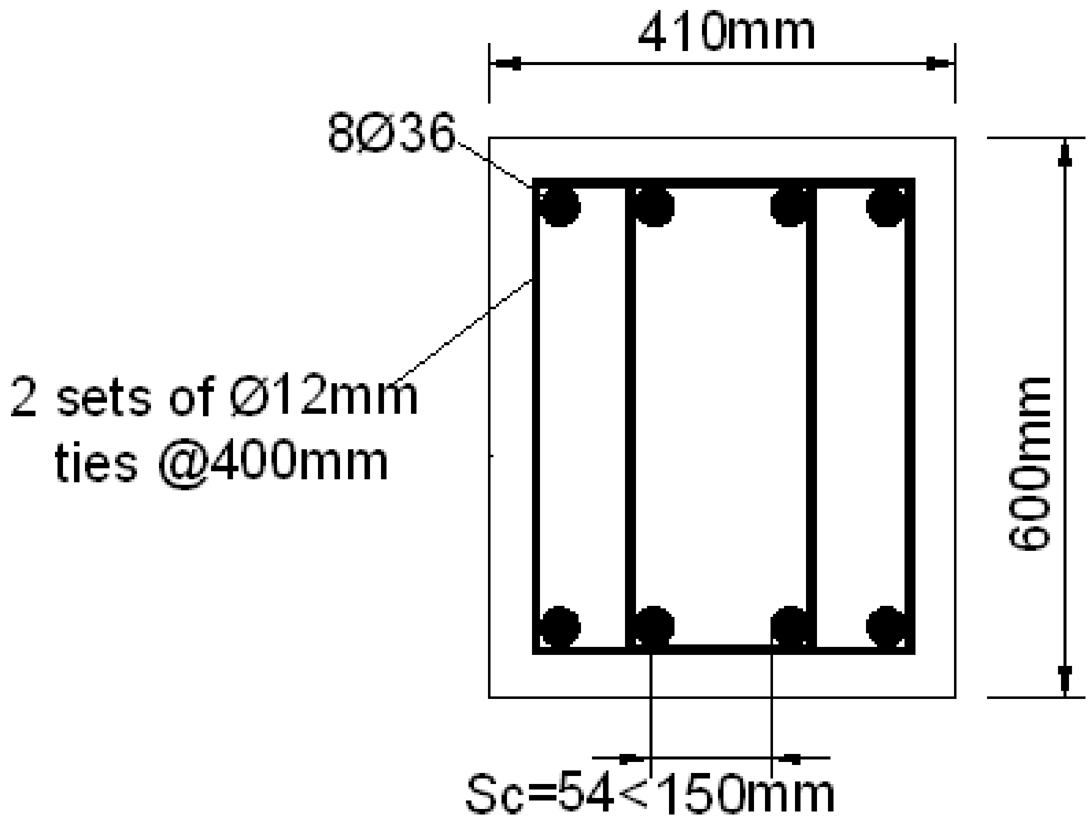
$$for\ db = 36mm > 32mm, use\ tie\ \emptyset 12mm\ @$$

$$min\begin{cases} 16d_b = 16 * 36 = 576mm \\ 48d_{tie} = 48 * 12 = 576mm \\ least\ dimension\ of\ \ cross\ section = 410mm \end{cases}$$

$$use\ tie\ \emptyset 12mm\ @400mm\ c/c$$

$$s_c = \frac{410 - 2 * 40 - 4 * 36 - 2 * 12}{4 - 1} = 54mm$$

$$\geq max\begin{cases} 1.5db = 1.5 * 36 = 54mm \\ 40mm \end{cases}\ \ O.K$$

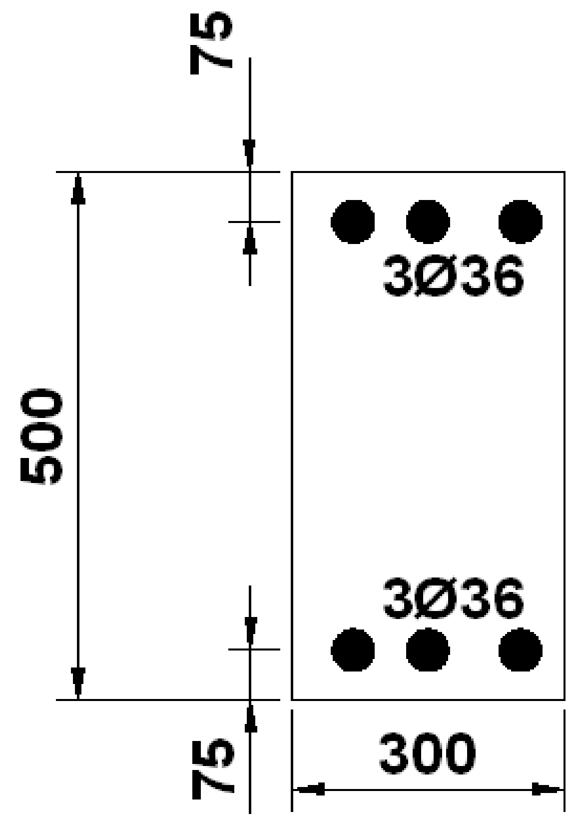

410mm
8Ø36
2 sets of Ø12mm
ties @400mm
600mm
Sc=54<150mm

Example3: analysis of short tied column.

b=300mm, h=500mm, d'=75mm,

e=180mm, fy=400MPa, fc'=28MPa.

If the bending about strong axis, find Mu, Pu.



Solution:

$$\rho_g = \frac{Ast}{Ag} = \frac{6 * \frac{36^2}{4} * \pi}{500 * 300} = 0.041$$

$$\gamma = \frac{h - 2d'}{h} = \frac{500 - 2 * 75}{500} = 0.7$$

$$\frac{e}{h} = \frac{0.18}{0.5} 0.36$$

Graph A10→Kn=0.66

$$Kn = \frac{Pu}{\emptyset f c'. Ag} \rightarrow 0.66 = \frac{Pu}{0.65 * 28000 * 0.3 * 0.5} \rightarrow Pu$$

$$= 1802kN$$

$$Pu = \emptyset Pn \rightarrow Pn = \frac{1802}{0.65} = 2772kN$$

$$Mu = Pu * e = 1802 * 0.18 = 324kN.m$$

Note: compare with $Pu = 2869\ kN$, calculate from example1 , the difference 3%

Example4: design of short spiral circular column.

Short columns, ultimate load(Pu=2800kN, Mu=135kN.m,), d'=60mm, column diameter=450mm, fy=400MPa, fc'=28MPa.

Required: Area of steel, As:

Solution:

e=Mu/Pu=135/2800=0.048m , $e/h = 0.048/0.45 = 0.107$

$$\gamma = \frac{D-2d'}{D} = \frac{450-2*60}{450} = 0.73$$

$$Kn = \frac{Pn}{fc'.Ag} = \frac{Pu}{\emptyset fc'.Ag} = \frac{2800}{0.7 * 28000 * 0.225^2 * \pi}$$

$$= 0.90$$

$$Rn = \frac{Pn}{fc'.Ag}\frac{e}{h} = \frac{Pu}{\emptyset fc'.Ag}\frac{e}{h} = Kn\frac{e}{h} = 0.9 * 0.107$$

$$= 0.096,$$

From graphs: For γ=0.7 (graph A14)→ρg=0.03

For γ=0.8 (graph A15)→ρg=0.025

For γ=0.73 (use

interpolation)→ $\rho g = 0.028 \begin{cases} < \rho_{max} = 0.08 \\ > \rho_{min} = 0.01 \end{cases} \quad O.K$

As= ρg*Ag=0.028*$0.225^2 * \pi$ =4453 mm^2

Use 8Ø28mm(As=4928 mm^2),   Use spiral Ø10mm(79mm^2)

dc=450-2*40=370mm

Ac=$\frac{\pi(370)^2}{4} = 107521$ mm^2

$$\rho_{s,min} \geq 0.45\left(\frac{Ag}{Ac} - 1\right)\frac{fc'}{fy} = 0.45\left(\frac{225^2 * \pi}{107521} - 1\right)\frac{28}{400}$$

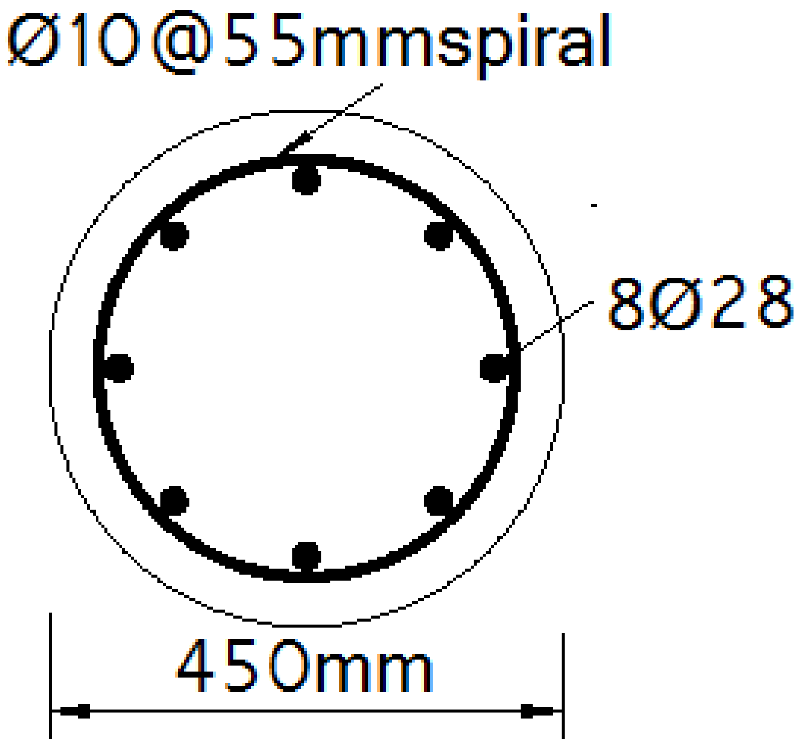
$$= 0.0151$$

$$\rho_s = \frac{4Asp}{d_c.s} = \frac{4 * 79}{370.s} = 0.0151 \rightarrow s = 56mm \begin{cases} < 75mm \\ > 25mm \end{cases} O.K$$

$$use\ spiral\ \emptyset 10mm\ @55mm\ pitch$$

$$s_c = \frac{\pi(370 - 2 * 10 - 2 * \frac{28}{2})}{8} = 126mm$$

$$\geq max \begin{cases} 1.5db = 1.5 * 28 = 42mm \\ 40mm \end{cases} \quad O.K$$


Ø10@55mmspiral
8Ø28
450mm

Sway(unbraced) and nonsway(braced) frames

In actual structures, a frame are seldom either completely braced or completely unbraced. It is necessary, therefore, to determine in advance if bracing provided by shear walls, elevator and utility shafts, stair walls, or other elements is adequate to restrain the frame against significant sway effect.

ACI code permitted to assume a *column* in a structure is no sway if the increase in column ends moments due to second-order effects (P-Δ moment) does not exceed 5% of the first-order (analysis of a frame under gravity loads) ends moments.

It also be permitted to assume a *story* within a structure is nonsway if: $Q \leq 0.05$

$$Q = \frac{(\sum Pu)\Delta_\circ}{V_u l_c}$$

Q: stability index for a story.

$\sum Pu$, V_u : total vertical load and the horizontal *story* shear, respectively.

$\Delta_\circ$: relative deflection between the top and bottom of that *story* due to V_u

l_c :length of column (c/c).

Slender columns (long columns):

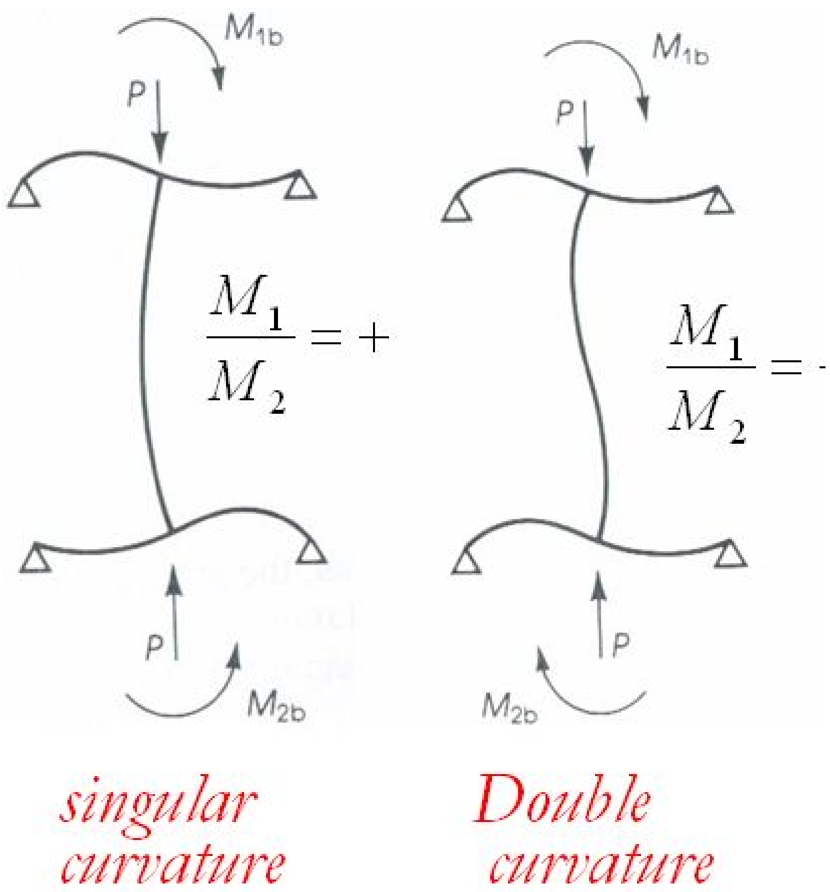
A column is to be said slender if its cross section dimensions are small compared with its length.

- Slender columns-nonsway frames.
- Slender columns-sway frames.

Slender columns-nonsway frames:

For compression member braced against side sway, effects of slenderness may be ignored when:

$$\frac{kl_u}{r} < 34 - 12\left(\frac{M1}{M2}\right),\ 34 - 12\left(\frac{M1}{M2}\right) \leq 40$$



M1: value of smaller factored end moment.

M2: value of larger factored end moment.

l_u : unsupported length of compression member, defined in ACI code 10.11.3 as clear distance between floor slabs, beams or other members capable of providing lateral support.

r : radius of gyration ($\sqrt{\frac{I}{A}}$)

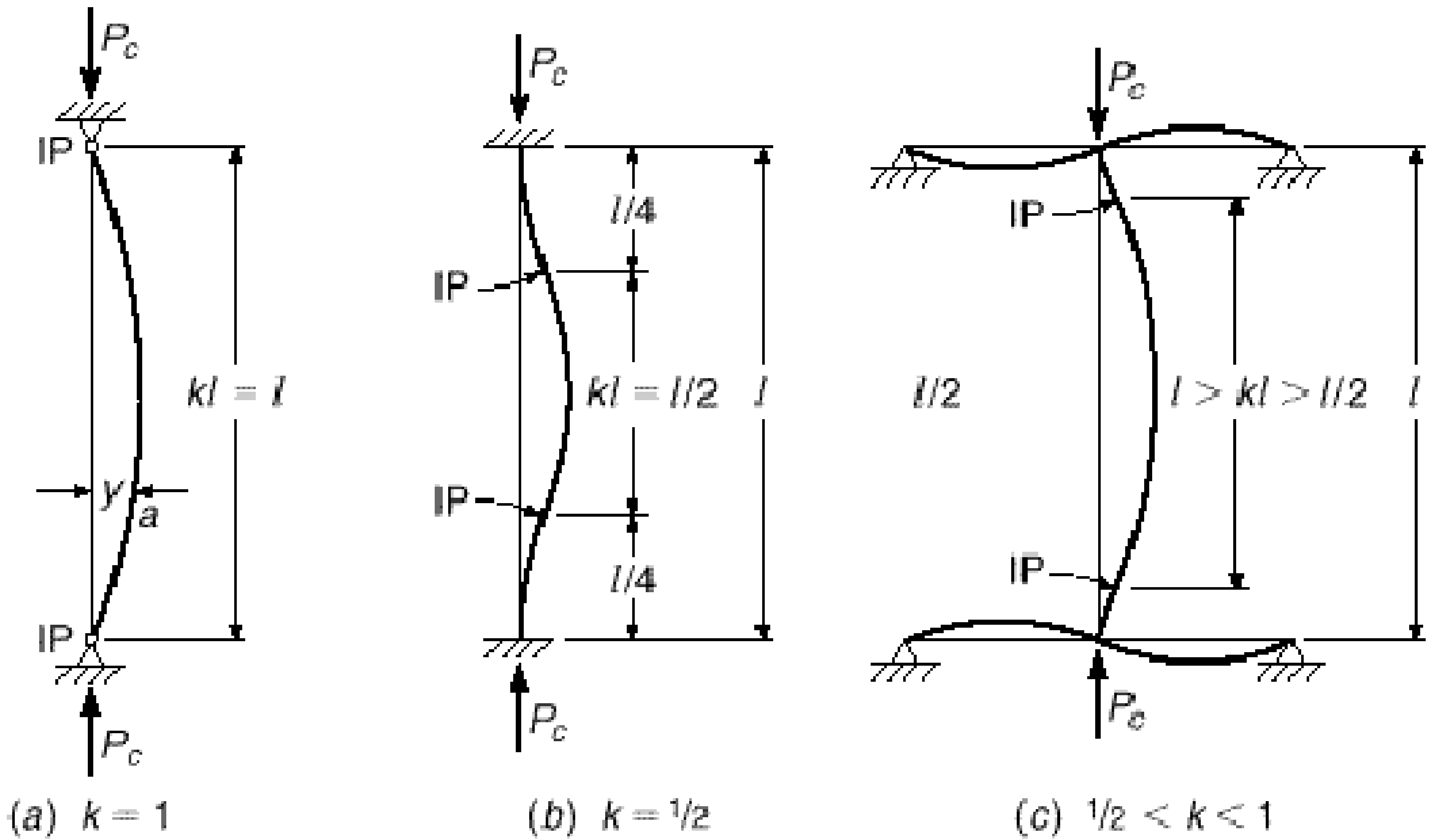
r=0.3h for rectangular section.

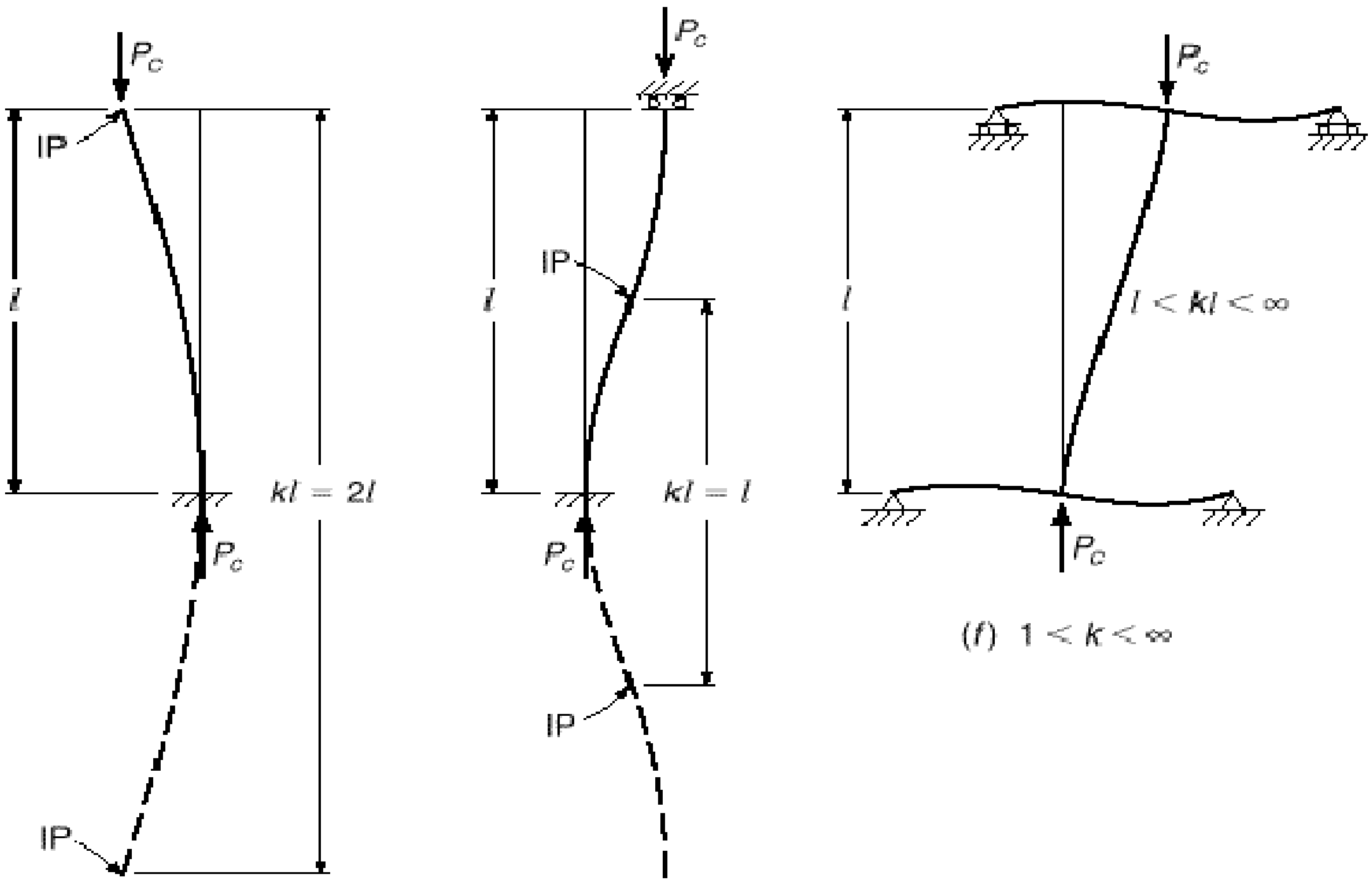
h: dimension perpendicular to axis of bending.

r=0.25D for circular section.

D: diameter of column.

k :effective length factor, depends on the end-restrained coefficients (ψ) at both ends of column.





(d) $k = 2$ (e) $k = 1$ (f) $1 < k < \infty$

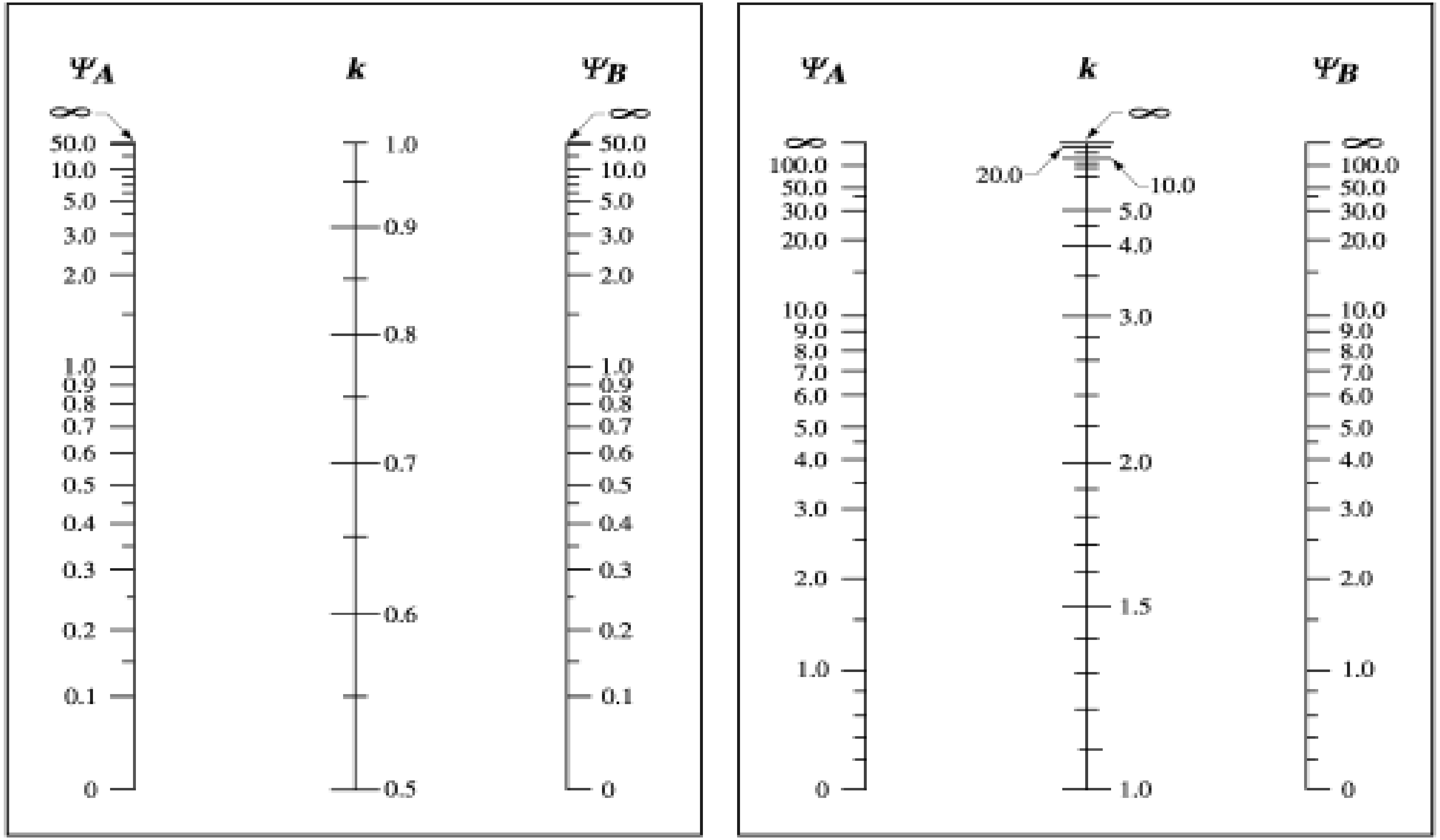
$$\psi = \frac{\sum\left(\frac{EI}{l}\right)_{columns}}{\sum\left(\frac{EI}{l}\right)_{beams}}, \quad l: length\ of\ beam\ or\ column\ c\ to\ c.$$

I: moment of inertia:

I=0.35 Ig for rectangular beam

I=0.7 Ig for T- beam (Ig for web)

I=0.7 Ig for column



(a)
Nonsway Frames

(b)
Sway Frames

Ψ = ratio of Σ(EI/ℓ$_c$) of compression members to Σ(EI/ℓ) of flexural members in a plane at one end of a compression member

ℓ = span length of flexural member measured center to center of joints

Fig. R10.12.1—Effective length factors, k

For columns $\frac{kl_u}{r} > 34 - 12\left(\frac{M1}{M2}\right)$, (i.e slender column):

$$M_c = \delta_{ns} M_2 \geq \delta_{ns} M_{2,min}$$

$$M_{2,min} = \mathrm{Pu} * (15 + 0.03\mathrm{h}), \qquad \mathrm{h, in\ mm}$$

$$\delta_{ns} = moment\ magnification\ factor$$

$$\delta_{ns} = \frac{\mathrm{c_m}}{1 - \frac{P_u}{0.75 P_{cr}}} \geq 1.0$$

$$\mathrm{c_m} = 0.6 + 0.4\left(\frac{M1}{M2}\right) \geq 0.4$$

$$P_{cr} = \frac{\pi^2 EI}{(kl_u)^2}$$

$$EI = \frac{0.4Ec.Ig}{1 + \beta_d}$$

Mc: factored moment amplified for the effects of member curvature.

Pu: factored axial load.

Pcr: critical buckling load.

β_d : ratio of maximum factored axial sustained load to maximum factored axial load associated with the same load combination($\frac{Pu_{\text{sustained}}}{Pu_{\text{total}}}$).

M2: larger factored end moment on column.

$M2_{,min}$ =Pu*(15+0.03h), h,in mm.

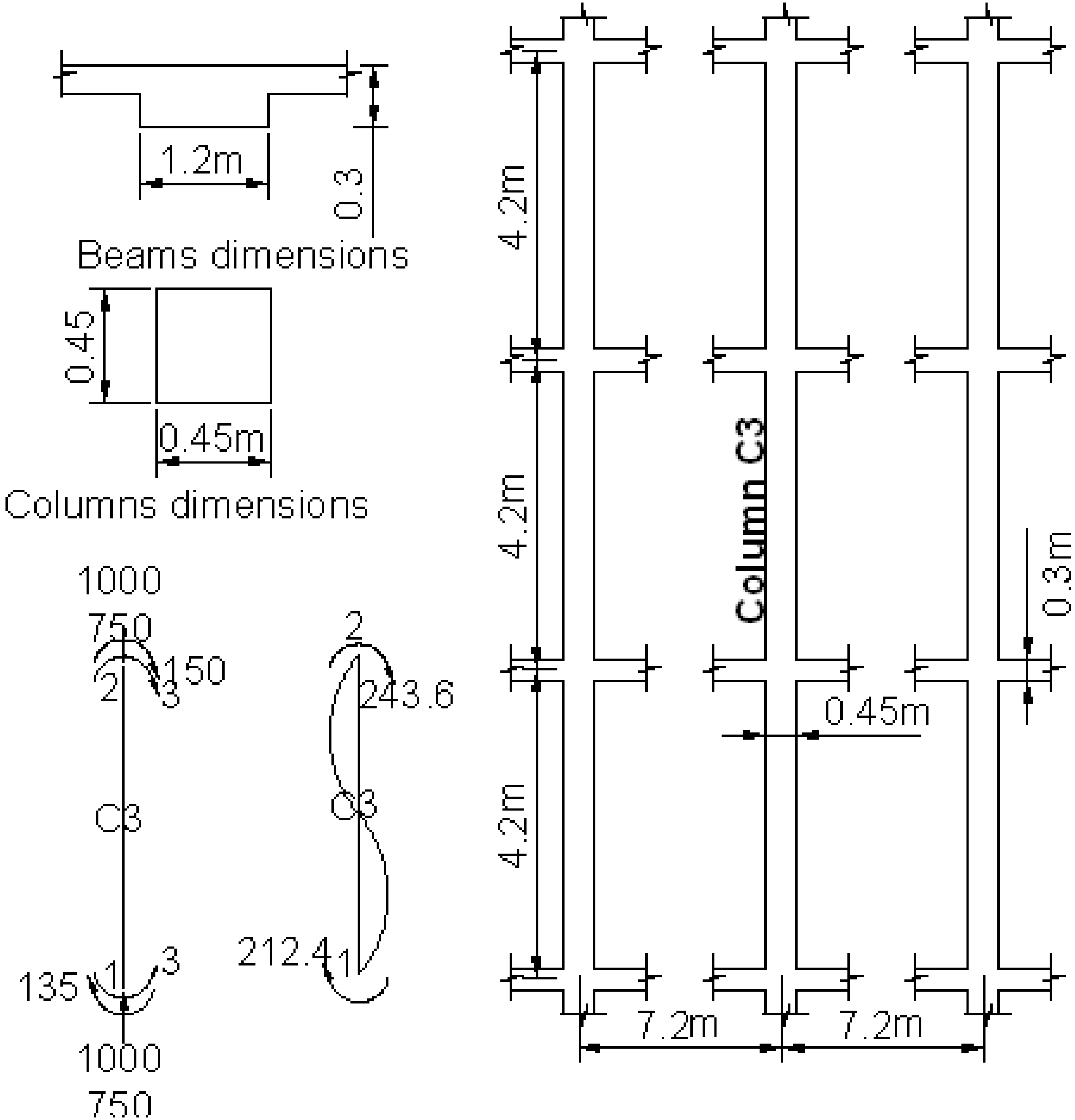
$$\psi = \begin{cases} \infty, & in\ practice = 10\ \ for\ hinged\ support \\ 0, & in\ practice = 1\ for\ fixed\ support \end{cases}$$

- Where M1/M2 is positive is bent in single curvature. For members with transverse loads between supports, Cm shall be taken as 1.0.

Example1:

- Fy=300MPa, fc'=30MPa
- Beams dimensions(bw=1.2m, h=0.3m)
- Columns dimensions(0.45*0.45m).
- Braced against side sway by stair and elevator shafts having concrete walls that are monolithic with floors. ***Required: design of column C3***

	D.L	L.L
P(kN)	1000	750
M2(kN.m)	3	150
M1(kN.m)	-3	135


1.2m
0.3
Beams dimensions
0.45
0.45m
Columns dimensions
1000
750
150
2
3
C3
135
1
3
1000
750
2
243.6
C3
212.4
1
4.2m
4.2m
4.2m
Column C3
0.3m
0.45m
7.2m
7.2m

Solution:

Pu=1.2*1000+1.6*750=2400kN

Mu2=1.6*150+1.2*3=243.6kN.m

Mu1=1.6*135+1.2*(-3)=212.4 kN.m

$$I_b = 0.7Ig_{web} = 0.7 * \frac{1.2 * 0.3^3}{12} = 1.89 * 10^{-3}m^4$$

$$I_c = 0.7Ig = 0.7 * \frac{0.45 * 0.45^3}{12} = 2.392 * 10^{-3}m^4$$

$$\psi_{top} = \psi_{bot} = \frac{\sum\left(\frac{EI}{l}\right)_{columns}}{\sum\left(\frac{EI}{l}\right)_{beams}} = \frac{Ec(2 * \frac{2.392*10^{-3}}{4.2})}{Ec(2 * \frac{1.89*10^{-3}}{4.2})} = 2.17$$

Braced column → graph →k=0.87

$$l_u = 4.2 - 0.3 = 3.9m$$

$$\frac{kl_u}{r} = \frac{kl_u}{0.3h} = \frac{0.87 * 3.9}{0.3 * 0.45} = 25.1$$

$$34 - 12\left(\frac{M1}{M2}\right) = 34 - 12\left(\frac{-212.4}{243.6}\right) = 44.4 > 40$$

$$\rightarrow 34 - 12\left(\frac{M1}{M2}\right) = 40$$

$$\frac{kl_u}{r} = 25.1 < 34 - 12\left(\frac{M1}{M2}\right) = 40 \rightarrow \therefore short\ column$$

Design values: Pu=2400kN, Mu=243.6kN

e=Mu/Pu=243.6/2400=0.1015 m

$$\gamma = \frac{h - 2d'}{h} = \frac{450 - 2 * 65}{450} = 0.7$$

$$\frac{e}{h} = \frac{0.1015}{0.45} 0.225$$

$$Kn = \frac{Pu}{\emptyset fc'.Ag} = \frac{2400}{0.65 * 30000 * 0.45 * 0.45} = 0.61$$

$$Rn = Kn\frac{e}{h} = 0.61 * 0.225 = 0.14$$

$$\rho g = 0.013 \begin{cases} < \rho_{max} = 0.08 \\ > \rho_{min} = 0.01 \end{cases} \quad O.K$$

Since $\varepsilon t < 0.002 \rightarrow Ø = 0.65$

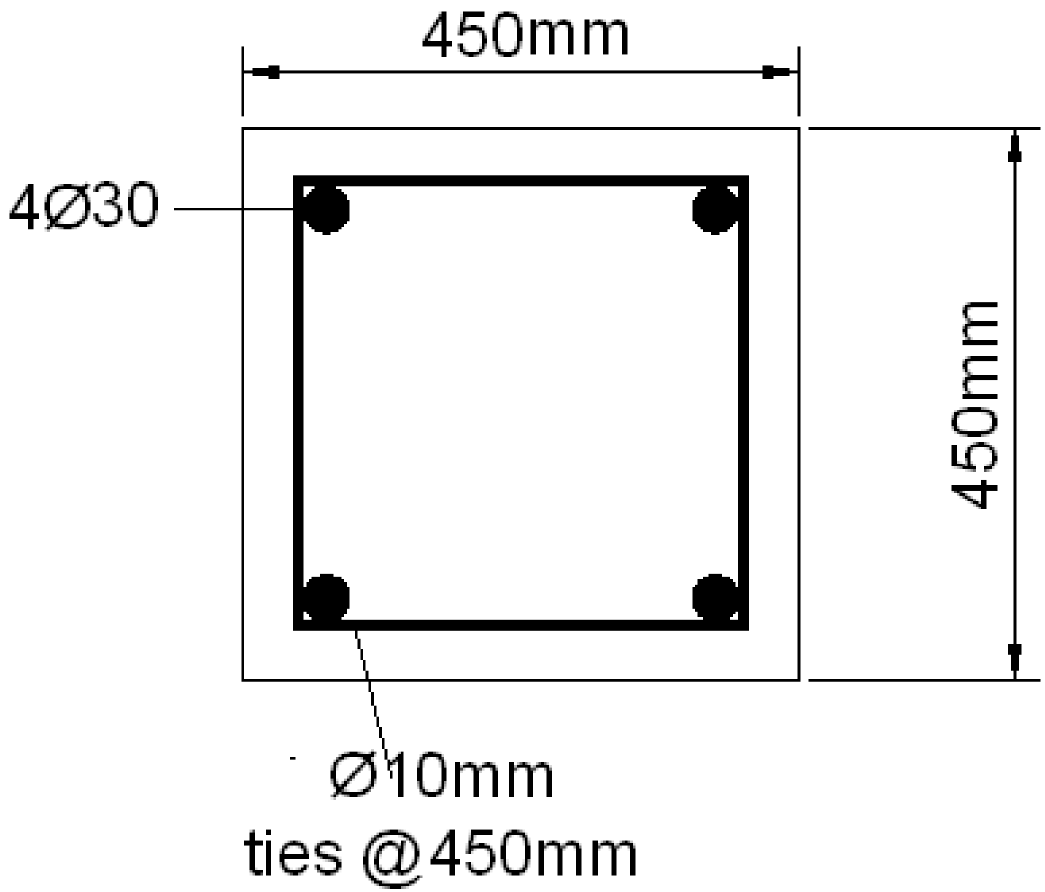
As= $\rho g * Ag = 0.013*450*450 = 2633 \ mm^2$

Use 4Ø30mm(As=2827 mm^2)

$for\ db = 30mm < 32mm, use\ tie\ Ø10mm\ @$

$$min\begin{cases} 16d_b = 16*30 = 480\ mm \\ 48d_{tie} = 48*10 = 480\ mm \\ least\ dimension\ of\ column\ cross\ section = 450mm \end{cases}$$

$use\ tie\ Ø10mm\ @450mm\ c/c$

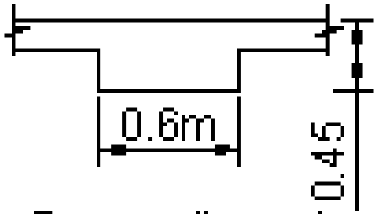

450mm
4Ø30
450mm
Ø10mm
ties @450mm

Example2:

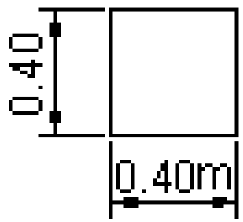
- fy=300MPa, fc'=30MPa
- Beams dimensions(bw=0.6 m, h=0.45 m)
- Columns dimensions(0.4*0.4m).

	D.L	L.L
P(kN)	620	410
M2(kN.m)	83	55
M1(kN.m)	-55	-36

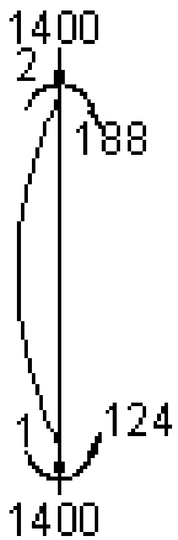
Required: design of column AB.

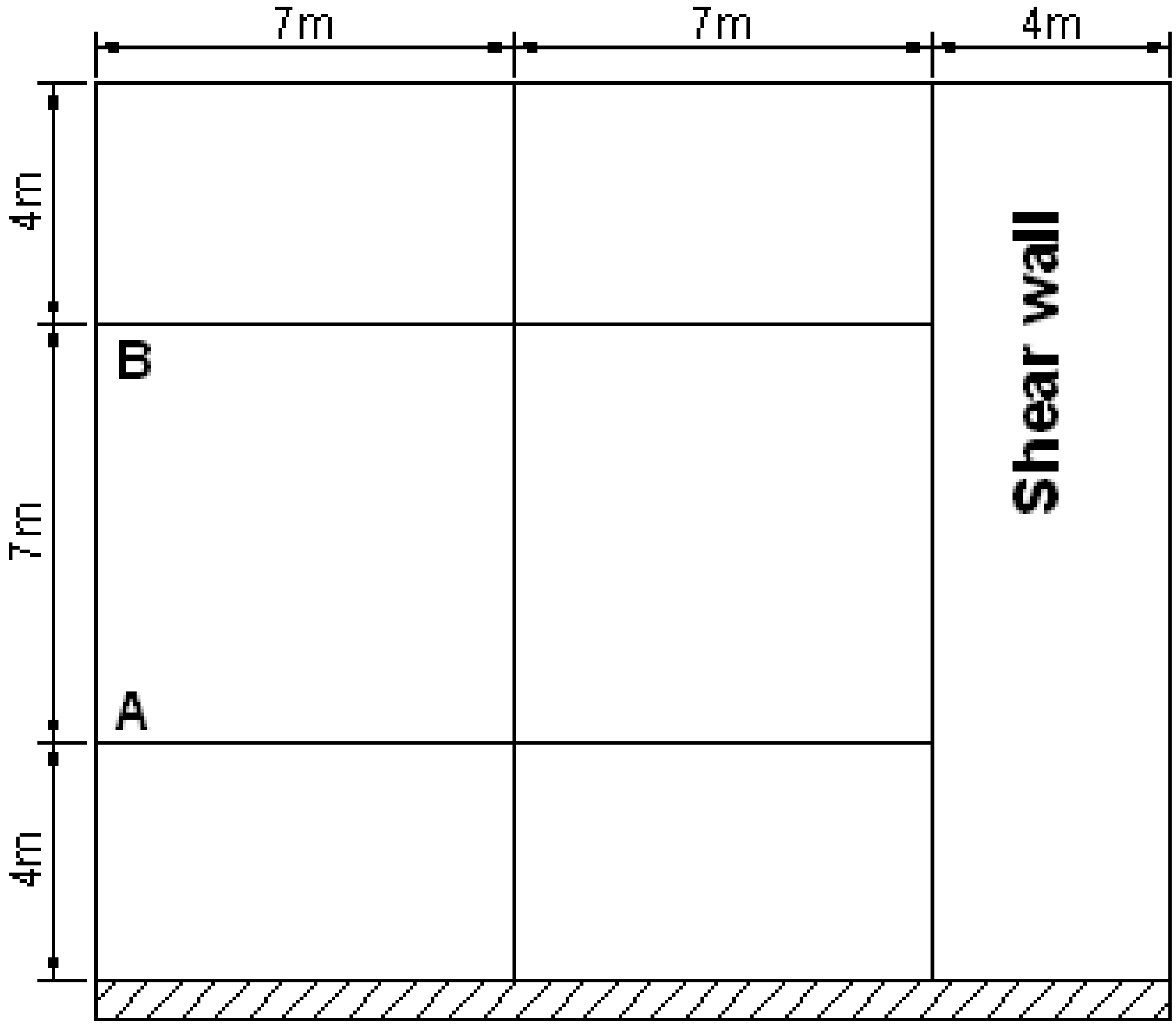


Beams dimensions



Columns dimensions





Solution:

Pu=1.2*620+1.6*410=1400 kN

Mu2=1.6*83+1.2*55=188 kN.m

Mu1=1.6*(-55)+1.2*(-36)=-124 kN.m

$$I_b = 0.7Ig_{web} = 0.7 * \frac{0.6 * 0.45^3}{12} = 3.189 * 10^{-3}m^4$$

$$I_c = 0.7Ig = 0.7 * \frac{0.4 * 0.4^3}{12} = 1.493 * 10^{-3}m^4$$

$$\psi_{top} = \psi_{bot} = \frac{\sum\left(\frac{EI}{l}\right)_{columns}}{\sum\left(\frac{EI}{l}\right)_{beams}} = \frac{Ec(\frac{1.493*10^{-3}}{7} + \frac{1.493*10^{-3}}{4})}{Ec(\frac{3.189*10^{-3}}{7})}$$

$$= 1.29$$

Braced column → graph →k=0.81

$$l_u = 7 - 0.45 = 6.55m$$

$$\frac{kl_u}{r} = \frac{kl_u}{0.3h} = \frac{0.81 * 6.55}{0.3 * 0.4} = 44.21$$

$$34 - 12\left(\frac{M1}{M2}\right) = 34 - 12\left(\frac{124}{188}\right) = 26 < 40$$

$$\frac{kl_u}{r} = 44.21 > 34 - 12\left(\frac{M1}{M2}\right) = 26 \rightarrow \therefore\ slender\ column$$

$$\mathrm{c_m} = 0.6 + 0.4(\frac{M1}{M2}) \geq 0.4$$

$$\mathrm{c_m} = 0.6 + 0.4\left(\frac{124}{188}\right) = 0.864 \geq 0.4\ O.K$$

$$\beta_\mathrm{d} = \frac{Pu_{\mathrm{sustained}}}{Pu_{\mathrm{total}}} = \frac{1.2 * 620}{1400} = 0.53$$

$$\mathrm{EI} = \frac{0.4Ec.Ig}{1 + \beta_\mathrm{d}} = \frac{0.4 * 4700 * \sqrt{30} * \frac{0.4 * 0.4^3}{12}}{1 + 0.53}$$

$$= 14.358\ MN.m^2$$

$$P_{cr} = \frac{\pi^2 EI}{(kl_u)^2} = \frac{\pi^2 * 14.358}{(0.81 * 6.55)^2} = 5.034\ MN$$

$$\delta_{ns} = \frac{\mathrm{c_m}}{1 - \frac{P_u}{0.75 P_{cr}}} = \frac{0.864}{1 - \frac{1400}{0.75 * 5034}} = 1.373 > 1.0$$

$$M_c = \delta_{ns} M_2 \geq \delta_{ns} M_{2,min}$$

$$M_c = 1.373 * 188 = 258\ kN.m$$

$$\geq 1.373 * 1400 \left(\frac{15}{1000} + 0.03 * 0.4 \right) = 52 kNm$$

$$\therefore M_c = 258\ kN.m$$

Design values: Pu=1400 kN, Mu=258 kN.m

e=Mu/Pu=258/1400=0.184 m

$$\gamma = \frac{h - 2d'}{h} = \frac{400 - 2 * 70}{400} = 0.65$$

$$\frac{e}{h} = \frac{0.184}{0.4} = 0.46$$

$$Kn = \frac{Pu}{\emptyset fc'.Ag} = \frac{1400}{0.65 * 30000 * 0.4 * 0.4} = 0.45$$

$$Rn = Kn\frac{e}{h} = 0.45 * 0.46 = 0.207$$

From graphs: For γ=0.6 (graph)→ρg=0.03

For γ=0.7 (graph)→ρg=0.023

For $\gamma = 0.65, \rightarrow \ \rho g = 0.0265 \begin{cases} < \rho_{max} = 0.08 \\ > \rho_{min} = 0.01 \end{cases} \ O.K$

Since εt<0.002→Ø=0.65

As= 0.0265*Ag=0.013*400*400=4240 mm^2

Use 8Ø28 mm(As=4926 mm^2)

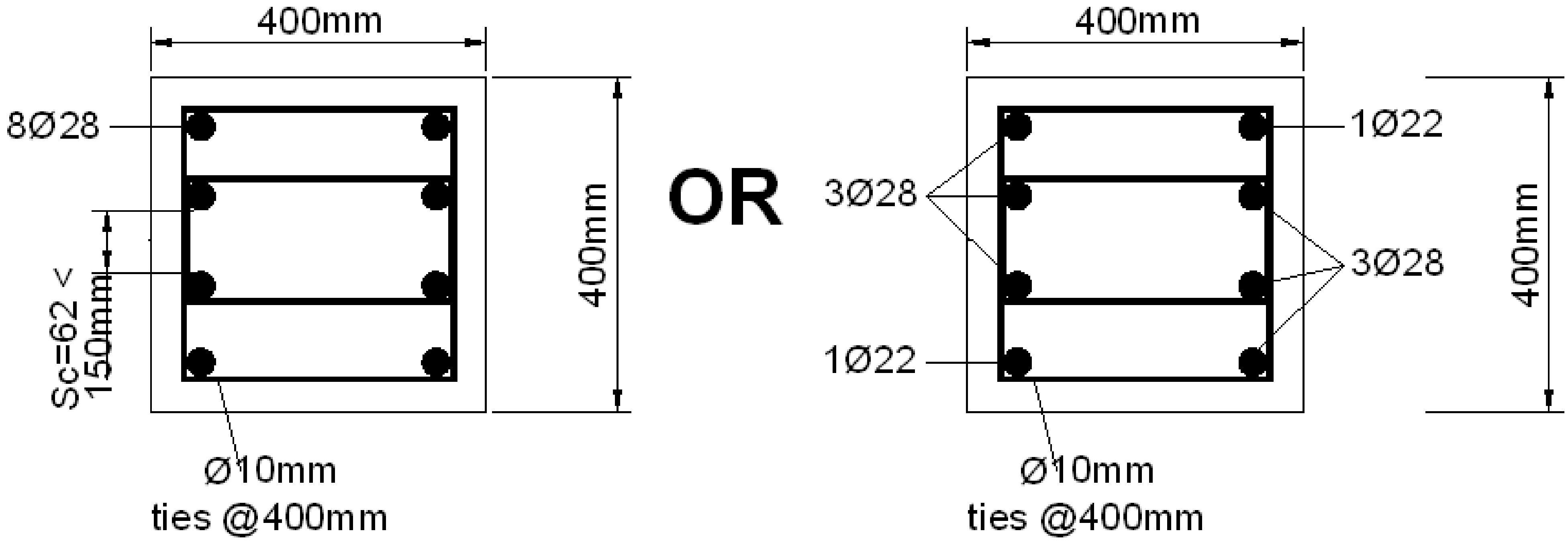
$for\ db = 28mm < 32mm, use\ tie\ \emptyset 10mm\ @$

$$min \begin{cases} 16d_b = 16 * 28 = 448\ mm \\ 48d_{tie} = 48 * 10 = 480\ mm \\ least\ dimension\ of\ column\ cross\ section = 400mm \end{cases}$$

$$use\ tie\ \emptyset 10mm\ @400mm\ c/c$$

$$s_c = \frac{400 - 2*40 - 2*10 - 4*28}{4-1} = 62\ mm$$

$$\geq max\begin{cases} 1.5db = 1.5*28 = 42mm \\ 40mm \end{cases}\quad O.K$$


400mm
8Ø28
Sc=62 < 150mm
400mm
Ø10mm
ties @400mm
OR
400mm
1Ø22
3Ø28
3Ø28
1Ø22
400mm
Ø10mm
ties @400mm

Slender columns-sway frames:

- Stability index, $Q = \frac{(\sum Pu)\Delta_{\circ}}{V_u l_c} > 0.05 \rightarrow sway\ frame$
- $sway\ frame, if\ \frac{kl_u}{r}\begin{cases} < 22.0 \rightarrow short\ column \\ \geq 22.0 \rightarrow long\ column \end{cases}$
- $1.0 \leq k \leq \infty$

The moments M1&M2 at the ends of an individual compression member shall be taken as:

$M1 = M1_{ns} + \delta_s M1_s$ ACI 10-15

$M2 = M2_{ns} + \delta_s M2_s$ ACI 10-16

$M1_{ns}$:factoered end moment on a compression member at the end at which M1 acts, due to loads that cause no appreciable side sway.

$M2_{ns}$:factoered end moment on a compression member at the end at which M2 acts, due to loads that cause no appreciable side sway.

$M1_{s}$:factoered end moment on a compression member at the end at which M1 acts, due to loads that cause appreciable side sway.

$M2_s$:factoered end moment on a compression member at the end at which M2 acts, due to loads that cause appreciable side sway.

δ_s: moment magnification factor for frames not braced against side sway.

$$\delta_s = \frac{1}{1-Q}\begin{cases} \geq 1.0 \\ \leq 1.5 \end{cases} \quad \text{ACI 10-17}$$

OR

$$\delta_s = \frac{1}{1-\frac{\sum Pu}{0.75\sum Pcr}}\begin{cases} \geq 1.0 \\ \leq 2.5 \end{cases} \quad \text{ACI 10-18}$$

$\sum Pu$: summation for all the factored vertical loads in a story.

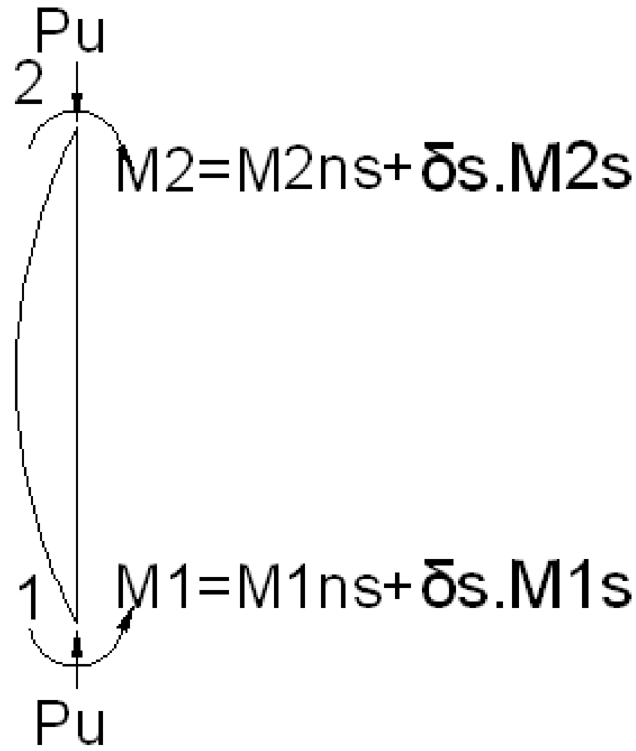
$\sum Pcr$: summation for all the critical buckling load for all columns in a story.

$$P_{cr} = \frac{\pi^2 EI}{(kl_u)^2}$$

$$\mathrm{EI} = \frac{0.4Ec.Ig}{1 + \beta_d}$$

β_d : ratio of maximum factored sustained shear within a story to the maximum(i.e total) factored shear in that story.

For each individual member, if $\frac{l_u}{r} > \frac{35}{\sqrt{\frac{Pu}{fc\prime Ag}}} \rightarrow M_c = \delta_{ns} M_2 = \delta_{ns}(M2_{ns} + \delta_s M2_s)$



Braced column

Example:

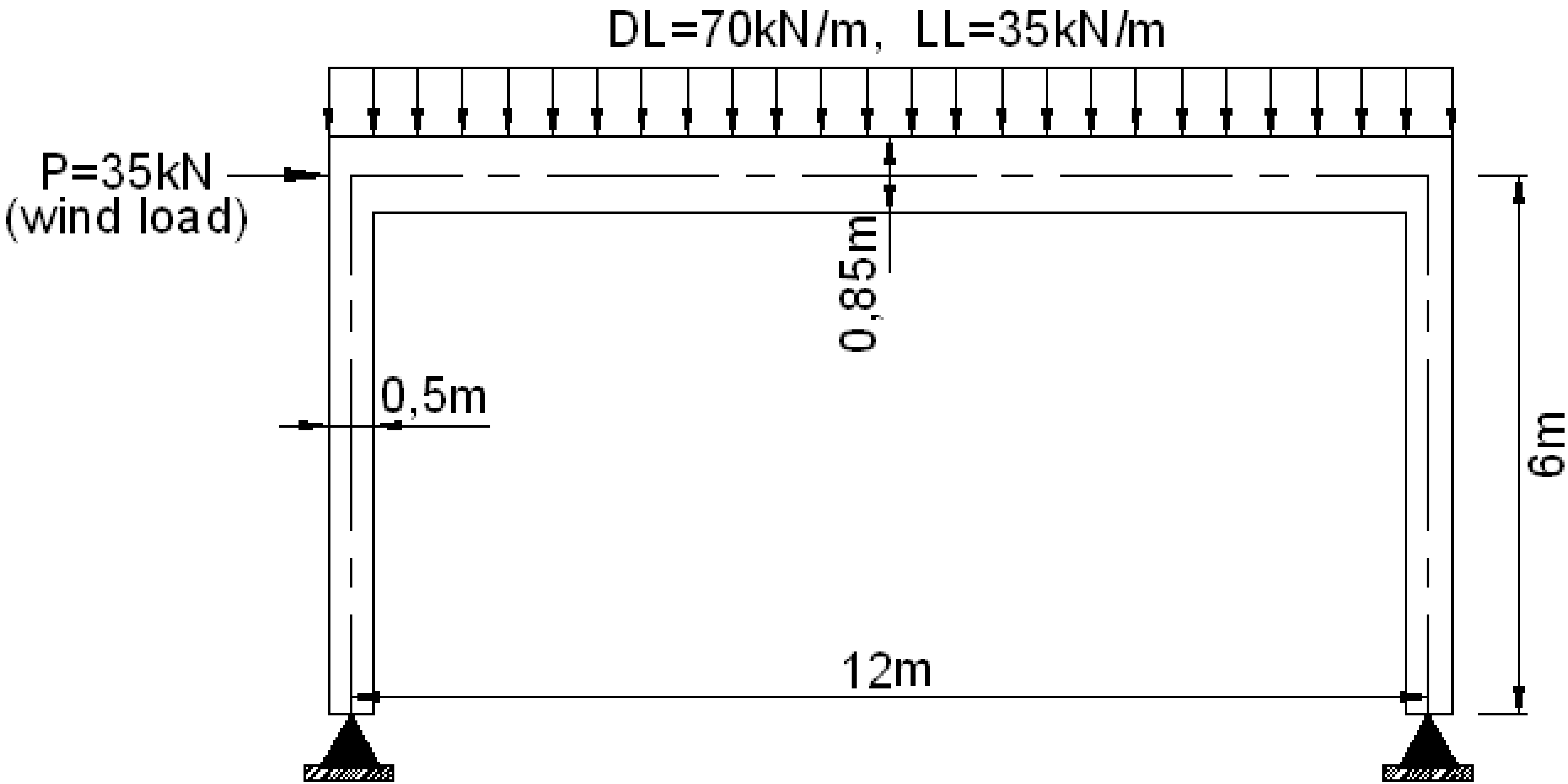
fc'=30 MPa, fy=400MPa

columns dimensions (500*500 mm)

beam dimensions (500mm width, 850mm depth)

service load(DL=70 kN/m, LL=35 kN/m, wind load, w=35 kN applied at top of frame).

Required: columns design .


DL=70kN/m, LL=35kN/m
P=35kN
(wind load)
0,85m
0,5m
6m
12m

Solution:

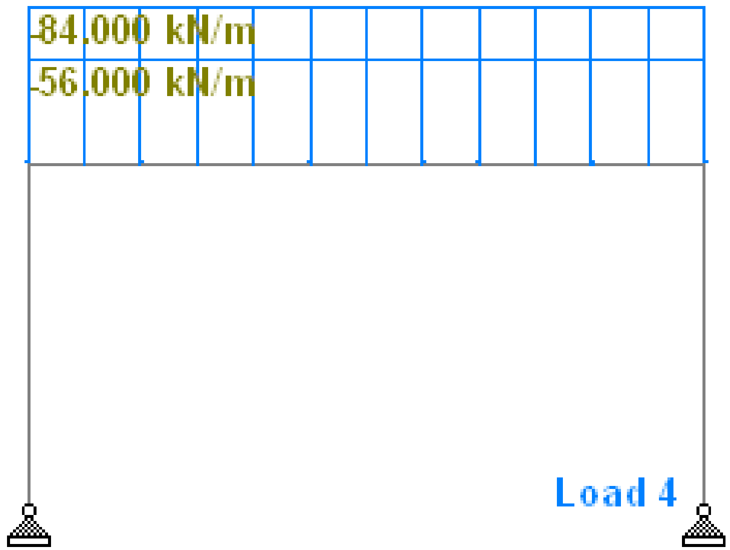
Since wind effect are included in the design, three possible factored load combinations are to be applied:

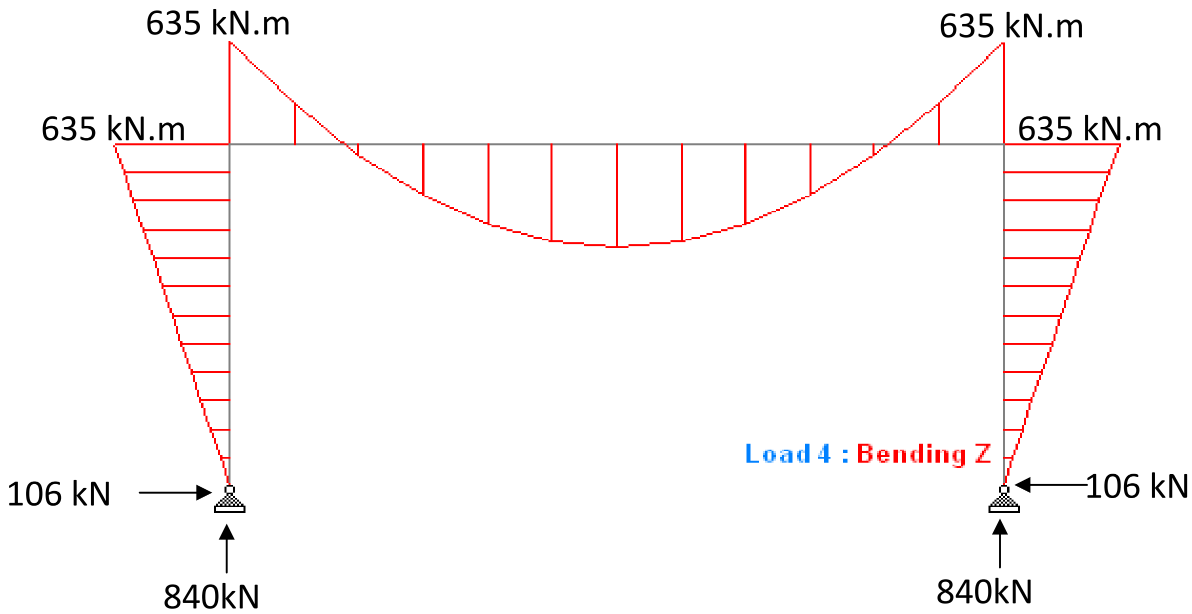
1. U=1.2D+1.6L
2. U=1.2D+1.0L+1.6W
3. U=0.9D+1.6W

Design for load case 1 (U=1.2D+1.6L):

U_D=1.2*70=84kN/m

U_L=1.6*35=56kN/m





Check if the story sway or non-sway:

The story is non-sway (braced), since no horizontal(wind) load applied.

1. Check slenderness for columns(braced)

Pu=840 kN (support reaction)

Mu2=635 kN.m

Mu1=0

$$I_b = 0.35Ig_{web} = 0.35 * \frac{0.5 * 0.85^3}{12} = 8.95 * 10^{-3}m^4$$

$$I_c = 0.7Ig = 0.7 * \frac{0.5 * 0.5^3}{12} = 3.64 * 10^{-3}m^4$$

$$\psi_{top} = \frac{\sum\left(\frac{EI}{l}\right)_{columns}}{\sum\left(\frac{EI}{l}\right)_{beams}} = \frac{Ec(\frac{3.64*10^{-3}}{6})}{Ec(\frac{8.95*10^{-3}}{12})} = 0.81$$

ψ_{bot} =10 (hinge support)

Braced column → graph →k=0.72

$$l_u = 6 - \left(\frac{0.85}{2}\right) = 5.57\, m$$

$$\frac{kl_u}{r} = \frac{kl_u}{0.3h} = \frac{0.72 * 5.57}{0.3 * 0.5} = 26.7$$

$$34 - 12\left(\frac{M1}{M2}\right) = 34 - 12\left(\frac{0}{M2}\right) = 34 < 40$$

$$\frac{kl_u}{r} = 26.7 < 34 - 12\left(\frac{M1}{M2}\right) = 34 \rightarrow \therefore \textit{short column}$$

2. Calculate steel reinforcement

Design values: Pu=840 kN, Mu=635 kN.m

e=Mu/Pu=635/840=0.756 m

$$\gamma = \frac{h - 2d'}{h} = \frac{500 - 2 * 70}{500} = 0.72$$

$$\frac{e}{h} = \frac{0.756}{0.5} = 1.51$$

$$Kn = \frac{Pu}{\emptyset fc'.Ag} = \frac{840}{0.65 * 30000 * 0.5 * 0.5} = 0.172$$

$$Rn = Kn\frac{e}{h} = 0.172 * 1.51 = 0.26$$

For γ=0.7 (graph)→ρg=0.038

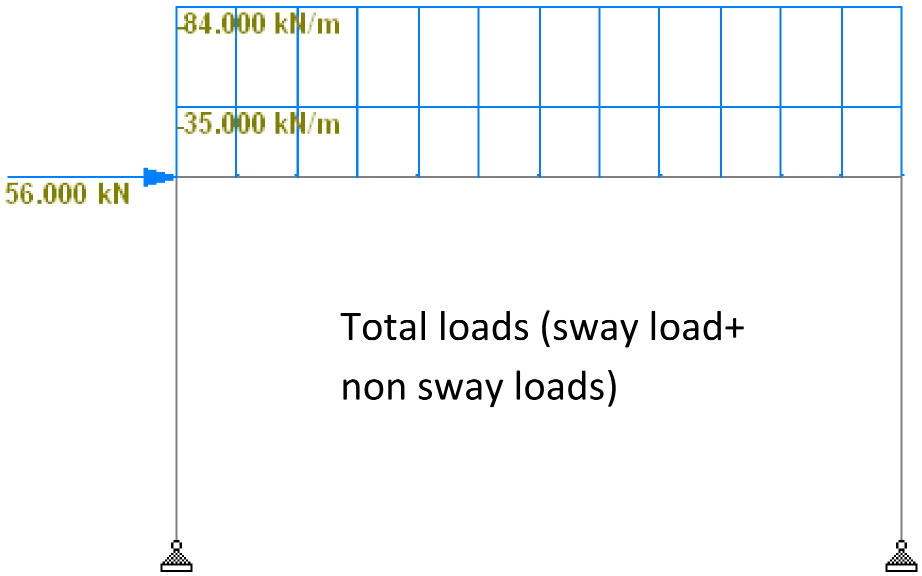
For γ=0.8 (graph)→ρg=0.032

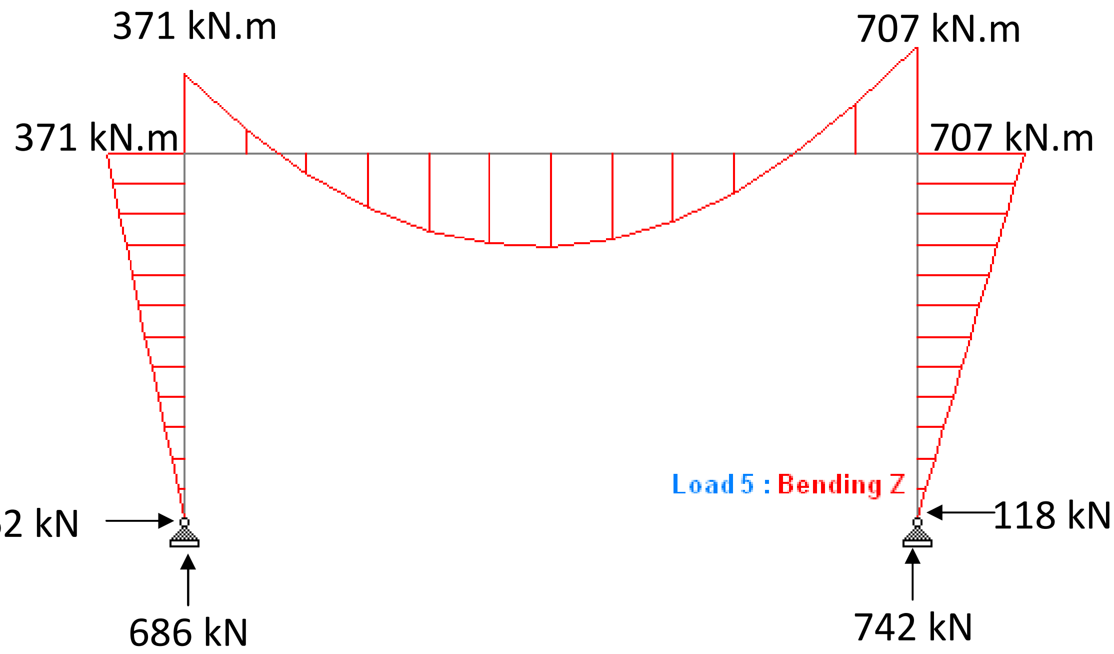
For γ=0.72→ρg=0.0368$\begin{cases} < \rho_{max} = 0.08 \\ > \rho_{min} = 0.01 \end{cases}$ $O.K$

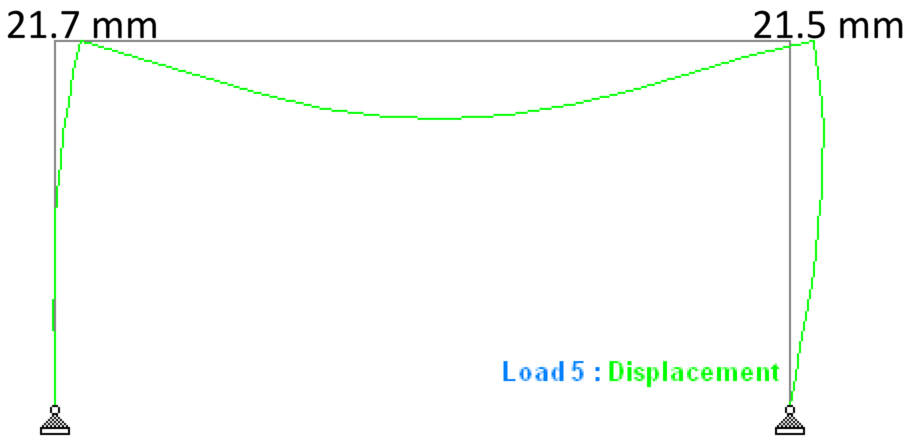
As= ρg *Ag=0.0368*400*700=9200 mm^2

Design for load case 2(U=1.2D+1.0L+1.6W):

$$U = \begin{pmatrix} \mathrm{U_D} = 1.2 * 70 = 84\,\frac{kN}{m} \\ \mathrm{U_L} = 1.0 * 35 = 35\,\frac{kN}{m} \\ \mathrm{U_W} = 1.6 * 35 = 56 kN \end{pmatrix}$$


84.000 kN/m
35.000 kN/m
56.000 kN
Total loads (sway load+
non sway loads)


371 kN.m
707 kN.m
371 kN.m
707 kN.m
Load 5 : Bending Z
62 kN
118 kN
686 kN
742 kN


21.7 mm
21.5 mm
Load 5 : Displacement

1. Check if the story sway or non-sway:

$$\sum Pu = (1.2 * 70 + 1 * 35) * 12 = 1428\ kN$$

$$OR \sum support\ reactions = 686 + 742 = 1428\ kN$$

Vu=1.6*W=1.6*35=56 kN OR$\sum support\ reactions = 118 - 62 = 56\ kN$

Δo=21.7 mm

Lc=6 m

$$Q = \frac{(\sum Pu)\Delta_{\circ}}{V_u l_c} = \frac{1428 * 0.0217}{56 * 6} = 0.092 > 0.05 \rightarrow$$

$\therefore$ *sway frame*

2. Check slenderness for columns(braced)

$$I_b = 0.7 Ig_{web} = 0.35 * \frac{0.5 * 0.85^3}{12} = 8.95 * 10^{-3} m^4$$

$$I_c = 0.7 Ig = 0.7 * \frac{0.5 * 0.5^3}{12} = 3.64 * 10^{-3} m^4$$

$$\psi_{top} = \frac{\sum\left(\frac{EI}{l}\right)_{columns}}{\sum\left(\frac{EI}{l}\right)_{beams}} = \frac{Ec\left(\frac{3.64*10^{-3}}{6}\right)}{Ec\left(\frac{8.95*10^{-3}}{12}\right)} = 0.81$$

ψ_{bot} =10 (hinge support)

unbraced column → graph →k=1.85

$$l_u = 6 - \left(\frac{0.85}{2}\right) = 5.57\ m$$

$$\frac{kl_u}{r} = \frac{kl_u}{0.3h} = \frac{1.85 * 5.57}{0.3 * 0.5} = 68.7 > 22.0 \rightarrow$$

$$\therefore slender(long)column$$

$$\delta_s = \frac{1}{1 - Q} = \frac{1}{1 - 0.092} = 1.1 \begin{cases} > 1.0 \\ < 1.5 \end{cases}$$

OR

$$\beta_d = \frac{Vu_{\text{sustained}}}{Vu_{\text{total}}} = 0 \qquad\qquad (wind\ load)$$

$$EI = \frac{0.4Ec.Ig}{1 + \beta_d} = \frac{0.4 * 4700 * \sqrt{30} * \frac{0.5 * 0.5^3}{12}}{1 + 0}$$

$$= 53.631\ MN.m^2$$

$$P_{cr} = \frac{\pi^2 EI}{(kl_u)^2} = \frac{\pi^2 * 53631}{(1.85 * 5.57)^2} = 4.985\ MN$$

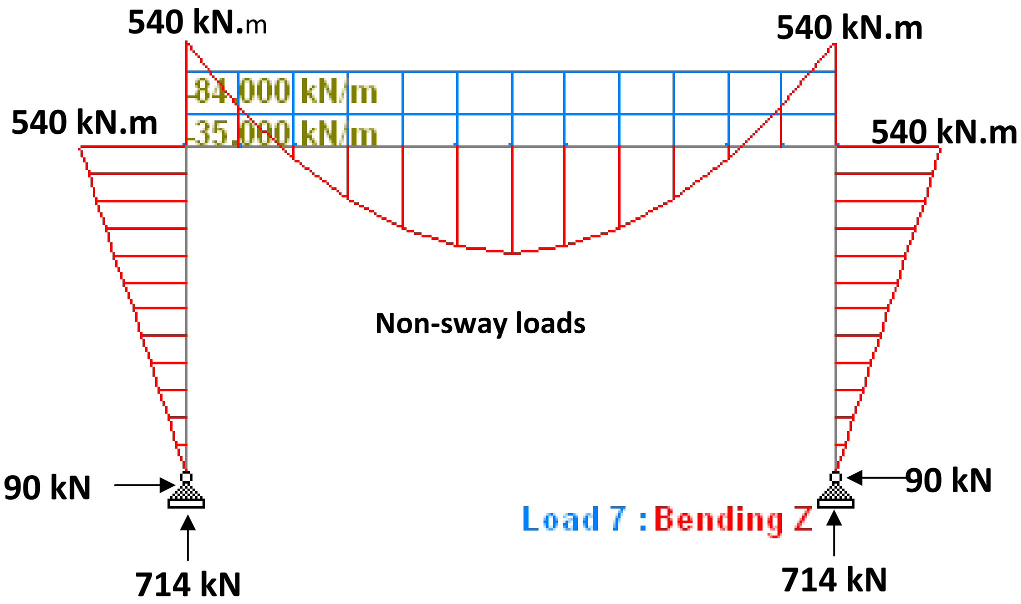
$$\delta_s = \frac{1}{1 - \frac{\sum P_u}{0.75 \sum P_{cr}}} = \frac{1}{1 - \frac{686+742}{0.75*2*4985}} = 1.23 \begin{cases} > 1.0 \\ < 2.5 \end{cases}$$

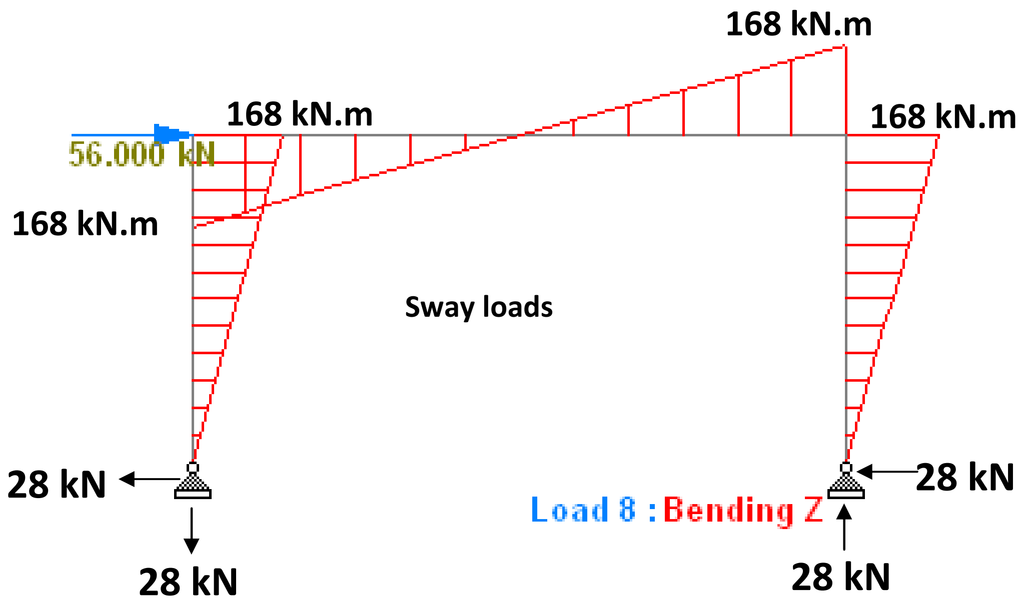
Use any value of δ_s above, $\delta_s = 1.23$

The ultimate load, U divided into: (non-sway load and sway load)

$$U = \begin{pmatrix} \mathrm{U_D} = 1.2 * 70 = 84\,\frac{kN}{m} \\ \mathrm{U_L} = 1.0 * 35 = 35\,\frac{kN}{m} \\ \mathrm{U_W} = 1.6 * 35 = 56kN \end{pmatrix}$$

$$\rightarrow \begin{cases} nonsway\ load \begin{cases} \mathrm{U_D} = 1.2 * 70 = 84\,\frac{kN}{m} \\ \mathrm{U_L} = 1.0 * 35 = 35\,\frac{kN}{m} \end{cases} \\ sway\ load, \mathrm{U_W} = 1.6 * 35 = 56kN \end{cases}$$


540 kN.m
540 kN.m
84.000 kN/m
35.000 kN/m
540 kN.m
540 kN.m
Non-sway loads
90 kN
90 kN
Load 7 : Bending Z
714 kN
714 kN


168 kN.m
168 kN.m
168 kN.m
56.000 kN
168 kN.m
Sway loads
28 kN
28 kN
Load 8 : Bending Z
28 kN
28 kN

	Left column	Right column
M1s (kN.m)	0	0
M1ns (kN.m)	0	0
M2s (kN.m)	-168	-168
M2ns (kN.m)	540	-540
Pu (kN)	686	742
Amplifier moments	M1= M1ns+δ_sM1s=0 M2= M2ns+δ_sM2s M2=540+1.23*(-168) =333 kN.m	M1= M1ns+δ_sM1s=0 M2= M2ns+δ_sM2s M2=-540+1.23*(-168) = -747 kN.m

For each column, check

Right column:

$$\frac{l_u}{r} = \frac{5.57}{0.3*0.5} = 37.1 < \frac{35}{\sqrt{\frac{Pu}{fc'A}}} = \frac{35}{\sqrt{\frac{742}{3 * {}^{3} * .5 * 5}}} = 111 \rightarrow$$

$$not\ need\ to\ M_c = \delta_{ns} M_2 = \delta_{ns}(M2_{ns} + \delta_s M2_s) = \delta_{ns} * 747$$

Left column: $\frac{l_u}{r} = \frac{5.57}{0.3*0.5} = 37.1 < \frac{35}{\sqrt{\frac{Pu}{fc'Ag}}} = \frac{35}{\sqrt{\frac{686}{30*10^3*0.5*05}}} =$

$$116 \rightarrow not\ need\ to\ M_c = \delta_{ns} M_2 = \delta_{ns}(M2_{ns} + \delta_s M2_s) =$$

$$\delta_{ns} * 333$$

3. Calculate steel reinforcement

Right column more critical.

Design values:Pu=742 kN, Mu=747 kN.m

e=Mu/Pu=747/742=1.006 m

$$\gamma = \frac{h - 2d'}{h} = \frac{500 - 2 * 70}{500} = 0.72$$

$$\frac{e}{h} = \frac{1.006}{0.5} = 2.013$$

$$Kn = \frac{Pu}{\emptyset f c'.Ag} = \frac{742}{0.65 * 30000 * 0.5 * 0.5} = 0.15$$

$$Rn = Kn\frac{e}{h} = 0.15 * 2.013 = 0.3$$

For γ=0.7 (graph)→ρg=0.047

For γ=0.8 (graph)→ρg=0.04

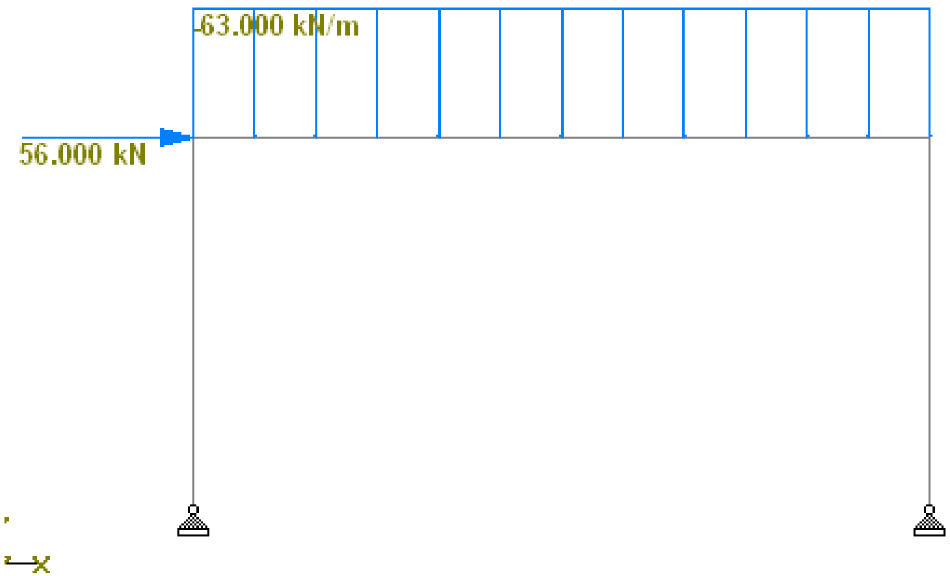
For γ=0.72→ρg=0.0456$\begin{cases} < \rho_{max} = 0.08 \\ > \rho_{min} = 0.01 \end{cases}$ $O.K$

As= ρg *Ag=0.0456*500*500=11400 mm^2

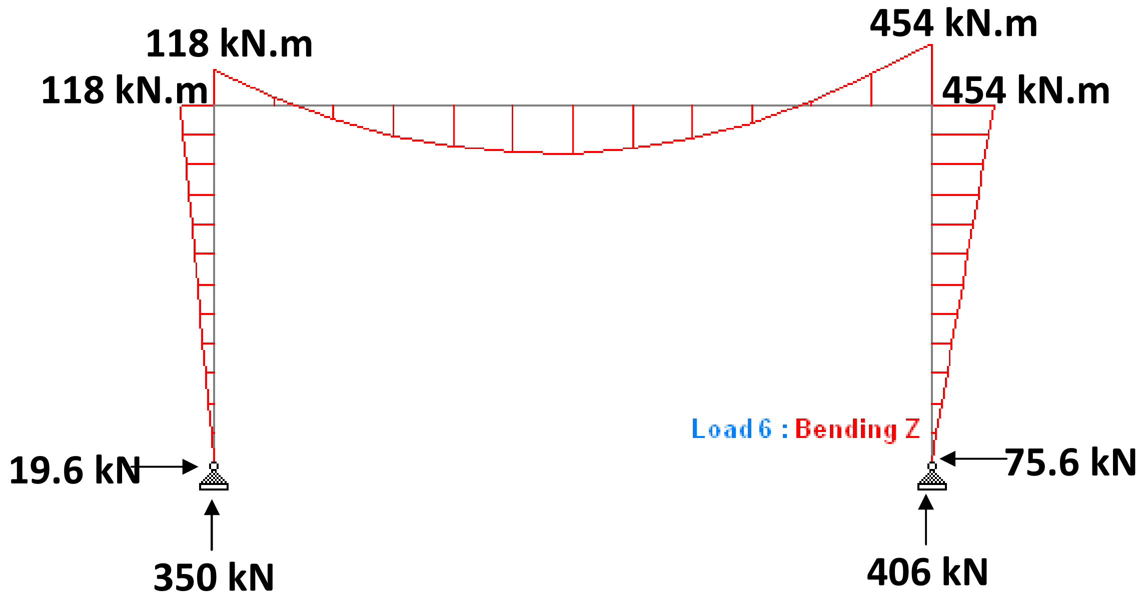
Design for load case 3 (U=0.9D+1.6W):

U_D=0.9*70=63 kN/m

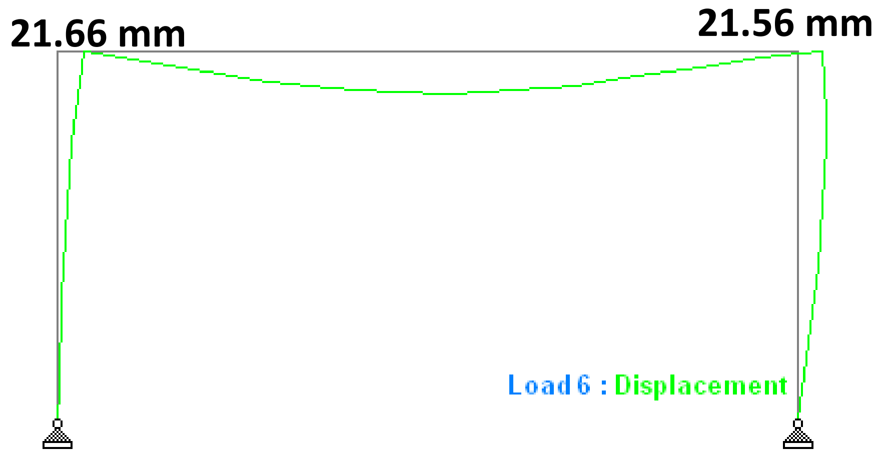
U_W=1.6*35=56 kN



63.000 kN/m
56.000 kN
X



118 kN.m
118 kN.m
454 kN.m
454 kN.m
Load 6 : Bending Z
19.6 kN
75.6 kN
350 kN
406 kN



21.66 mm
21.56 mm
Load 6 : Displacement

1. Check if the story sway or non-sway:

$$\sum Pu = 0.9 * 70 * 12 = 756\ kN$$

$$OR \sum support\ reactions = 350 + 406 = 756\ kN$$

Vu=1.6*W=1.6*35=56 kN OR $\sum support\ reactions = 75.6 - 19.6 = 56\ kN$

Δo=21.66 mm

Lc=6 m

$$Q = \frac{(\sum Pu)\Delta_\circ}{V_u l_c} = \frac{756 * 0.02166}{56 * 6} = 0.0487 < 0.05 \rightarrow$$

$\therefore$ *nonsway frame*

2. Check slenderness for columns(braced)

Short columns (from load case 1)

3. Calculate steel reinforcement

	Left column	Right column
M1	0	0
M2	118	-454
Pu	350	406

Right column more critical.

<u>Design values: Pu=406 kN, Mu=454 kN.m</u>

e=Mu/Pu=454/406=1.118 m

$$\gamma = \frac{h - 2d'}{h} = \frac{500 - 2 * 70}{500} = 0.72$$

$$\frac{e}{h} = \frac{1.118}{0.5} = 2.236$$

$$Kn = \frac{Pu}{\emptyset fc'.Ag} = \frac{406}{0.65 * 30000 * 0.5 * 0.5} = 0.08$$

$$Rn = Kn\frac{e}{h} = 0.08 * 2.236 = 0.18$$

For γ=0.7 (graph)→ρg=0.028

For γ=0.8 (graph)→ρg=0.024

For γ=0.72→ρg=0.027 $\begin{cases} < \rho_{max} = 0.08 \\ > \rho_{min} = 0.01 \end{cases} O.K$

As= ρg *Ag=0.027*500*500=6750 mm^2

From the above three load combinations, the maximum

$$\begin{cases} \text{As} = 11400 \text{ mm}^2 \text{ for right column} \ldots \ldots . load\ case\ 2 \\ \text{As} = 9200 \text{ mm}^2 \quad \text{for left column} \ldots \ldots . load\ case\ 1 \end{cases}$$

Due to reversible action of wind, both columns should provided with As=11400 mm^2, Use 8Ø44(12164 mm^2)

$$for\ db = 44mm > 32mm, use\ tie\ \emptyset 12mm\ @$$

$$min\begin{cases} 16d_b = 16 * 44 = 704\ mm \\ 48d_{tie} = 48 * 12 = 576\ mm \\ least\ dimension\ of\ column\ cross\ section = 500mm \end{cases}$$

$$use\ tie\ \emptyset 12mm\ @500mm\ c/c$$

$$s_c = \frac{500 - 2 * 40 - 2 * 12 - 4 * 44}{4 - 1} = 73\ mm$$

$$\geq max\begin{cases} 1.5db = 1.5 * 44 = 66mm \\ 40mm \end{cases}\ O.K$$

Check column for shear

$$Vu = 118kN \quad load\ case\ 2$$

$$Vc = 0.17\sqrt{30} * 0.5 * 0.43 * 1000$$

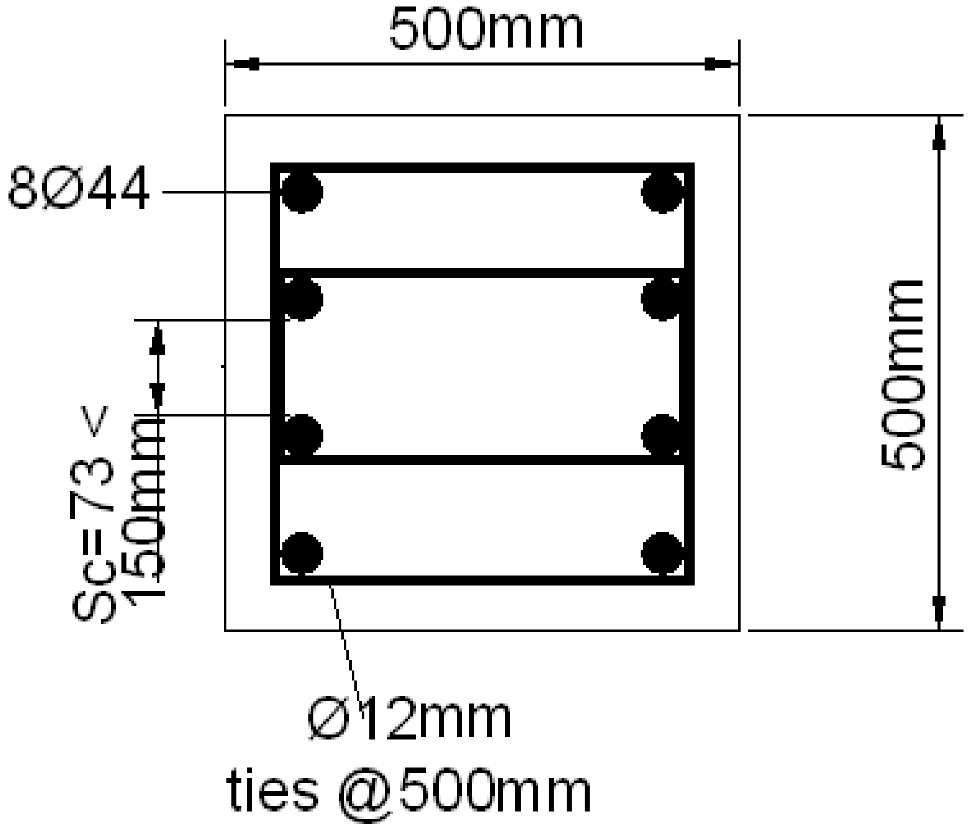
$$= 200\ kN\ (effect\ of\ axial\ force\ ignored)$$

$$\frac{Vu}{\emptyset} = \frac{118}{0.75} = 157\ kN < Vc = 200kN \quad O.K$$

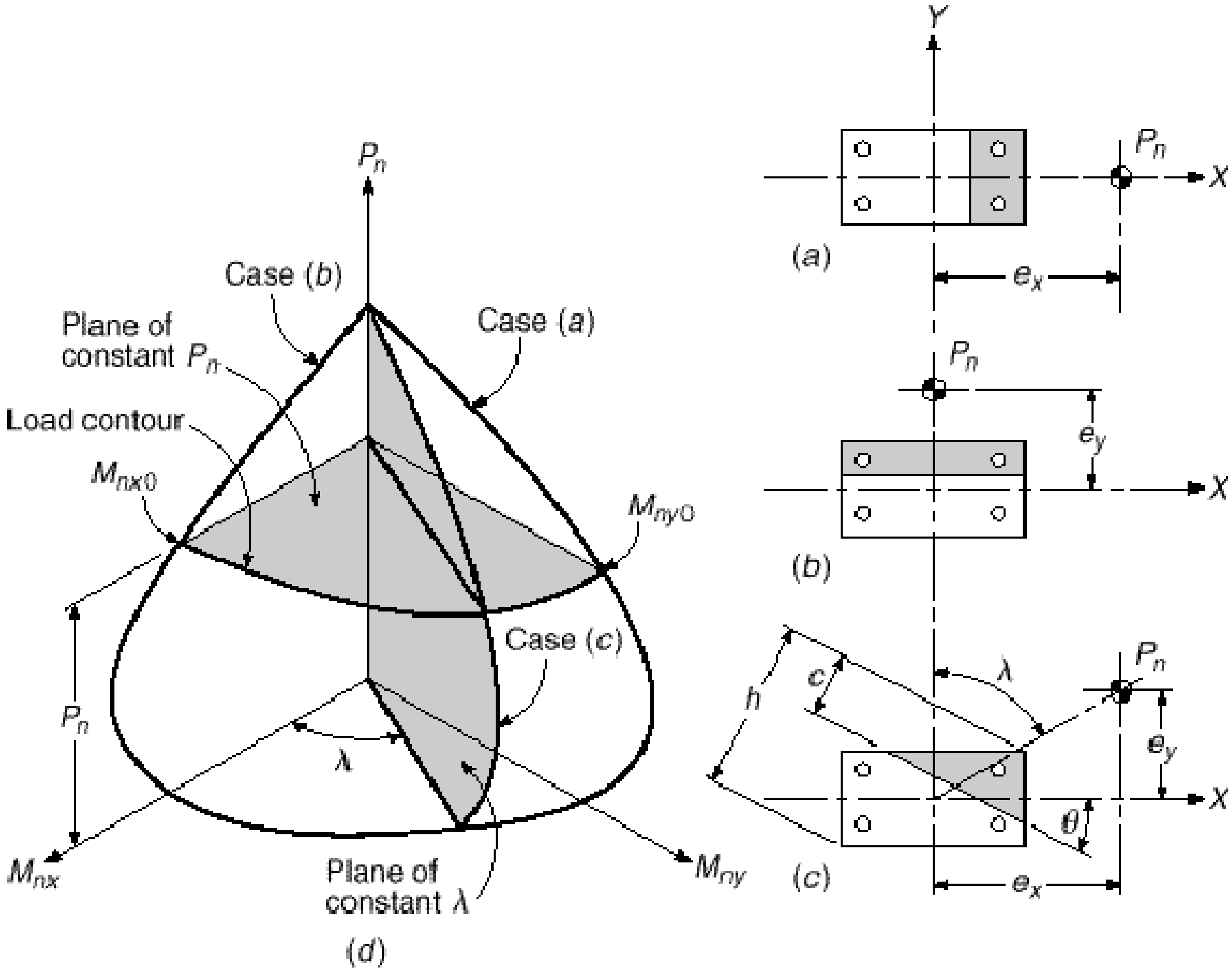
$$\frac{Vu}{\emptyset} = \frac{118}{0.75} = 157\ kN > \frac{Vc}{2} = 100kN \quad \rightarrow use\ Av_{min}$$

$$Av_{min} = \frac{b_w s}{3fy} \rightarrow 2 * 113 = \frac{500 * s}{3 * 400} \rightarrow s = 542mm >$$

$$tie\ spacing = 500mm \rightarrow \therefore use\ tie\ \emptyset 12mm\ @500mm\ c/c$$



Biaxial bending plus compression force:



- Fig.(a), the section subject to bending about y-axis only, with load eccentricity (e_x)measured in x-direction. The corresponding strength interaction curve is shown as case(a) in the three-dimensional sketch in fig(d) and is drawn in the plane defined by the axes P_n and M_{ny}. Such a curve can be established by the usual methods for uniaxial bending.
- Similarly, fig(b) bending about x-axis only, with load eccentricity (e_y)measured in x-direction. The corresponding interaction curve is shown as case(b) in plane of P_n and M_{nx} in fig(d).

- For case(c), which combines x and y axis bending, the orientation of the resultant eccentricity is defined by the angle λ:

$$\lambda = \tan^{-1}\frac{ex}{ey} = \tan^{-1}\frac{M_{ny}}{M_{nx}}$$

Axis of bending is defined by the angle θ with respect to x-axis. The angle λ in fig(c) establishes a plane in fig(d), passing through the vertical P_n axis and making an angle λ with the M_{nx} axis.

- Case(c) represent the interaction curve for this value of λ.

- For other values of λ, similar curves are obtained to define a *failure surface* for axial load plus biaxial bending. The surface is exactly analogous to the *interaction curve* for axial plus uniaxial bending.

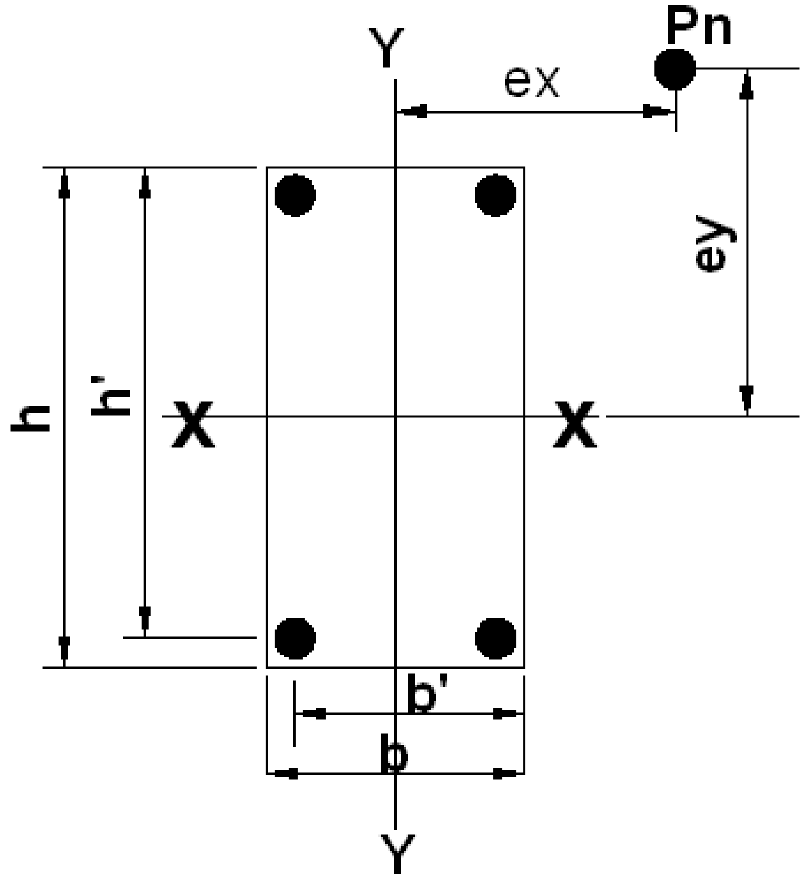
If rectangular column subject to biaxial bending moment, the following simple method may be used.

1. If $\frac{M_{nx}}{h'} > \frac{M_{ny}}{b'}$, $then$

$$M'_{nx} = M_{nx} + \frac{\beta h'}{b'} M_{ny}$$

2. If $\frac{M_{ny}}{b'} > \frac{M_{nx}}{h'}$, $then$

$$M'_{ny} = M_{ny} + \frac{\beta b'}{h'} M_{nx}$$



β: factor ranged 0.3-1.0,can be take 0.75

M_{nx}: the nominal *biaxial* moment strength about x-axis($M_{nx} = Pn * e_y$)

M_{ny}: the nominal *biaxial* moment strength about y-axis($M_{ny} = Pn * e_x$)

M'_{nx}: the *uniaxial* nominal moment strength about x-axis.

M'_{ny}: the *uniaxial* nominal moment strength about y-axis.

The biaxial rectangular column designed using above method should be checked by the following Breslers reciprocal load equation:

$$\frac{1}{P_n} \leq \frac{1}{P_{nxo}} + \frac{1}{P_{nyo}} - \frac{1}{P_{no}}$$

P_n:approximate value of nominal load in biaxial bending with eccentricities e_x *and* e_y.

P_{nyo}:nominal load when eccentricity e_x is present ($e_y = 0$)(bending about y-axis).

P_{nxo}:nominal load when eccentricity e_y is present ($e_x = 0$)(bending about x-axis).

P_{no}:nominal load for concentrically loaded column ($e_x = e_y = 0$).

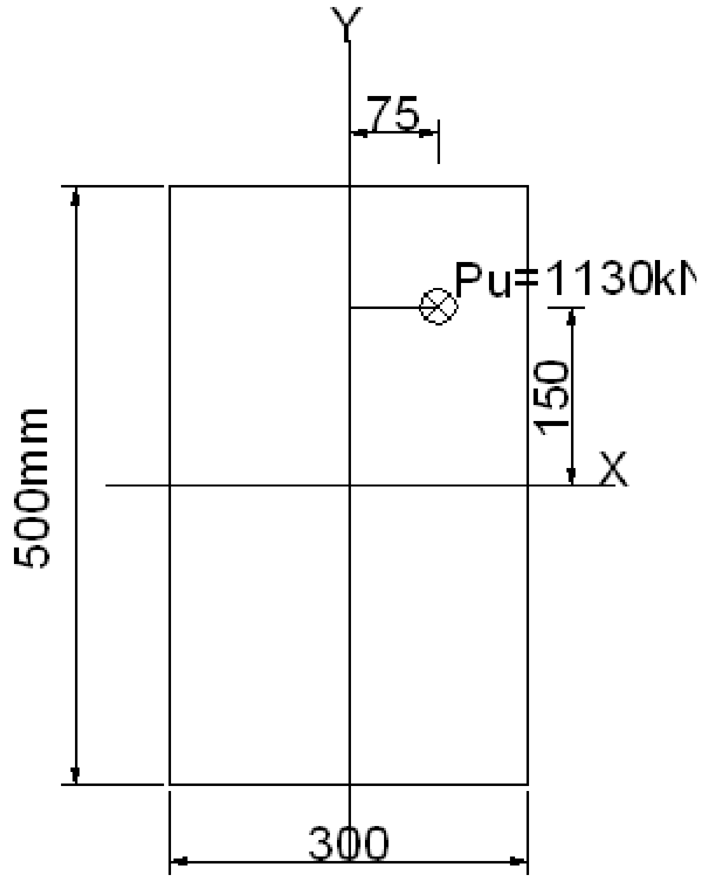
** Bresler method to be acceptably accurate for design when* $P_n \geq 0.1\, \boldsymbol{P_{n0}}$

Example1:

Short column, fy=400MPa, fc'=28MPa,

$P_u = 1130kN\ with\ e_x = 75mm, e_y = 150mm$. A complete design of is required.

Solution:



$$P_n = \frac{P_u}{\emptyset} = \frac{1130}{0.65} = 1738\ kN$$

$$M_{nx} = P_n * e_y = 1738 * 0.15$$

$$= 261\ kN.m$$

$$M_{ny} = P_n * e_x = 1738 * 0.075 = 130\ kN.m$$

$$b' = 300 - 65 = 235mm$$

$$h' = 500 - 65 = 435mm$$

$$\frac{M_{nx}}{h'} > \frac{M_{ny}}{b'}$$

$$\frac{261}{435} = 0.6 > \frac{130}{235} = 0.55 \rightarrow M'_{nx} = M_{nx} + \frac{\beta h'}{b'}\ M_{ny}$$

$$\beta = 0.75$$

$$M'_{nx} = 261 + \frac{0.75 * 435}{235} * \ 130 = 441\ kN.m$$

Uniaxial column $M'_{nx} = 441\ kN.m$, Pn=1738kN, h=500mm

e=Mn/Pn=441/1738=0.254 m

$$\gamma = \frac{h - 2d'}{h} = \frac{500 - 2 * 65}{500} = 0.74$$

$$\frac{e}{h} = \frac{0.254}{0.5} = 0.507$$

$$Kn = \frac{Pu}{\emptyset fc'.Ag} = \frac{1130}{0.65 * 28000 * 0.3 * 0.5} = 0.41$$

$$Rn = Kn\frac{e}{h} = 0.41 * 0.507 = 0.21$$

For γ=0.7 (graph)→ρg=0.031

For γ=0.8 (graph)→ρg=0.027

For γ=0.74→ρg=0.0294 $\begin{cases} < \rho_{max} = 0.08 \\ > \rho_{min} = 0.01 \end{cases}$ $O.K$

As= ρg *Ag=0.0294*300*500= 4410 mm^2

Use 8Ø28mm (As= 4920 mm^2)

$$for\ db = 28mm < 32mm, use\ tie\ Ø10mm\ @$$

$$min \begin{cases} 16d_b = 16 * 28 = 448\ mm \\ 48d_{tie} = 48 * 10 = 480\ mm \\ least\ dimension\ of\ column\ cross\ section = 300mm \end{cases}$$

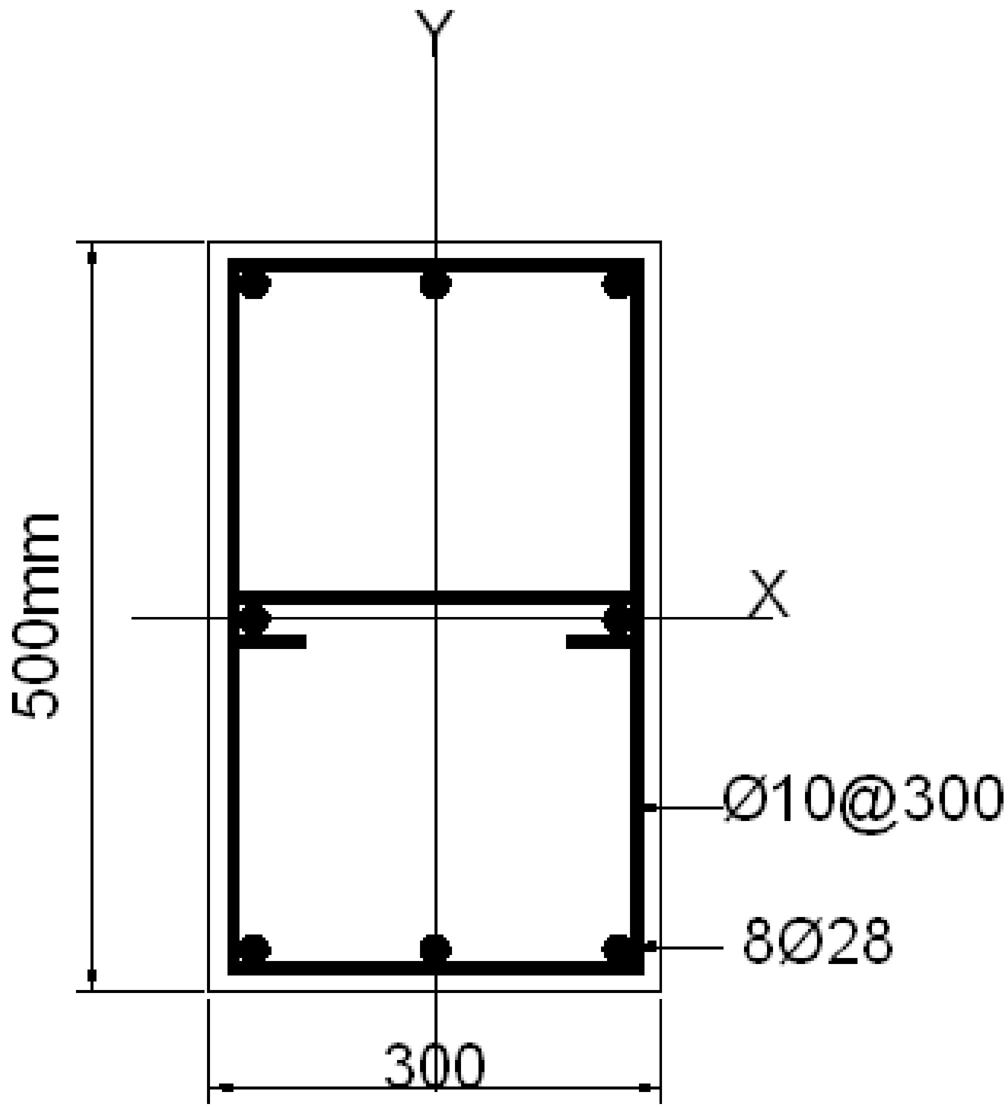
$$use\ tie\ Ø10mm\ @300mm\ c/c$$

$$s_c = \frac{500 — 2 * 40 - 2 * 10 - 3 * 28}{3 - 1} = 158\, mm$$

$$\geq max \begin{cases} 1.5db = 1.5 * 28 = 42\, mm \\ 40mm \end{cases} \quad O.K$$

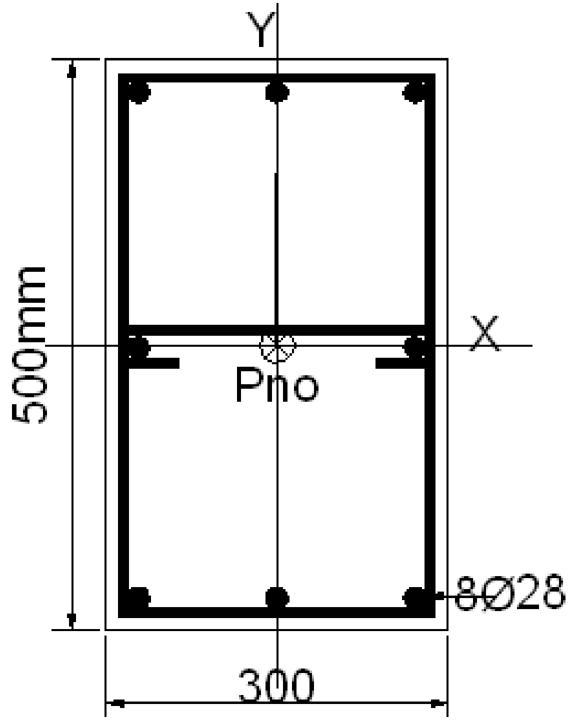
$$s_c = \frac{300 — 2 * 40 - 2 * 10 - 3 * 28}{3 - 1} = 58\, mm$$

$$\geq max \begin{cases} 1.5db = 1.5 * 28 = 42\, mm \\ 40mm \end{cases} \quad O.K$$


Y
X
500mm
Ø10@300
8Ø28
300

Check the design using Breslers reciprocal load equation

$$\frac{1}{P_n} \le \frac{1}{P_{nxo}} + \frac{1}{P_{nyo}} - \frac{1}{P_{no}}$$



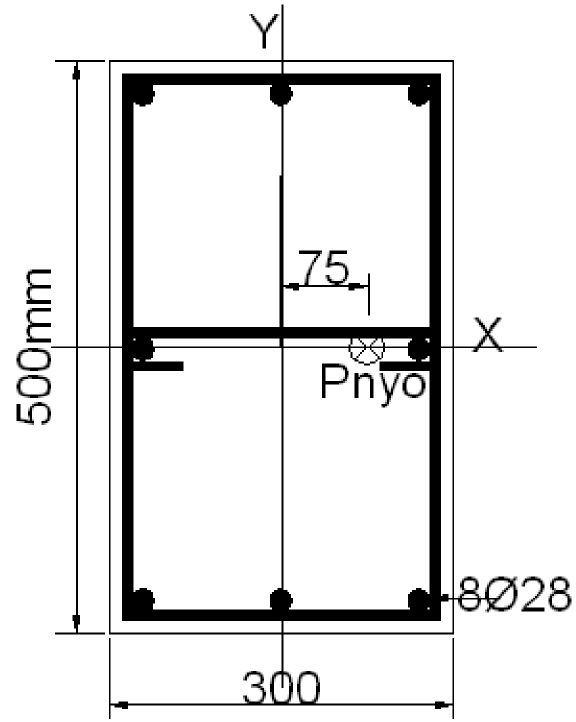
1. To find P_{no} $(e_x = e_y = 0\ )$.

$$P_{no} = 0.85fc'(Ag - Ast) + Ast\ fy$$

$$P_{no} = 0.85 * 28000(0.5 * 0.3 - 4920 * 10^{-6}) + 4920 * 10^{-6} * 400000 = 5423\ kN$$

$$P_n = 1728\ kN\ > 0.1P_{no} = 0.1 * 5423 = 542.3\ kN\ \ O.K$$

2. To find P_{nyo} $(bending\ about\ y-axis\ ,\quad e_y=0\quad)$, h=300mm.



$$\gamma=\frac{h-2d'}{h}=\frac{300-2*65}{300}$$

$$=0.57\ say\ 0.6$$

$$\rho_g=\frac{As}{Ag}=\frac{4920}{300*500}=0.033$$

$$\frac{e}{h}=\frac{75}{300}=0.25$$

For γ=0.6, $\rho_g=0.033, \frac{e}{h}=0.25$ →graph A5→kn=0.65

$$Kn = \frac{Pu}{\emptyset fc'.Ag} \rightarrow 0.65 = \frac{P_{nyo}}{28000 * 0.3 * 0.5} \rightarrow$$

$$P_{nyo} = 2730\ kN$$

3. To find P_{nxo} $(bending\ about\ x-axis\ ,\ \ e_x=0\ )$, h=500mm.

$$\gamma=\frac{h-2d'}{h}=\frac{500-2*65}{500}=0.74$$

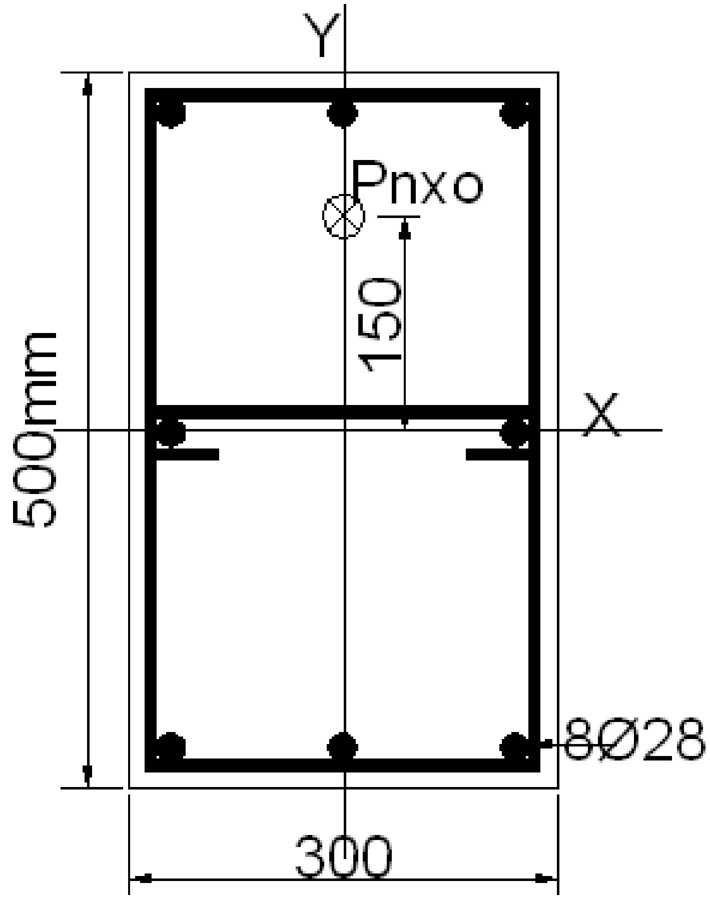
$$\rho_g=\frac{As}{Ag}=\frac{4920}{300*500}=0.033$$

$$\frac{e}{h}=\frac{150}{500}=0.3$$

For γ=0.7→graph A6→kn=0.63

For γ=0.8→graph A7→kn=0.65

For γ=0.74 →kn=0.64



$$Kn = \frac{Pu}{\emptyset fc'.Ag} \rightarrow 0.64 = \frac{P_{nyo}}{28000 * 0.3 * 0.5} \rightarrow$$

$$P_{nxo} = 2688\ kN$$

$$\frac{1}{P_n} = \frac{1}{P_{nxo}} + \frac{1}{P_{nyo}} - \frac{1}{P_{no}}$$

$$\frac{1}{P_n} = \frac{1}{2688} + \frac{1}{2730} - \frac{1}{5423} \rightarrow P_{n,max} = 1805\ kN > P_{n,applied}$$

$$= 1738\ kN\ \ O.K$$

Hence, the column safe to carry load, $P_u = 1130kN$ ($P_n = 1738\ kN$) $with\ e_x = 75mm, e_y = 150mm$.

Example2:Short column, fy=400MPa, fc'=28MPa,

$find\ maximum\ safe\ value\ of P_u.$

Solution:

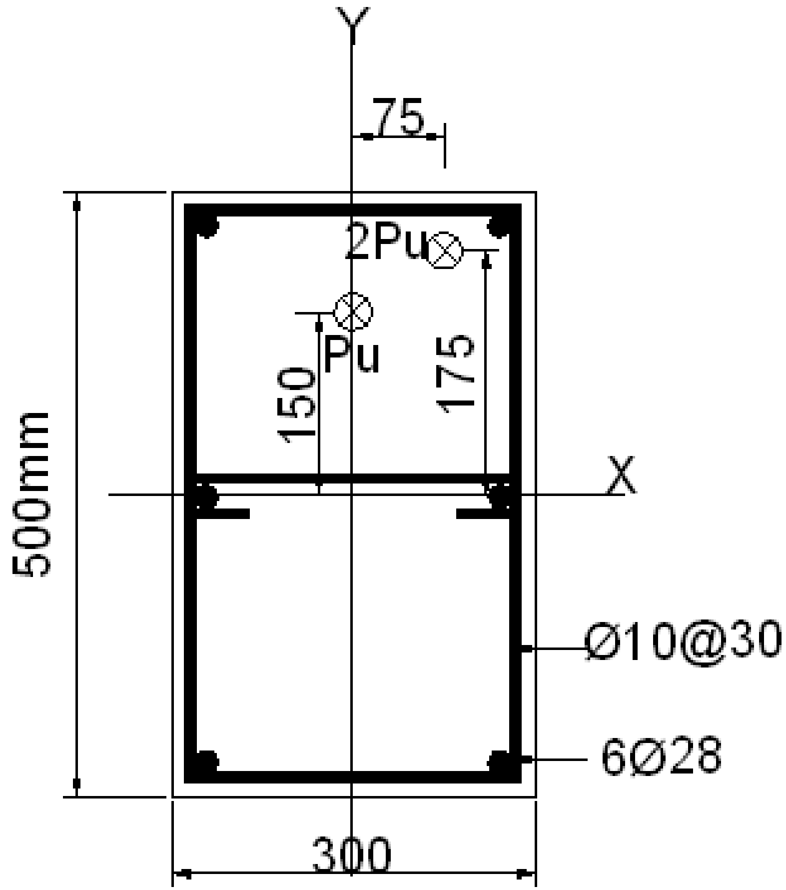
$$\sum x - x = 0$$

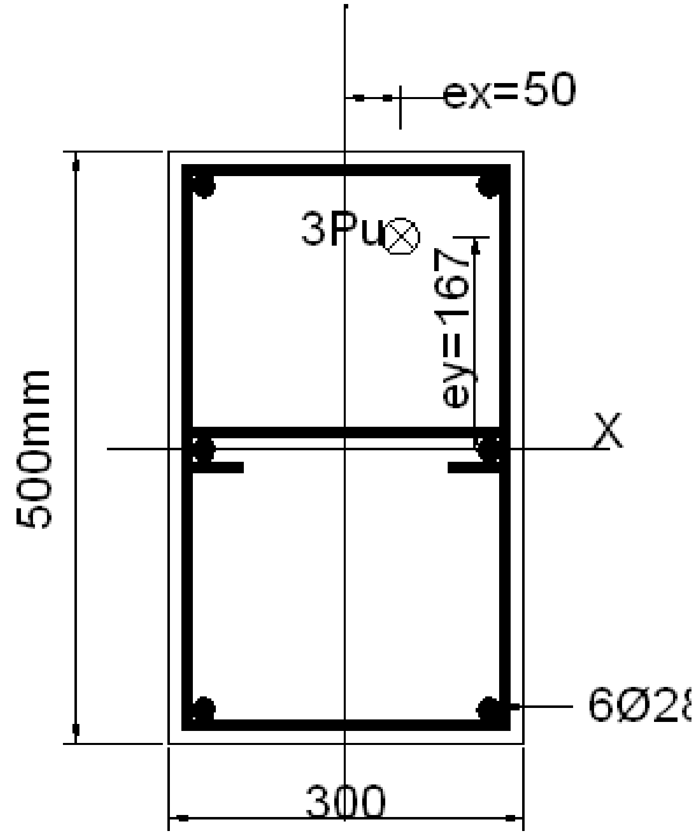
$$P_u * 150 + 2P_u * 175 = 3P_u * e_y$$

$$\rightarrow e_y = 167\ mm$$

$$\sum y - y = 0$$

$$2P_u * 75 = 3P_u * e_x \rightarrow e_x = 50\ mm$$



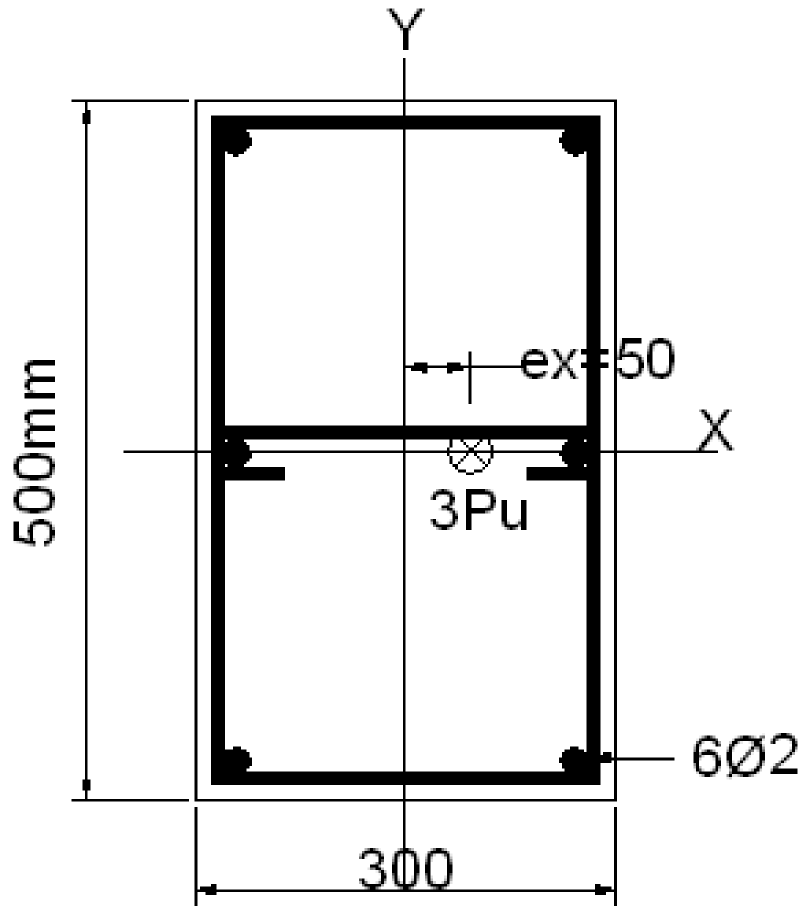


1. To find P_{no} ($e_x = e_y = 0$).

$$P_{no} = 0.85fc'(Ag - Ast) + Ast\ fy$$

$$P_{no} = 0.85 * 28000(0.5 * 0.3 - 3690 * 10^{-6}) + 3690 * 10^{-6} * 400000 = 4958\ kN$$

2. To find P_{nyo} $(bending\ about\ y - axis\ ,\ \ e_y = 0\ )$, h=300mm.



$$\gamma = \frac{h - 2d'}{h} = \frac{300 - 2 * 65}{300}$$

$$= 0.57\ say\ 0.6$$

$$\rho_g = \frac{As(6Ø28)}{Ag} = \frac{3690}{300 * 500}$$

$$= 0.025$$

$$\frac{e}{h} = \frac{50}{300} = 0.17$$

For γ=0.6, $\rho_g = 0.025, \frac{e}{h} = 0.25$ →graph A9→kn=0.65

$$Kn = \frac{Pu}{\emptyset fc'.Ag} \rightarrow 0.65 = \frac{P_{nyo}}{28000 * 0.3 * 0.5} \rightarrow$$

$$P_{nyo} = 2730\ kN$$

3. To find P_{nxo} $(bending\ about\ x-axis\ ,\quad e_x=0\quad)$, h=500mm.

$$\gamma=\frac{h-2d'}{h}=\frac{500-2*65}{500}=0.74$$

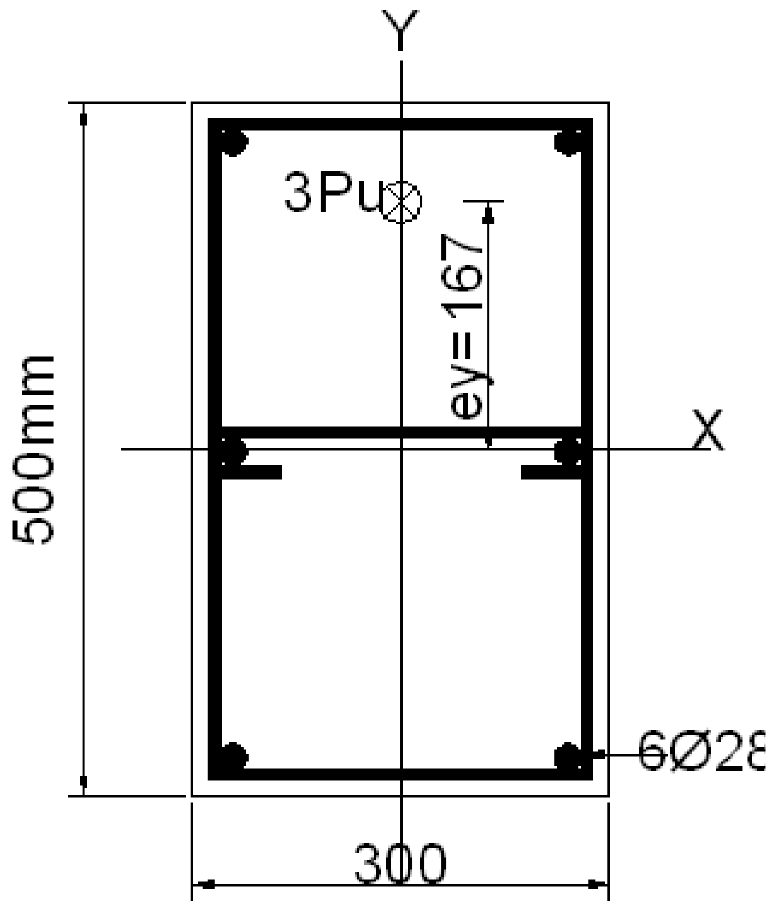
$$\rho_g=\frac{As(4\emptyset28)}{Ag}=\frac{2460}{300*500}$$

$$=0.016$$

$$\frac{e}{h}=\frac{167}{500}=0.334$$

For γ=0.7→graph A10→kn=0.51

For γ=0.8→graph A11→kn=0.52



For γ=0.74 →kn=0.515

$$Kn = \frac{Pu}{\emptyset fc'.Ag} \rightarrow 0.515 = \frac{P_{nyo}}{28000 * 0.3 * 0.5} \rightarrow$$

$$P_{nxo} = 2163\ kN$$

$$\frac{1}{P_n} = \frac{1}{P_{nxo}} + \frac{1}{P_{nyo}} - \frac{1}{P_{no}}$$

$$\frac{1}{P_n} = \frac{1}{2163} + \frac{1}{2730} - \frac{1}{4958} \rightarrow P_{n,max} = 1595\ kN \quad O.K$$

$$P_{n,max} = 1595\ kN = P_{n,applied} = \frac{3P_u}{\emptyset} \rightarrow P_u = 345.6\ kN$$

$$P_n = 1595\ kN > 0.1P_{no} = 0.1 * 4958 = 495.8\ kN \quad O.K$$