ALMUSTAQBAL UNIVERSITY

DEPARTMENT OF BUILDING & CONSTRUCTION ENGINEERING TECHNOLOGY

ANALYSIS AND DESIGN OF REINFORCED CONCRETE STRUCTURES II

DIRECT DESIGN METHOD FOR SLABS WITH INTERIOR BEAMS

SLAB SYSTEM WITH INTERIOR BEAMS:

- Aspect ratio $\left(\frac{l_2}{l_1}\right)$ where:

  l_1: *span centre to centre of supports in the direction of the frame*

  l_2: *span centre to centre of supports in the perpindicular direction of the frame*

- $\boldsymbol{\alpha_f \frac{l_2}{l_1}}$
- $\boldsymbol{\beta_t}$**:** torsional member (**8.10.5.2a**)

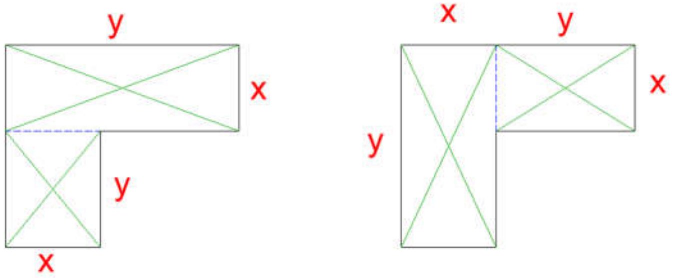
  $\beta_t = \frac{C}{2I_s}, I_S = \frac{l_2.t^3}{12}$ (where $\boldsymbol{l_2}$: *centre to centre of supports*)

  Where:

  $C = \sum\left(1 - 0.63\frac{x}{y}\right)\frac{x^3 y}{3}$ …..(**8.10.5.2*b***) **, *x*:** small dimension**, *y*:** large dimension

  For the edge beam (torsional member):

  - Check the two possibilities for the ***C*** value for the edge beam and take the largest.



  - $\boldsymbol{\alpha = \frac{I_B}{I_S}}, \quad \boldsymbol{I_S = \frac{l_2.t^3}{12}},$ l_2: *width of frame*
  - Aspect ratio= $\frac{l_2}{l_1}$, $\boldsymbol{l_2}$: *centre to centre of supports.*
  - $\boldsymbol{\beta_t = \frac{C}{2I_s}, I_S = \frac{l_2.t^3}{12}}$, $\boldsymbol{l_2}$: *centre to centre of supports.*

MOMENT ON BEAMS

$$M_{beam} = 0.85\left(\alpha_{f1}\frac{l_2}{l_1}\right) \times M_{c.s.} \leq 0.85\ M_{c.s.}$$
$$M_{slab} = M_{c.s.} - M_{beam}$$
$$M_{m.s.} = M^{\mp} - M_{c.s.}$$

DESIGN PROCEDURE

1. **Calculate** W_u, $\quad W_u = 1.2\,DL + 1.6LL$

2. $M_o = \frac{W_u l_2 {l_n}^2}{8}$

3. $M^{\mp} = coeff. \times M_o \begin{cases} \quad for\ interior\ span\ coeff. \begin{cases} 0.35\ M^{+} \\ 0.65 M^{-} \end{cases} \\ for\ exterior\ span \ldots \ldots .\,Table\ 8.10.4.2 \end{cases}$

4. $M_{c.s.} = coeff.\,M^{\mp} \qquad coeff. \begin{cases} 8.10.5.1 \\ 8.10.5.2 \\ 8.10.5.5 \end{cases}$

   - ○ $M_{beam} = \ 0.85\left(\alpha_{f1}\frac{l_2}{l_1}\right) \times M_{c.s.} \leq 0.85\ M_{c.s.}$
   - ○ $M_{slab} = M_{c.s.} - M_{beam}$

5. $M_{m.s.} = M^{\mp} - M_{c.s.}$
6. **Reinforcement design:**

M_u: *known* ➡ **compute** $A_s, \rho, and\ S$

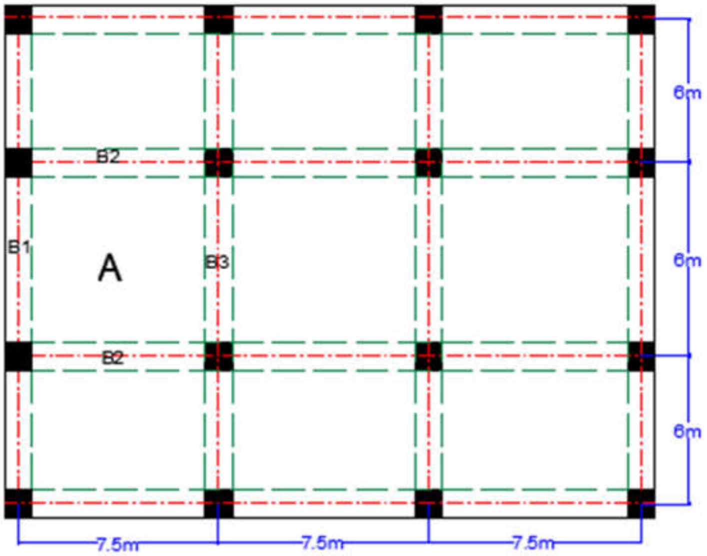
$\rho = \frac{1-\sqrt{1-2.622 M_u/f_{c'} b\, d^2}}{1.18 f_y/f_{c'}}$ ………… **Mu=× 10^6**, $f'_c \& f_y = MPa,\ \ b\&d = mm$

$A_s = \rho.b.d \quad \ldots \ldots \ldots A_s = \frac{mm^2}{m} \geq A_{s\,min} = \rho_{min}.b.t$

$S_{max} = min\,(\frac{1000 A_{s\,bar}}{A_s}\,, 2t, 450mm)$

$\rho_{min} = \begin{cases} 0.002\ (f_y = 300 - 400MPa) \\ 0.018 \qquad\quad (f_y = 420MPa) \\ 0.018\frac{420}{f_y} \qquad (f_y > 420MPa) \end{cases}$

Example 6: A two way reinforced concrete floor system is composed of panels measuring 7.5x6m as shown below. Using concrete with $f_c' = 27.6MPa$ and steel having $f_y = 414MPa$, design panel (A) to carry a service live load of 6.9 kN/m^2 in addition to the slab's own weight. Use slab thickness of 170mm, $\emptyset_{bar} = 12mm$ for the slab's reinforcement, columns are $360 \times 360mm$ and the beams measure as $500 \times 360mm$ ($f_{exterior\ beam} = 1.5$ & $f_{interior\ beam} = 2.0$).



Solution:

1. Check the slab thickness.
   - **Beam B1:**

$$I_{Beam} = 1.5 \times \frac{bh^3}{12} = 1.5 \times \frac{0.36 \times 0.5^3}{12} = 0.00563m^4.$$

Slab width=$\frac{7.5m}{2} + \frac{0.36}{2} = 3.93m$

$$I_{slab} = \frac{l_2 t^3}{12} = \frac{3.93 \times 0.17^3}{12} = 0.001609m^4$$

$$\therefore \alpha_{B1} = \frac{I_{Beam}}{I_{slab}} = \frac{0.00563}{0.001609} = \mathbf{3.5}$$

   - **Beam B2:**

$$I_{Beam} = 2 \times \frac{bh^3}{12} = 2 \times \frac{0.36 \times 0.5^3}{12} = 0.0075m^4.$$

Slab width=6m.

$$I_{slab} = \frac{l_2 t^3}{12} = \frac{6 \times 0.17^3}{12} = 0.00246m^4$$

$$\therefore \alpha_{B2} = \frac{I_{Beam}}{I_{slab}} = \frac{0.0075}{0.00246} = \mathbf{3.05}$$

- **Beam B3:**

$$I_{Beam} = 2 \times \frac{bh^3}{12} = 2 \times \frac{0.36 \times 0.5^3}{12} = 0.0075m^4.$$

Slab width=7.5m.

$$I_{slab} = \frac{l_2 t^3}{12} = \frac{7.5 \times 0.17^3}{12} = 0.00307m^4$$

$$\therefore \alpha_{B3} = \frac{I_{Beam}}{I_{slab}} = \frac{0.0075}{0.00307} = \mathbf{2.44}$$

$$\boldsymbol{\alpha_{fm}} = \frac{\mathbf{3.5 + 3.05 + 3.05 + 2.44}}{\mathbf{4}} = \mathbf{3.01}$$

$$\beta = \frac{l_n}{S_n} = \frac{7.5 - 0.36}{6 - 0.36} = 1.27$$

Using table 8.3.1.2 for a slab with interior beams:

$$h = \frac{l_n(0.8 + \frac{fy}{1400})}{36 + 9\beta} = \frac{7.14(0.8 + \frac{414}{1400})}{36 + 9 \times 1.27} \times 10^3 = 165mm\ > 90mm$$

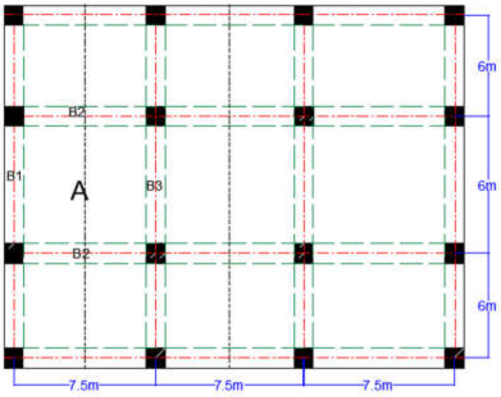
$$\therefore slab\ thickness\ (t) = 170mm.$$

2. Determine W_u.

$$DL = self\ weight\ of\ the\ slab = 0.17 \times 24 = 4.08kN/m^2$$

$$\therefore W_u = 1.2DL + 1.6LL = 1.2 \times 4.08 + 1.6 \times 6.9 = 16kN/m^2$$

3. Determine the moments for the transverse frames.



a. Interior Transverse Frame:-

$l_2 = 7.5m$

$l_n = 6 - 0.36 = 5.64m \qquad > 0.65l_1 = 0.65 \times 6 = 3.9m$

$$M_o = \frac{W_u l_2 {l_n}^2}{8} = \frac{16 \times 7.5 \times 5.46^2}{8} = 477kN.m$$

For interior span:

$-M = 0.65M_o = 0.65 \times 477 = 310kN.m$

$+M = 0.35M_o = 0.35 \times 477 = 167kN.m$

$$\frac{l_2}{l_1} = \frac{7.5}{6} = 1.25$$

$\alpha_{B3}\frac{l_2}{l_1} = 2.44 \times 1.25 = 3.1 \quad > 1 \ldots\ldots\ldots \therefore use\ \alpha_{B3}\frac{l_2}{l_1} = 1$

From ACI 8.10.5.1.

$$-M\ C.S.coeff. = 75 + 30\left(\alpha_{B3}\frac{l_2}{l_1}\right)\left(1 - \frac{l_2}{l_1}\right) = 75 + 30(1)(1 - 1.25) = 68\%$$

$-M\ C.S = 0.68 \times 310kN.m = 211kN.m$

$$M_{beam} = 0.85\left(\alpha_{f1}\frac{l_2}{l_1}\right) \times M_{c.s.} \leq 0.85\ M_{c.s.} = 0.85(3.1) \times 211 = 555.9kN.m\ > 0.85 \times 211 = 179.4kN.m$$

$\therefore M_{beam} = \mathbf{179.4kN.m}$

$M_{slab}\ C.S. = MC.S. - M_{beam} = 211 - 179.4 = \mathbf{32kN.m}$

$M\ M.S. = -M - MC.S = 310 - 211 = \mathbf{99kN.m}$

$$+M\ C.S.coeff. = 60 + 30\left(\alpha_{B3}\frac{l_2}{l_1}\right)\left(1.5 - \frac{l_2}{l_1}\right) = 60 + 30(1)(1.5 - 1.25) = 68\%$$

$+M\ C.S = 0.68 \times 167 = 114kN.m$

$M_{Beam} = 0.85(+M\ C.S) = 0.85 \times 114 = \mathbf{97kN.m}$

$M_{slab}CS = +MCS - M_{Beam} = 114 - 97 = \mathbf{17kN.m}$

$M\ M.S. = +M - MC.S = 167 - 114 = \mathbf{53kN.m}$

b. Exterior Transverse Frame:-

$$l_2 = \frac{7.5}{2} + \frac{0.36}{2} = 3.93m$$

$l_n = 6 - 0.36 = 5.64m \qquad > 0.65l_1 = 0.65 \times 6 = 3.9m$

$$M_o = \frac{W_u l_2 {l_n}^2}{8} = \frac{16 \times 3.93 \times 5.46^2}{8} = 250kN.m$$

For interior span:

$-M = 0.65M_o = 0.65 \times 250 = 163kN.m$

$+M = 0.35M_o = 0.35 \times 250 = 88kN.m$

$\frac{l_2}{l_1} = \frac{7.5}{6} = 1.25$

$\alpha_{B1}\frac{l_2}{l_1} = 3.5 \times 1.25 = 4.38 \quad > 1 \ldots\ldots\ldots \therefore use\ \alpha_{B1}\frac{l_2}{l_1} = 1$

From ACI 8.10.5.1.

$-M\ C.S.coeff. = 75 + 30\left(\alpha_{B1}\frac{l_2}{l_1}\right)\left(1 - \frac{l_2}{l_1}\right) = 75 + 30(1)(1 - 1.25) = 68\%$

$-M\ C.S = 0.68 \times 163kN.m = 111kN.m$

$M_{beam} = 0.85\ M_{c.s.} = 0.85 \times 111 = \mathbf{94kN.m}$

$M_{slab}\ C.S. = MC.S. - M_{beam} = 111 - 94 = \mathbf{17kN.m}$

$M\ M.S. = -M - MC.S = 163 - 111 = \mathbf{52kN.m}$

$+M\ C.S.coeff. = 60 + 30\left(\alpha_{B1}\frac{l_2}{l_1}\right)\left(1.5 - \frac{l_2}{l_1}\right) = 60 + 30(1)(1.5 - 1.25) = 68\%$

$+M\ C.S = 0.68 \times 88 = 60kN.m$

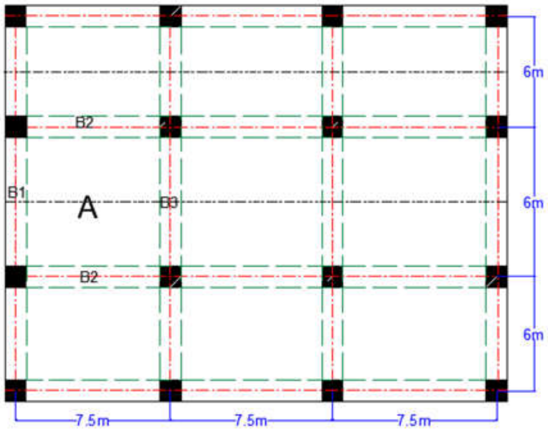
$M_{Beam} = 0.85(+M\ C.S) = 0.85 \times 60 = \mathbf{51kN.m}$

$M_{slab}CS = +MCS - M_{Beam} = 60 - 51 = \mathbf{9kN.m}$

$M\ M.S. = +M - MC.S = 88 - 60 = \mathbf{28kN.m}$

	Beam Moment	**Slab CS moment**	**MS moment**
Interior Transverse frame			
+	97	17	53
-	179	32	99
Exterior Transverse frame			
+	51	9	28
-	94	17	52

4. Determine the moments for the interior longitudinal frame.



$$l_2 = 6m$$

$$l_n = 7.5 - 0.36 = 7.14m \quad > 0.65 l_1 = 0.65 \times 7.5 = 4.9m$$

$$M_o = \frac{W_u l_2 {l_n}^2}{8} = \frac{16 \times 6 \times 7.14^2}{8} = 612kN.m$$

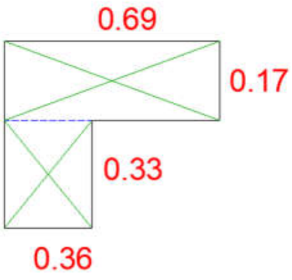
Longitudinal distribution of moments: (ACI 8.10.4.2)

$$exterior\ (-M) = 0.16M_o = 0.16 \times 612 = 98kN.m$$

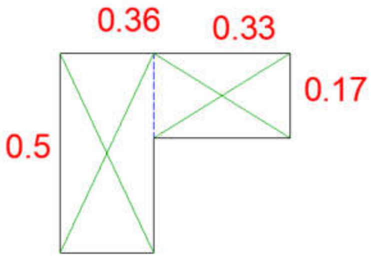
$$(+M) = 0.57M_o = 0.57 \times 612 = 349kN.m$$

$$interior\ (-M) = 0.7M_o = 0.7 \times 612 = 428kN.m$$

$$C = \sum \left(1 - 0.63\frac{x}{y}\right)\frac{x^3 y}{3}$$



Case 1



Case 2

Case 1:

$$C = \left(1 - 0.63\frac{0.33}{0.36}\right)\frac{0.33^3 \times 0.36}{3} + \left(1 - 0.63\frac{0.17}{0.69}\right)\frac{0.17^3 \times 0.69}{3} = 0.00277m^4$$

Case 2:

$$C=\left(1-0.63\frac{0.36}{0.50}\right)\frac{0.36^3\times 0.50}{3}+\left(1-0.63\frac{0.17}{0.33}\right)\frac{0.17^3\times 0.33}{3}=0.00461m^4$$

$\therefore C=0.00461m^4$

$\frac{l_2}{l_1}=\frac{6}{7.5}=0.8$, $\alpha_{B2}\frac{l_2}{l_1}=3.05\times 0.8=2.44$

$$I_{slab}=\frac{l_2t^3}{12}=\frac{6\times 0.17^3}{12}=0.0025m^4$$

$$\therefore \beta_t=\frac{C}{2I_{slab}}=\frac{0.00461}{2\times 0.0025}=0.922$$

$$exter-M\,C.S.coeff.=100-10\beta_t+12\beta_t\left(\alpha_{B2}\frac{l_2}{l_1}\right)\left(1-\frac{l_2}{l_1}\right)$$

$$=100-10\times 0.922+12\times 0.922(1)(1-0.8)=93\%$$

$-M\,C.S=0.93\times 98kN.m=91.14kN.m$

$M_{beam}=\ 0.85\,M_{c.s.}=0.85\times 91.14=\boldsymbol{78kN.m}$

$M_{slab}\,C.S.=MC.S.-M_{beam}=91.14-78=\boldsymbol{14kN.m}$

$exter\ (-M\,M.S.)=\ -M-MC.S=98-91.14=\boldsymbol{7kN.m}$

$$+M\,C.S.coeff.=60+30\left(\alpha_{B2}\frac{l_2}{l_1}\right)\left(1.5-\frac{l_2}{l_1}\right)=60+30(1)(1.5-0.8)=81\%$$

$+M\,C.S=0.81\times 349=282.7kN.m$

$M_{Beam}=0.85(+M\,C.S)=0.85\times 282.7=\boldsymbol{240.3kN.m}$

$M_{slab}CS=+MCS-M_{Beam}=282.7-240.3=\boldsymbol{42.4kN.m}$

$M\,M.S.=\ +M-MC.S=349-282.7=\boldsymbol{66.3kN.m}$

$$interior-M\,C.S.coeff.=75+30\left(\alpha_{B2}\frac{l_2}{l_1}\right)\left(1-\frac{l_2}{l_1}\right)=75+30(1)(1-0.8)=81\%$$

$-M\,C.S=0.81\times 428kN.m=346.7kN.m$

$M_{beam}=\ 0.85\,M_{c.s.}=0.85\times 346.7=\boldsymbol{294.7kN.m}$

$M_{slab}\,C.S.=MC.S.-M_{beam}=346.7-294.7=\boldsymbol{52kN.m}$

$Inter\ (-M\,M.S.)=\ -M-MC.S=428-346.7=\boldsymbol{81.3kN.m}$

	Beam Moment	**Slab CS moment**	**MS moment**
Interior Transverse frame			
Exterior -M	78	14	7
+M	240.3	42.4	66.3
Interior -M	294.7	52	81.3

$A_{s\,min} = \rho_{min} \times b \times t = 0.002 \times 1000 \times 170 = 340mm^2$

$d_{long} = 170 - 20 - 1.5(12) = 132mm$

$d_{short} = 170 - 20 - \frac{12}{2} = 144mm$

$\rho_{\min long} = \frac{A_{s\,min}}{b \times d_{long}} = \frac{340}{1000 \times 132} = 0.0025$

$\rho_{\min short} = \frac{A_{s\,min}}{b \times d_{short}} = \frac{340}{1000 \times 144} = 0.0023$

$\rho_{max} = 0.85\beta_1 \frac{fc'}{fy} \frac{3}{7} = 0.85 \times 0.85 \times \frac{27.6}{414} \frac{3}{7} = 0.0206$

$S_{max} = \min(2t, 450mm) = \min(2 \times 170 = 340mm, 450mm)$

$\therefore S_{max} = 340mm$

Check (d) for both directions:(use maximum Mu in each direction)

$M_u = \emptyset \rho_{max} b d^2 \left(1 - 0.59\rho \frac{fy}{fc'}\right) = 0.9 \times 0.0206 \times 414 \times 10^3 \times d^2 \left(1 - 0.59 \times 0.0206 \times \frac{414}{27.6}\right) =$

$\therefore d = \sqrt{\frac{M_u}{6276}}$

Long direction: $d = \sqrt{\frac{M_u}{6276}} = \sqrt{\frac{27 \times 10^6}{6276}} = 65.6mm \ < d_{long} = 132mm \quad O.K.$

short direction: $d = \sqrt{\frac{M_u}{6276}} = \sqrt{\frac{22 \times 10^6}{6276}} = 59.2mm \ < d_{short} = 144mm \quad O.K.$

Check shear requirements: *(always in the short direction)*

$V_u @\ d\ from\ the\ face\ of\ the\ longer\ beam$

$V_u = Wu\left(\frac{S_{span}}{2} - \frac{b_w}{2} - d_{short}\right) = 16\left(\frac{6}{2} - \frac{0.36}{2} - 0.144\right) = 42.8kN$

$\emptyset V_c = \emptyset 0.17\sqrt{fc'}\, b d_{short} = 0.75 \times 0.17 \times \sqrt{27.6} \times 1000 \times 144 \times 10^{-3} = 96.45kN$

$\therefore V_u < \emptyset V_c \qquad O.K$

		location	Mn	b (mm)	d (mm)	$Mu = \frac{Mn \times 10^3}{b}$	ρ	As (mm^2)	No. of bars
Long direction 7.5m	2 half CS	Ext. -M	14	2640	132	5.3	0.0025	871	8
		+M	42	2640	132	16	0.0025	871	8
		Int -M	52	2640	132	20	0.0031	1080	10
	2half MS	Ext. -M	7	3000	132	2.3	0.0025	990	9
		+M	66	3000	132	22	0.0035	1386	13
		Int -M	81	3000	132	27	0.0043	1702	15
Short direction 6m	Ext half CS	-M	17	1320	144	12.8	0.0023	437	4
		+M	9	1320	144	6.8	0.0023	437	4
	Ms	-M	99	4500	144	22	0.0023	1880	17
		+M	53	4500	144	11.7	0.0023	1361	14
	Int half CS	-M	16	1320	144	12.1	0.0023	437	4
		+M	8.5	1320	144	6.4	0.0023	437	4