وزارة التعليم العالي والبحث العلمي
جامعة المستقبل

LEC : TWO

Course Name : Fundamentals of Electricity
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Stage : First
Academic Year : 2023
Lecture Title : Series DC Circuits

SERIES DC CIRCUITS

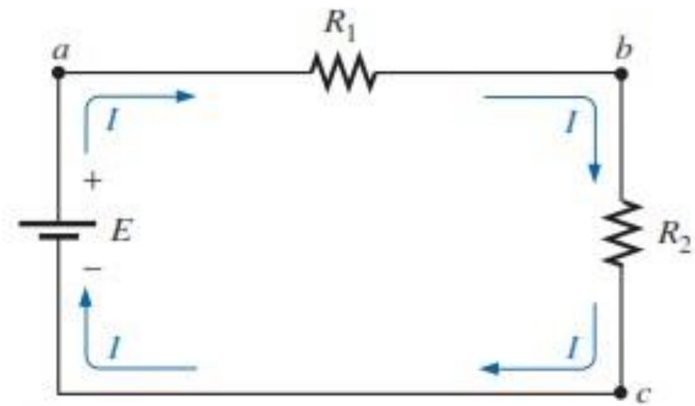
A **circuit** consists of any number of elements joined at terminal points, providing at least one closed path through which charge can flow. The circuit of Fig. 2.1(a) has three elements joined at three terminal points (*a, b, and c) to provide a closed path for the current I.*

Two elements are in series if

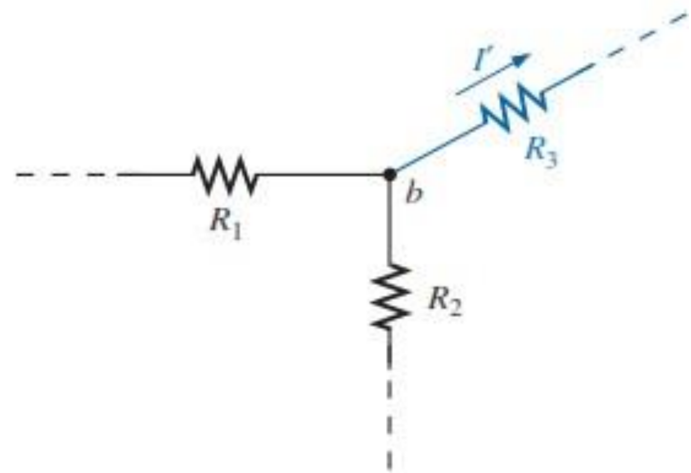
1.They have only one terminal in common (i.e., one lead of one is connected to only one lead of the other).

2.The common point between the two elements is not connected to another current-carrying element.

If the circuit of Fig. 2.1(a) is modified such that a current-carrying resistor R_3 is introduced, as shown in Fig. 2.1(b), the resistors R_1 *and* R_2 are no longer in series due to a violation of number 2 of the above definition of series elements.



(a) Series circuit



(b) R_1 and R_2 are not in series

Fig.2.1

(a) Series circuit; (b) situation in which R1 and R2 are not in series.

The total resistance of a series circuit is the sum of the resistance levels.

In general, to find the total resistance of N *resistors in series, the following* equation is applied:

$$\boxed{R_T = R_1 + R_2 + R_3 + \cdots + R_N} \qquad \text{(ohms, } \Omega)$$

Once R_T is known, the current drawn from the source can be determined using Ohm's law, as follows:

$$\boxed{I_s = \frac{E}{R_T}} \qquad \text{(amperes, A)}$$

The fact that the current is the same through each element of Fig. 2.1(a) permits a direct calculation of the voltage across each resistor using Ohm's law; that is,

$$\boxed{V_1 = IR_1, V_2 = IR_2, V_3 = IR_3, \ldots, V_N = IR_N} \qquad \text{(volts, V)}$$

The total power delivered to a resistive circuit is equal to the total power dissipated by the resistive elements.

$$\boxed{P_1 = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1}} \qquad \text{(watts, W)}$$

$$\boxed{P_{del} = P_1 + P_2 + P_3 + \cdots + P_N}$$

The power delivered by the source is

$$\boxed{P_{del} = EI} \qquad \text{(watts, W)}$$

EXAMPLE 2.1

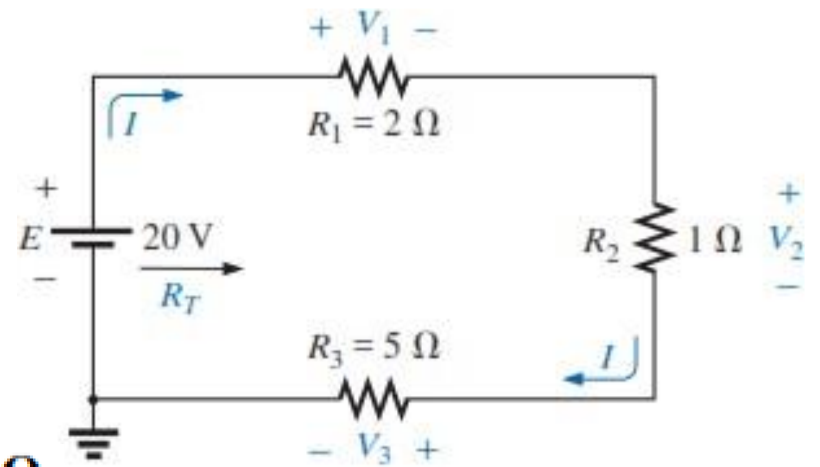
a. Find the total resistance for the series circuit shown.

b. Calculate the source current I.

c. Determine the voltages V_1, V_2, and V_3.

d. Calculate the power dissipated by R_1, R_2 and R_3.

e. Determine the power delivered by the source, and compare it to the sum of the power levels of part (d).



a. $R_T = R_1 + R_2 + R_3 = 2\ \Omega + 1\ \Omega + 5\ \Omega = \mathbf{8\ \Omega}$

b. $I_s = \dfrac{E}{R_T} = \dfrac{20\ \text{V}}{8\ \Omega} = \mathbf{2.5\ A}$

c. $V_1 = IR_1 = (2.5\ \text{A})(2\ \Omega) = \mathbf{5\ V}$

$V_2 = IR_2 = (2.5\ \text{A})(1\ \Omega) = \mathbf{2.5\ V}$

$V_3 = IR_3 = (2.5\ \text{A})(5\ \Omega) = \mathbf{12.5\ V}$

d. $P_1 = V_1I_1 = (5\ \text{V})(2.5\ \text{A}) = \mathbf{12.5\ W}$

$P_2 = I_2^2R_2 = (2.5\ \text{A})^2(1\ \Omega) = \mathbf{6.25\ W}$

$P_3 = V_3^2/R_3 = (12.5\ \text{V})^2/5\ \Omega = \mathbf{31.25\ W}$

e. $P_{\text{del}} = EI = (20\ \text{V})(2.5\ \text{A}) = \mathbf{50\ W}$

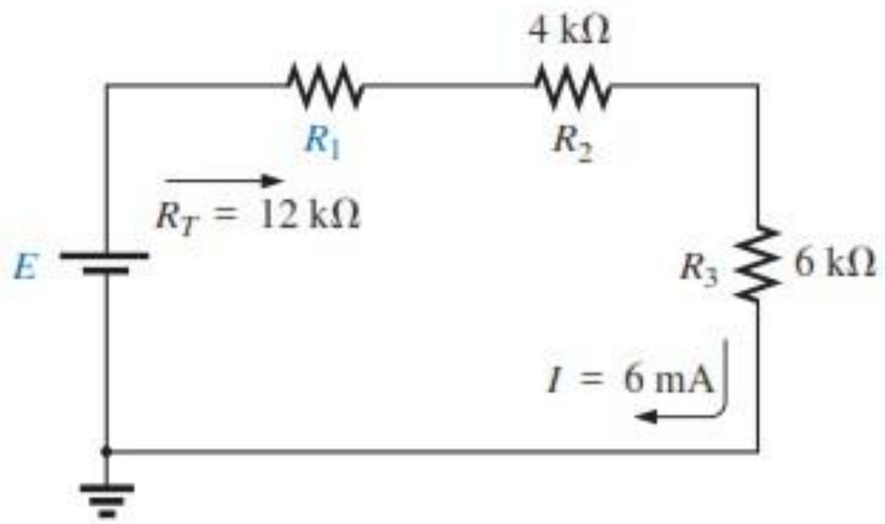
$P_{\text{del}} = P_1 + P_2 + P_3$

$50\ \text{W} = 12.5\ \text{W} + 6.25\ \text{W} + 31.25\ \text{W}$

$\underline{50\ \text{W} = 50\ \text{W} \quad \text{(checks)}}$

EXAMPLE 2.2 Given R_T and I, *calculate* R_1 and E *for the circuit shown.*

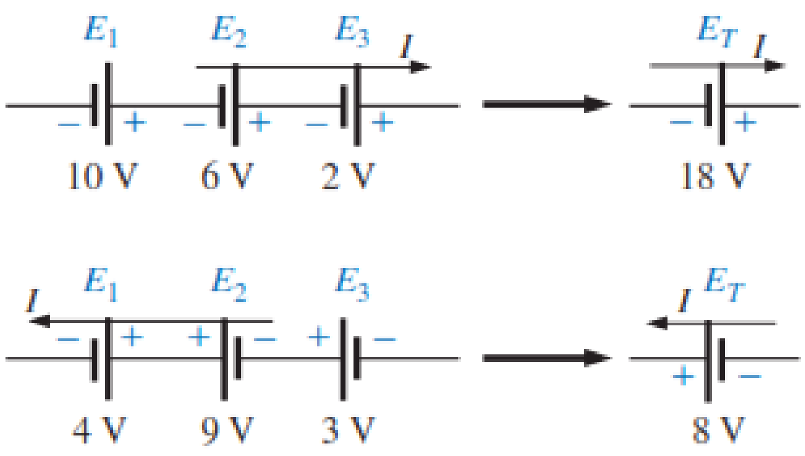
$$R_T = R_1 + R_2 + R_3$$
$$12\ \text{k}\Omega = R_1 + 4\ \text{k}\Omega + 6\ \text{k}\Omega$$
$$R_1 = 12\ \text{k}\Omega - 10\ \text{k}\Omega = \mathbf{2\ k\Omega}$$
$$E = IR_T = (6 \times 10^{-3}\ \text{A})(12 \times 10^{3}\ \Omega) = \mathbf{72\ V}$$



VOLTAGE SOURCES IN SERIES

$$E_T = E_1 + E_2 + E_3 = 10\ \text{V} + 6\ \text{V} + 2\ \text{V} = 18\ \text{V}$$

$$E_T = E_2 + E_3 - E_1 = 9\ \text{V} + 3\ \text{V} - 4\ \text{V} = 8\ \text{V}$$

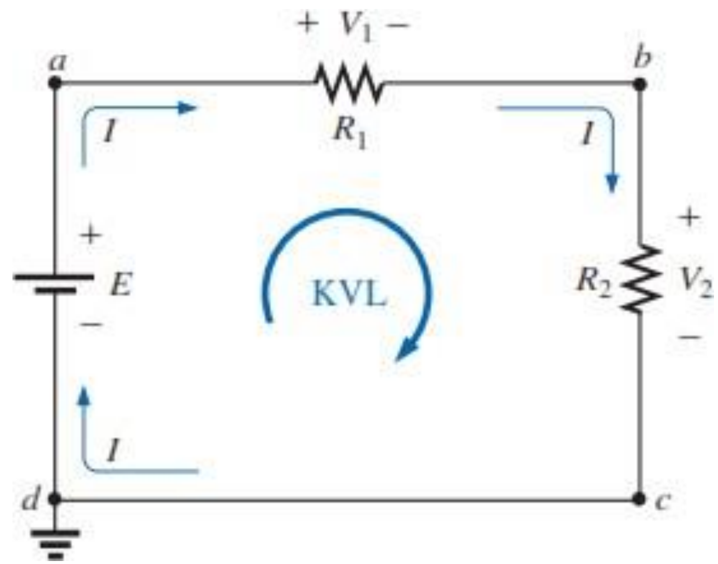


KIRCHHOFF’S VOLTAGE LAW

Kirchhoff’s voltage law (KVL) states that the algebraic sum of the potential rises and drops around a closed loop (or path) is zero.

A **closed loop** is any continuous path that leaves a point in one direction and returns to that same point from another direction without leaving the circuit.

$$\boxed{\Sigma_{\circlearrowright} V = 0}$$ (Kirchhoff’s voltage law in symbolic form)



Kirchhoff’s voltage law can also be stated in the following form:

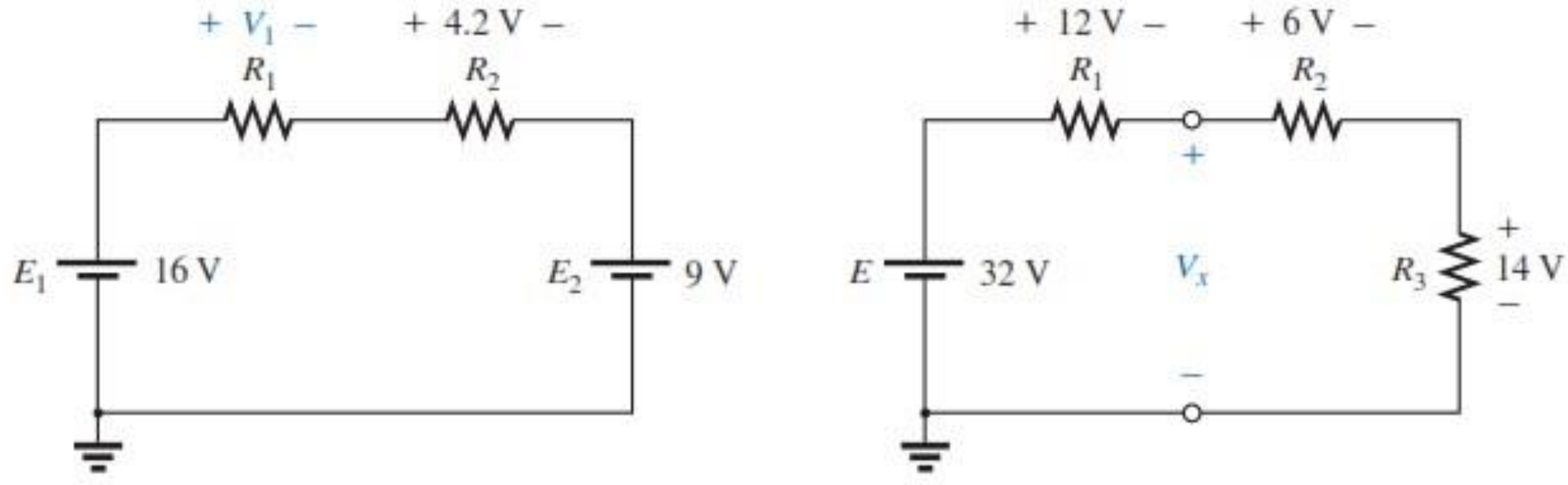
$$\boxed{\Sigma_{\circlearrowright} V_{\text{rises}} = \Sigma_{\circlearrowright} V_{\text{drops}}}$$

$$\Sigma_{\circlearrowright} V = 0$$

$$-E + V_2 + V_1 = 0$$

$$E = V_1 + V_2$$

EXAMPLE 2.3 Determine the unknown voltages for the networks shown.



(a) (b)

$$+E_1 - V_1 - V_2 - E_2 = 0$$

$$V_1 = E_1 - V_2 - E_2 = 16\text{ V} - 4.2\text{ V} - 9\text{ V}$$
$$= \mathbf{2.8\ V}$$

$$+E - V_1 - V_x = 0$$

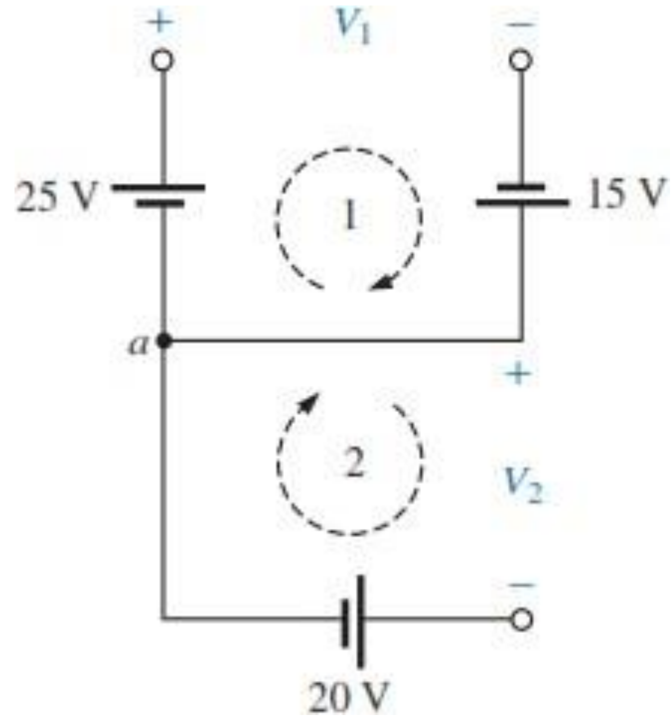
$$V_x = E - V_1 = 32\text{ V} - 12\text{ V}$$
$$= \mathbf{20\ V}$$

Using the clockwise direction for the other loop involving R_2 *and* R_3 will result in

$$+V_x - V_2 - V_3 = 0$$

$$V_x = V_2 + V_3 = 6\text{ V} + 14\text{ V}$$
$$= \mathbf{20\ V}$$

EXAMPLE 2.4 Find V_1 and V_2 for the network shown.



For path 1, starting at point *a in a clockwise direction:*

$$+25\text{ V} - V_1 + 15\text{ V} = 0$$

$$\text{and} \quad V_1 = \mathbf{40\ V}$$

For path 2, starting at point *a in a clockwise direction:*

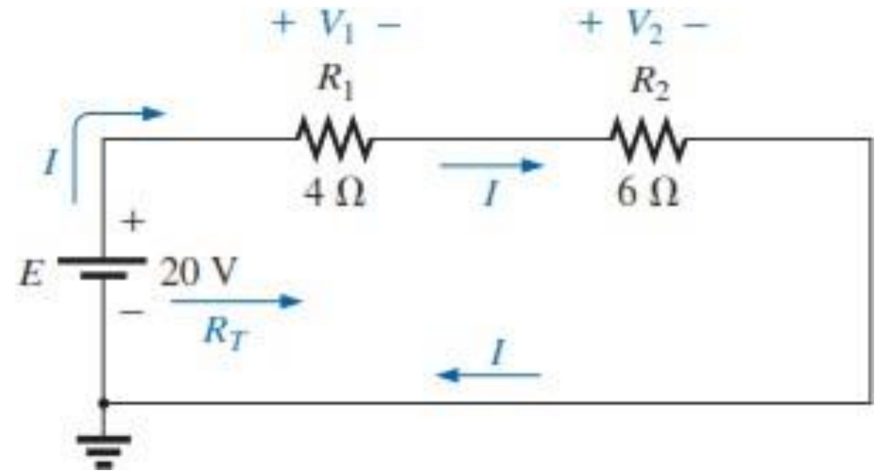
$$-V_2 - 20\text{ V} = 0$$

$$\text{and} \quad V_2 = \mathbf{-20\ V}$$

The minus sign simply indicates that the actual polarities of the potential difference are opposite the assumed polarity indicated in figure.

EXAMPLE 2.5 For the circuit shown.

a. Find R_T.
b. Find I.
c. Find V_T and V_1.
d. Find the power to the 4-Ω and 6-Ω resistors.
e. Find the power delivered by the battery, and compare it to that dissipated by the 4- Ω and 6- Ω resistors combined.
f. Verify Kirchhoff's voltage law (clockwise direction).



a. $R_T = R_1 + R_2 = 4\ \Omega + 6\ \Omega = \mathbf{10\ \Omega}$

b. $I = \dfrac{E}{R_T} = \dfrac{20\text{ V}}{10\ \Omega} = \mathbf{2\ A}$

c. $V_1 = IR_1 = (2\text{ A})(4\ \Omega) = \mathbf{8\ V}$
$V_2 = IR_2 = (2\text{ A})(6\ \Omega) = \mathbf{12\ V}$

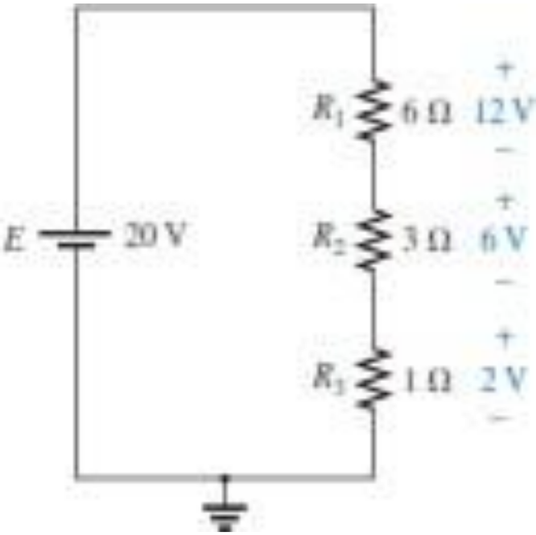
d. $P_{4\Omega} = \dfrac{V_1^2}{R_1} = \dfrac{(8\text{ V})^2}{4} = \dfrac{64}{4} = \mathbf{16\ W}$
$P_{6\Omega} = I^2R_2 = (2\text{ A})^2(6\ \Omega) = (4)(6) = \mathbf{24\ W}$

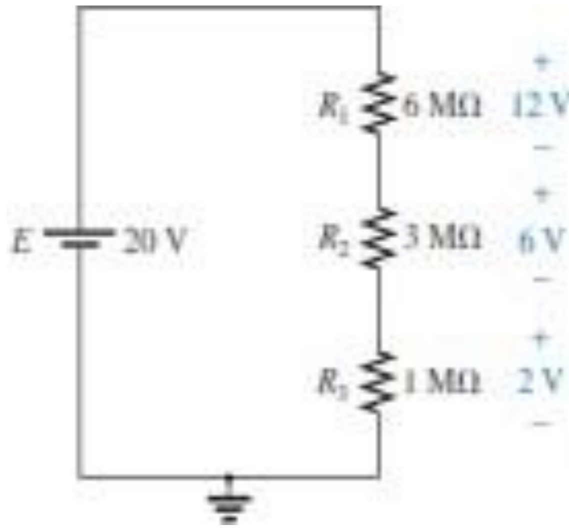
e. $P_E = EI = (20\text{ V})(2\text{ A}) = \mathbf{40\ W}$
$P_E = P_{4\Omega} + P_{6\Omega}$
$40\text{ W} = 16\text{ W} + 24\text{ W}$
$40\text{ W} = 40\text{ W}$ (checks)

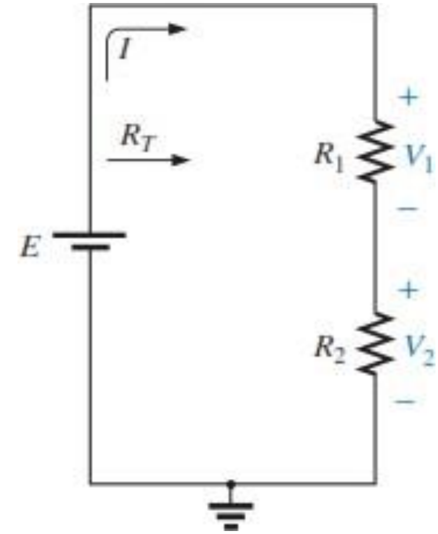
f. $\Sigma_{\circlearrowright} V = +E - V_1 - V_2 = 0$
$E = V_1 + V_2$
$20\text{ V} = 8\text{ V} + 12\text{ V}$
$20\text{ V} = 20\text{ V}$ (checks)

VOLTAGE DIVIDER RULE

In a series circuit, *the voltage across the resistive elements will divide as the magnitude of the resistance levels.*







$$R_T = R_1 + R_2$$

and $$I = \frac{E}{R_T}$$

$$\boxed{V_x = \frac{R_x E}{R_T}}$$

(voltage divider rule)

Applying Ohm's law:

$$V_1 = IR_1 = \left(\frac{E}{R_T}\right)R_1 = \frac{R_1 E}{R_T}$$

with $$V_2 = IR_2 = \left(\frac{E}{R_T}\right)R_2 = \frac{R_2 E}{R_T}$$

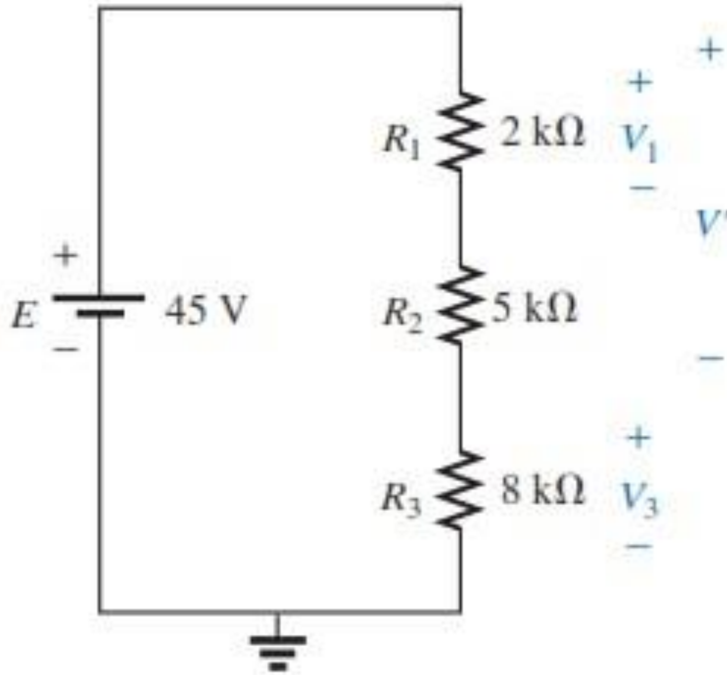
EXAMPLE 2.6 Using the voltage divider rule, determine the voltages V_1 and V_3 for the series circuit shown.

$$V_1 = \frac{R_1E}{R_T} = \frac{(2\ \text{k}\Omega)(45\ \text{V})}{2\ \text{k}\Omega + 5\ \text{k}\Omega + 8\ \text{k}\Omega} = \frac{(2\ \text{k}\Omega)(45\ \text{V})}{15\ \text{k}\Omega}$$

$$= \frac{(2 \times 10^3\ \Omega)(45\ \text{V})}{15 \times 10^3\ \Omega} = \frac{90\ \text{V}}{15} = \mathbf{6\ V}$$

$$V_3 = \frac{R_3E}{R_T} = \frac{(8\ \text{k}\Omega)(45\ \text{V})}{15\ \text{k}\Omega} = \frac{(8 \times 10^3\ \Omega)(45\ \text{V})}{15 \times 10^3\ \Omega}$$

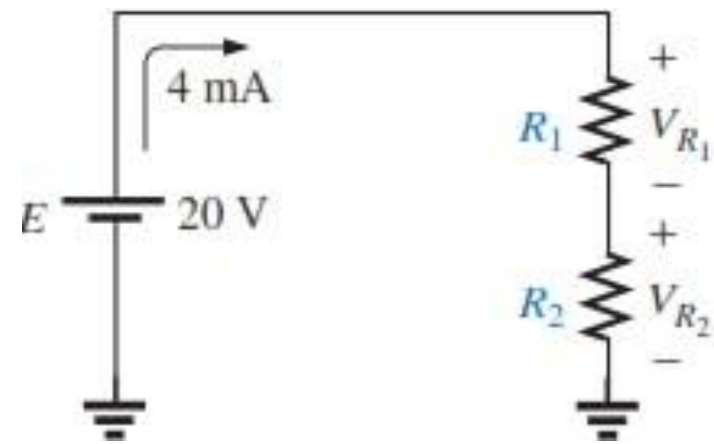
$$= \frac{360\ \text{V}}{15} = \mathbf{24\ V}$$



The rule can be extended to the voltage across two or more series elements if the resistance is expanded to include the total resistance of the series elements that the voltage is to be found across (R'); *that is,*

$$\boxed{V' = \frac{R'E}{R_T}} \qquad \text{(volts)}$$

EXAMPLE 2.7 Design the voltage divider shown such that $V_{R1} = 4V_{R2}$.



The total resistance is defined by

$$R_T = \frac{E}{I} = \frac{20\ \text{V}}{4\ \text{mA}} = 5\ \text{k}\Omega$$

Since $V_{R_1} = 4V_{R_2}$,

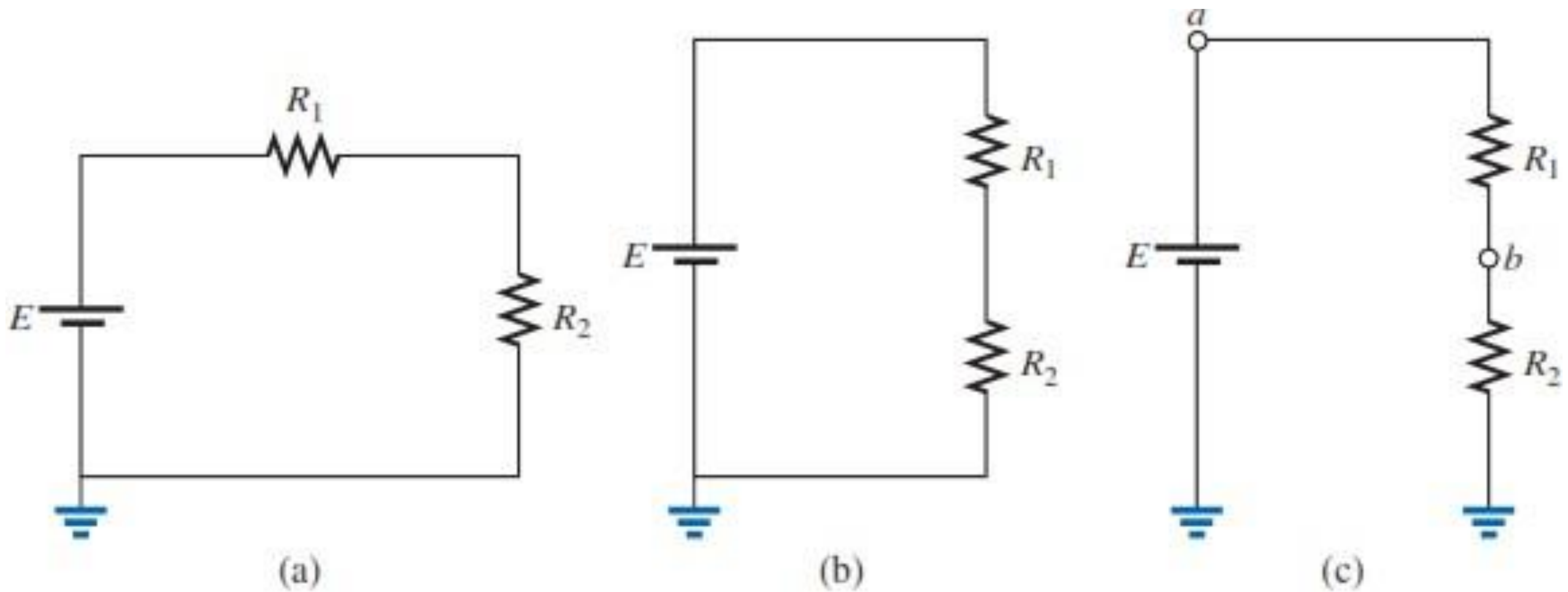
$$R_1 = 4R_2$$

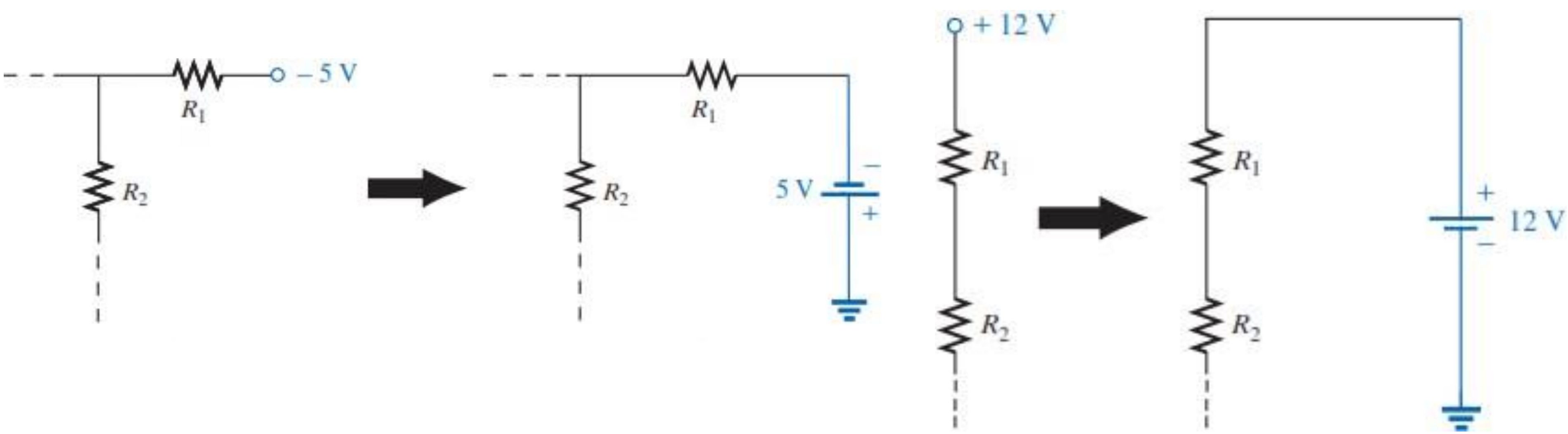
Thus $R_T = R_1 + R_2 = 4R_2 + R_2 = 5R_2$

$$5R_2 = 5\ \text{k}\Omega$$
$$R_2 = \mathbf{1\ k\Omega}$$

$$R_1 = 4R_2 = \mathbf{4\ k\Omega}$$

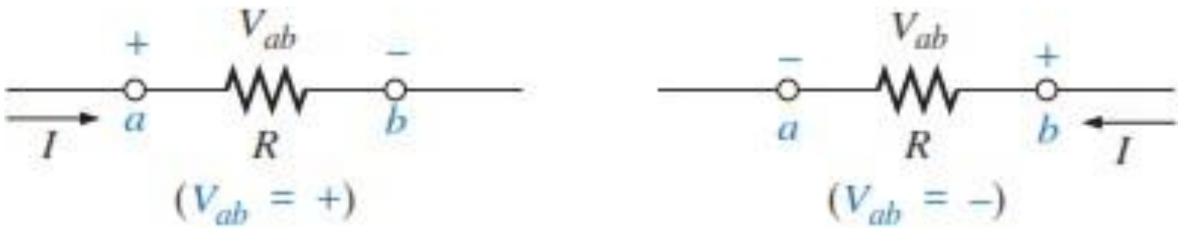
Voltage Sources and Ground





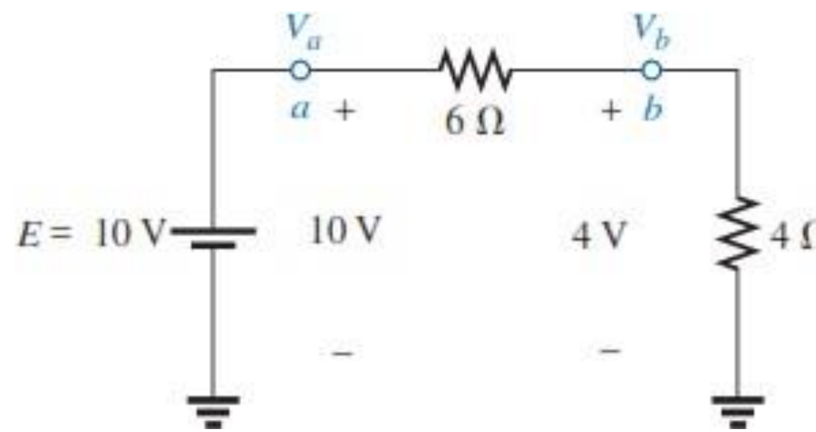
Double-Subscript Notation

The two points that define the voltage across the resistor *R are denoted by a and b. Since a* is the first subscript for *Vab*, point *a must have a higher potential than* point *b if Vab* is to have a positive value. If, in fact, point *b is at a higher* potential than point *a, Vab* will have a negative value, as indicated in Figure.



Single-Subscript Notation

If point *b of the notation Vab* is specified as ground potential (zero volts), then a single-subscript notation can be employed that provides the voltage at a point with respect to ground.

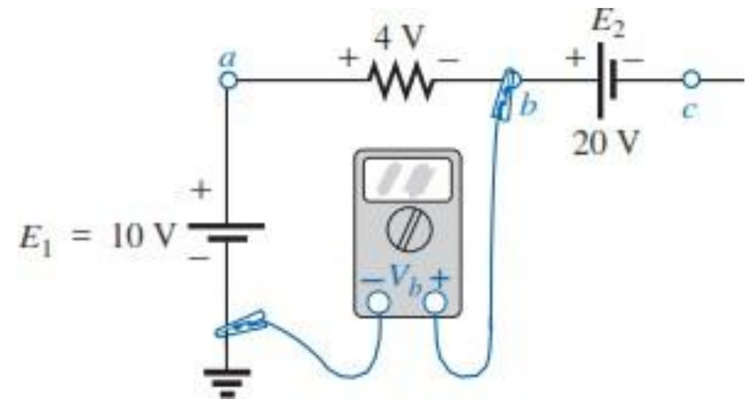


$$\boxed{V_{ab} = V_a - V_b}$$

$$V_{ab} = V_a - V_b = 10\text{ V} - 4\text{ V}$$
$$= 6\text{ V}$$

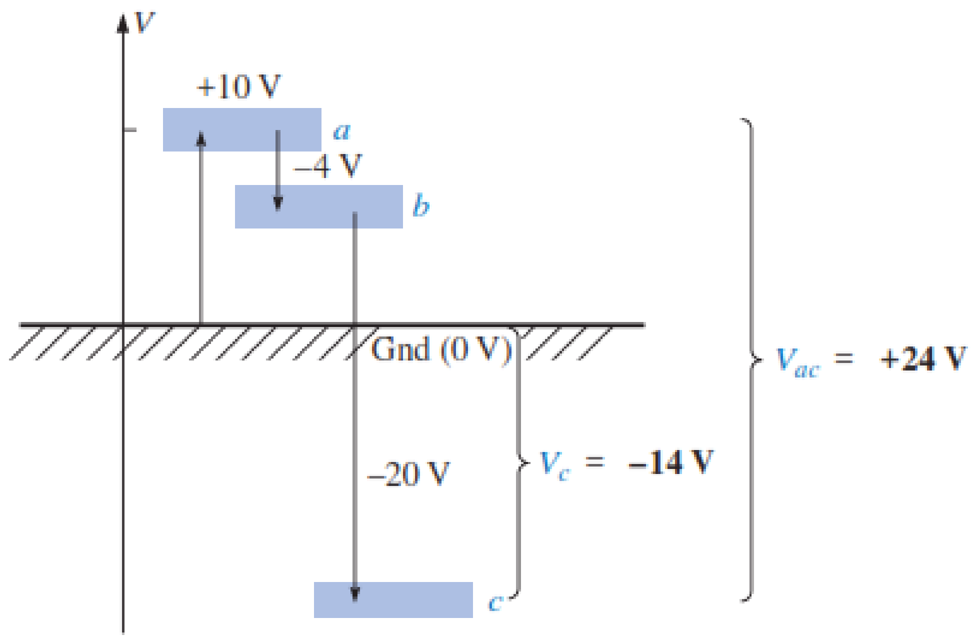
EXAMPLE 2.8 Find the voltages *Vb*, *Vc*, and *Vac* for the network shown.

$$V_b = +10\text{ V} - 4\text{ V} = \mathbf{6\ V}$$



$$V_c = V_b - 20\text{ V} = 6\text{ V} - 20\text{ V} = \mathbf{-14\ V}$$

$$V_{ac} = V_a - V_c = 10\text{ V} - (-14\text{ V}) = \mathbf{24\ V}$$



EXAMPLE 2.9 Determine *Vab*, *Vcb*, and *Vc* for the network shown.

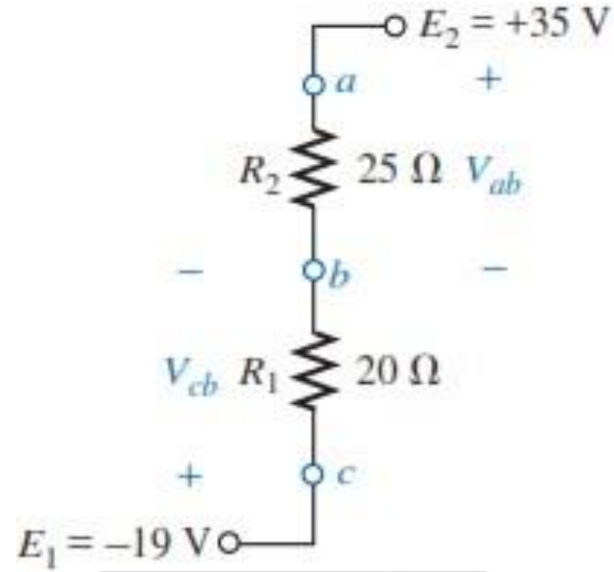
Note that there is a 54-V drop across the series resistors R_1 and R_2. The current can then be determined using Ohm's Law and the voltage levels as follows:

$$I = \frac{54\text{ V}}{45\ \Omega} = 1.2\text{ A}$$

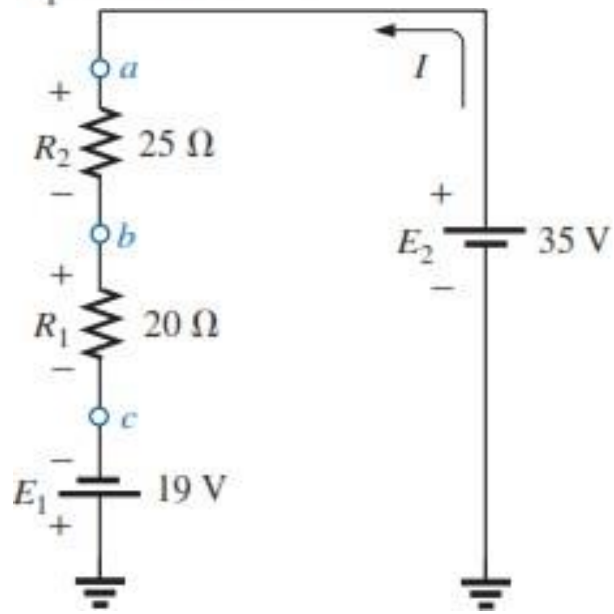
$$V_{ab} = IR_2 = (1.2\text{ A})(25\ \Omega) = \mathbf{30\ V}$$

$$V_{cb} = -IR_1 = -(1.2\text{ A})(20\ \Omega) = \mathbf{-24\ V}$$

$$V_c = E_1 = \mathbf{-19\ V}$$



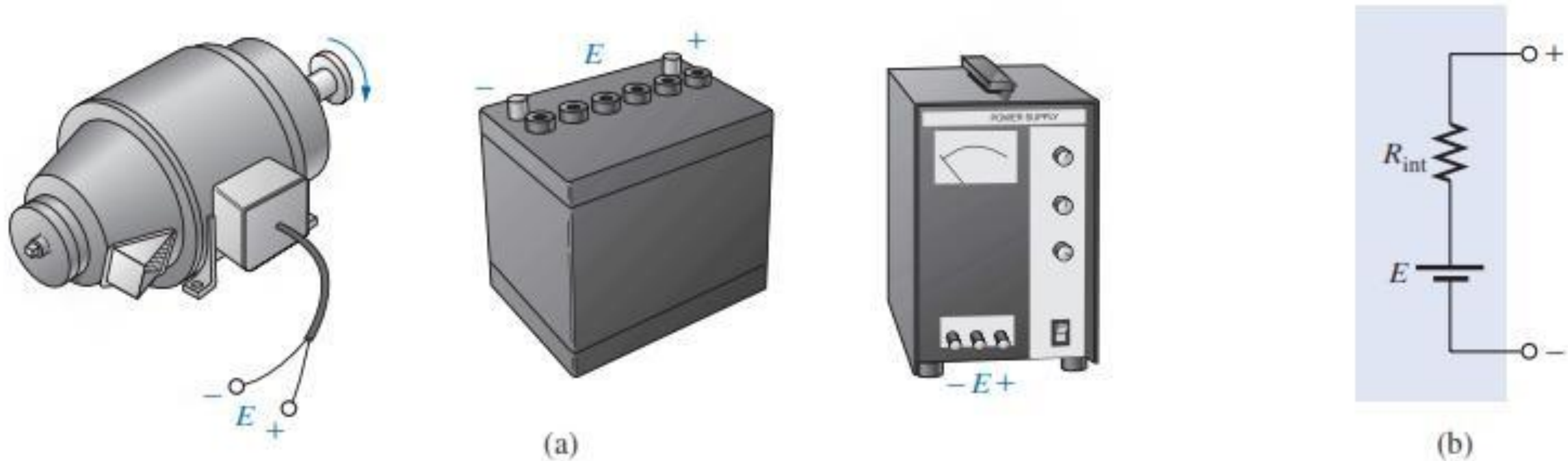
The other approach is to redraw the network as shown to clearly establish the aiding effect of *E*1 and *E2* and then solve the resulting series circuit.



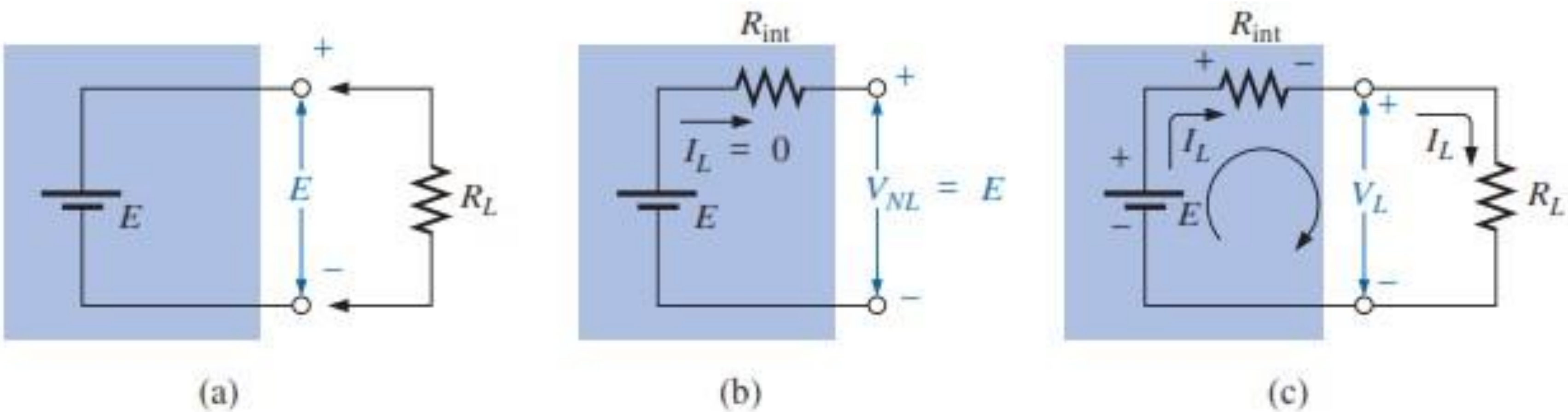
$$I = \frac{E_1 + E_2}{R_T} = \frac{19\text{ V} + 35\text{ V}}{45\ \Omega} = \frac{54\text{ V}}{45\ \Omega} = 1.2\text{ A}$$

$$V_{ab} = \mathbf{30\ V} \qquad V_{cb} = \mathbf{-24\ V} \qquad V_c = \mathbf{-19\ V}$$

INTERNAL RESISTANCE OF VOLTAGE SOURCES



(a) Sources of dc voltage; (b) equivalent circuit.

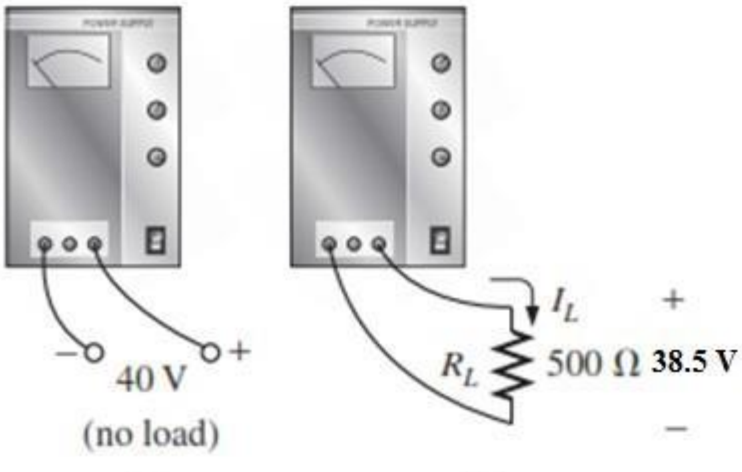


Voltage source: (a) ideal, $R_{int} = 0\,\Omega$; (b) determining V_{NL}; (c) determining R_{int}.

$$V_L = V_{NL} - I_L R_{\text{int}}$$

$$R_{\text{int}} = \frac{V_{NL}}{I_L} - R_L$$

EXAMPLE 2.10 Before a load is applied, the terminal voltage of the power supply of Fig.(a) is set to 40 V. When a load of 500Ω is attached, as shown in Fig. (b), the terminal voltage drops to 38.5 V. What happened to the remainder of the no-load voltage, and what is the internal resistance of the source?



(a) (b)

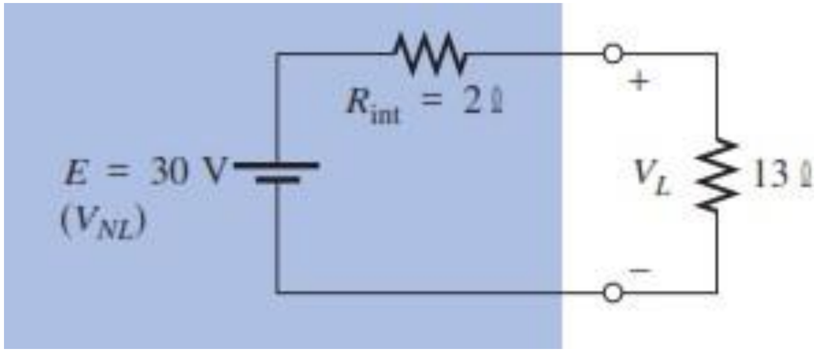
The difference of 40 V - 38.5 V = 1.5 V now appears across the internal resistance of the source. The load current is

38.5 V/0.5 k = 77 mA.

$$R_{\text{int}} = \frac{V_{NL}}{I_L} - R_L = \frac{40\text{ V}}{77\text{ mA}} - 0.5\text{ k}\Omega$$
$$= 519.48\ \Omega - 500\ \Omega = \mathbf{19.48\ \Omega}$$

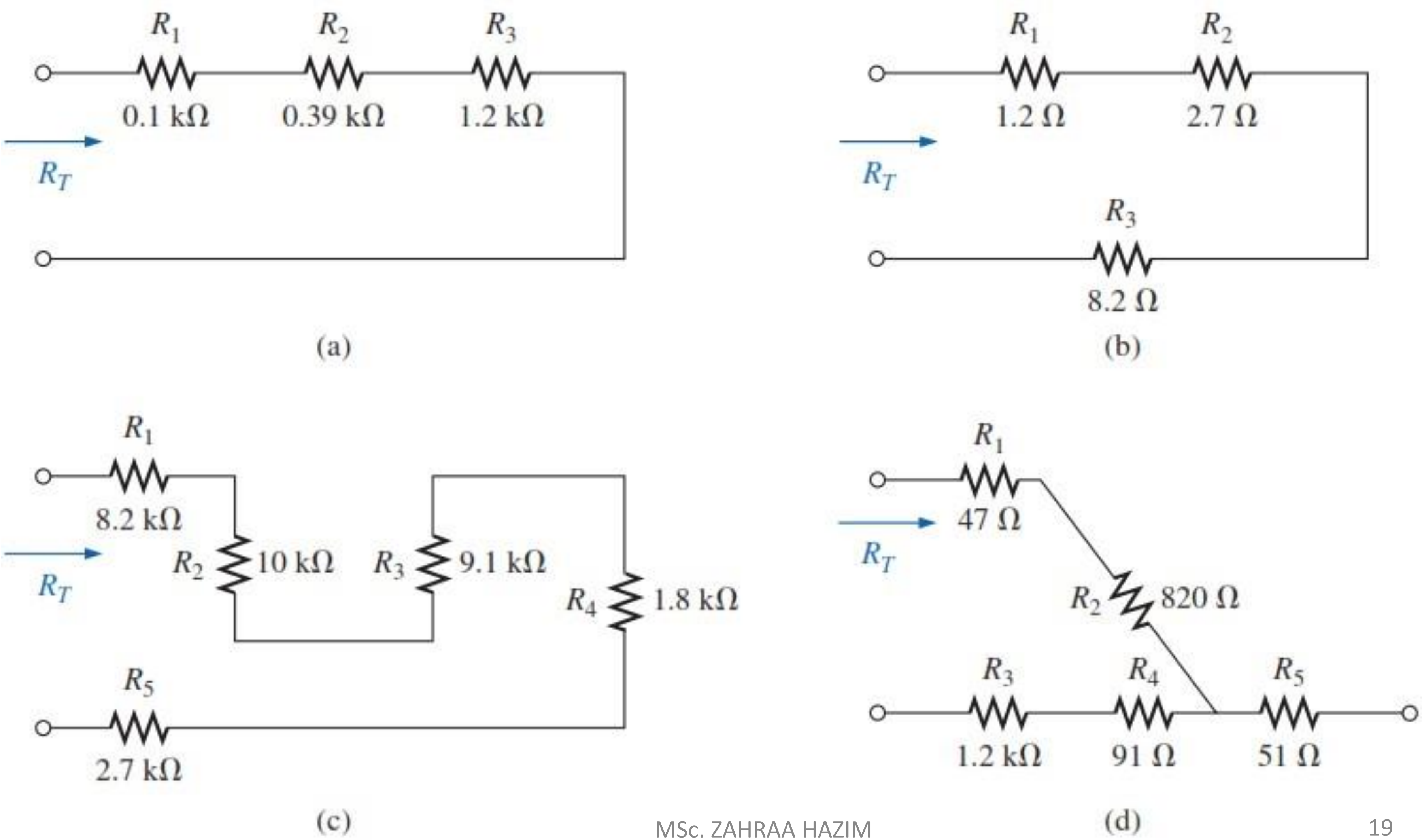
EXAMPLE 2.11 The battery of Fig. shown has an internal resistance of 2Ω . Find the voltage *VL* and the power lost to the internal resistance if the applied load is a 13-Ω resistor.

$$I_L = \frac{30\text{ V}}{2\ \Omega + 13\ \Omega} = \frac{30\text{ V}}{15\ \Omega} = 2\text{ A}$$
$$V_L = V_{NL} - I_L R_{\text{int}} = 30\text{ V} - (2\text{ A})(2\ \Omega) = \mathbf{26\ V}$$
$$P_{\text{lost}} = I_L^2 R_{\text{int}} = (2\text{ A})^2(2\ \Omega) = (4)(2) = \mathbf{8\ W}$$

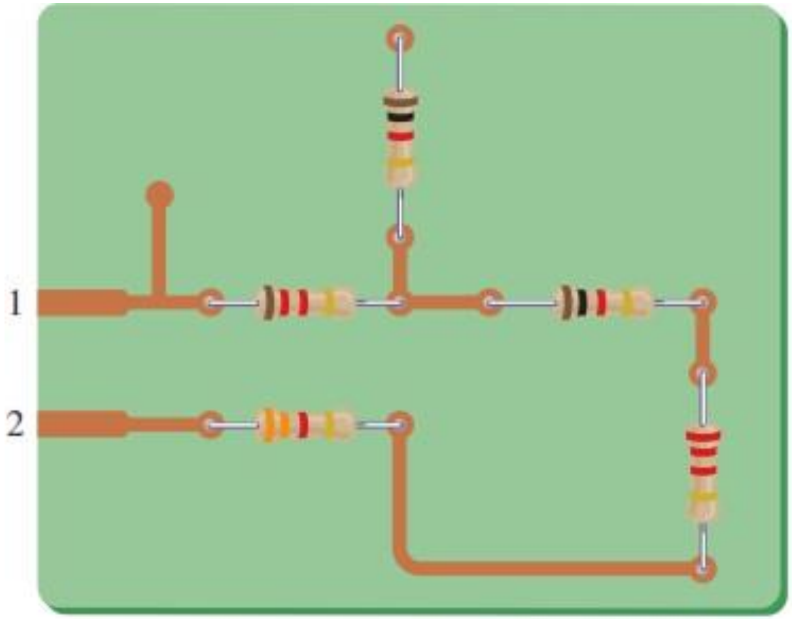


SECTION 5.2 Series Resistors

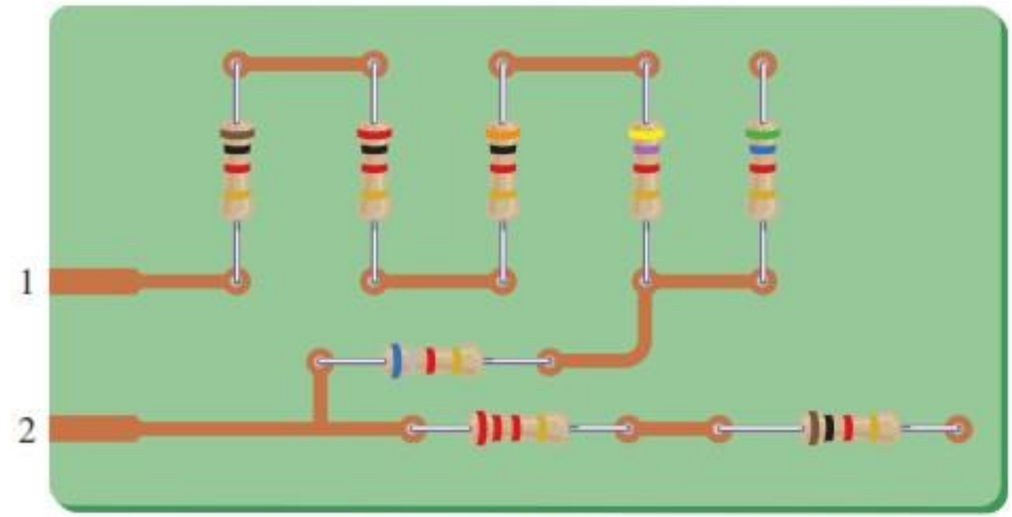
2. Find the total resistance R_T for each circuit shown.



3. For each circuit board in Fig., find the total resistance between connection tabs 1 and 2.



(a)



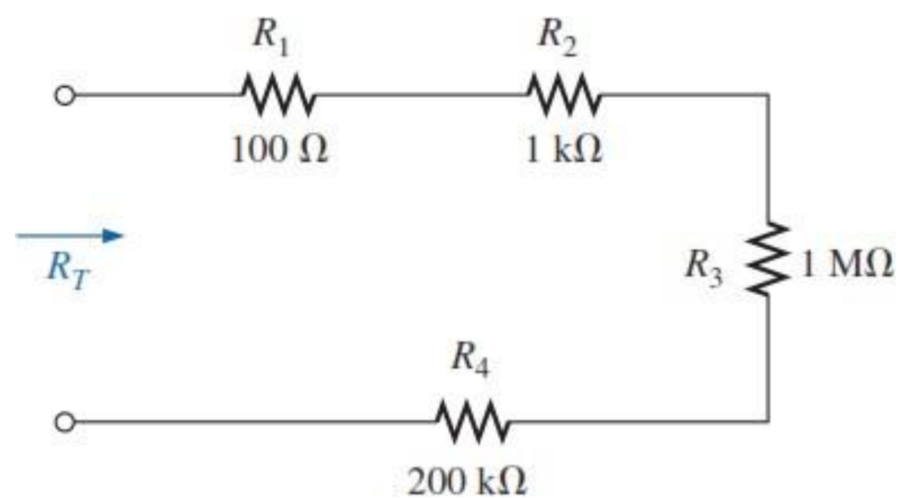
(b)

4. For the circuit in Fig., composed of standard values:

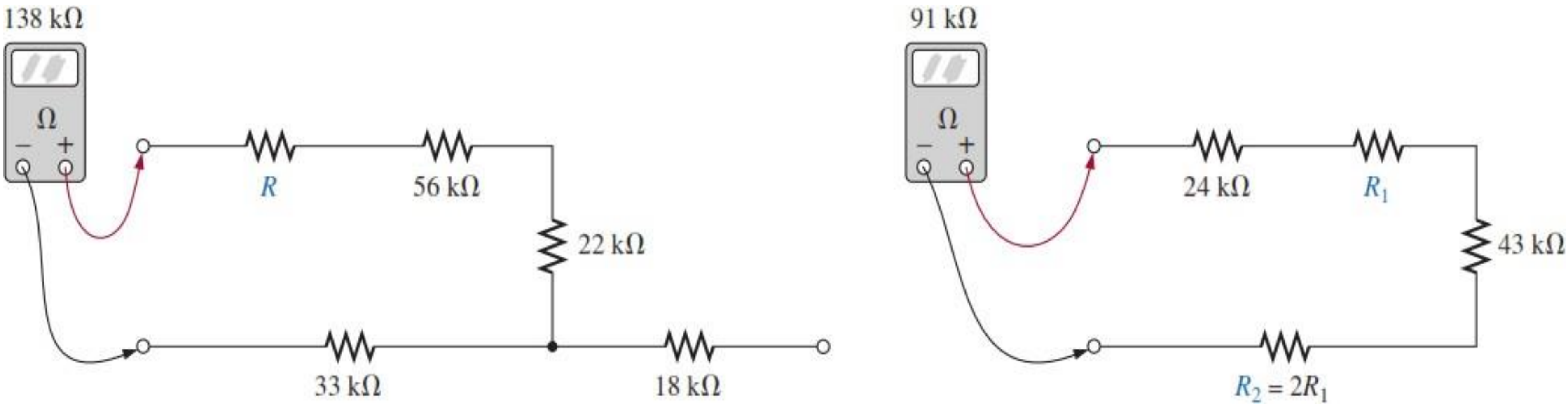
a. Which resistor will have the most impact on the total resistance?

b. On an approximate basis, which resistors can be ignored when determining the total resistance?

c. Find the total resistance, and comment on your results for parts (a) and (b).



5. For each configuration in Fig., find the unknown resistors using the ohmmeter reading.

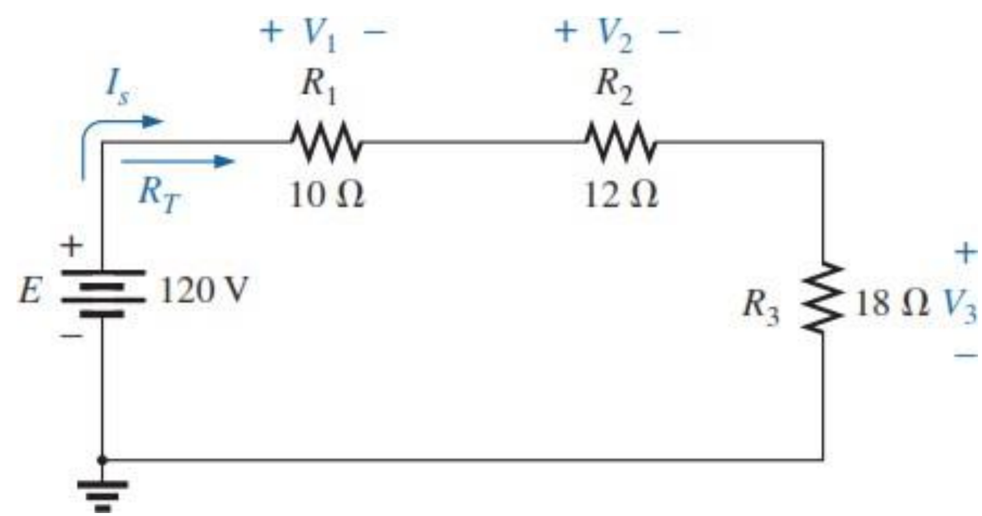


7. For the series configuration in Fig.:

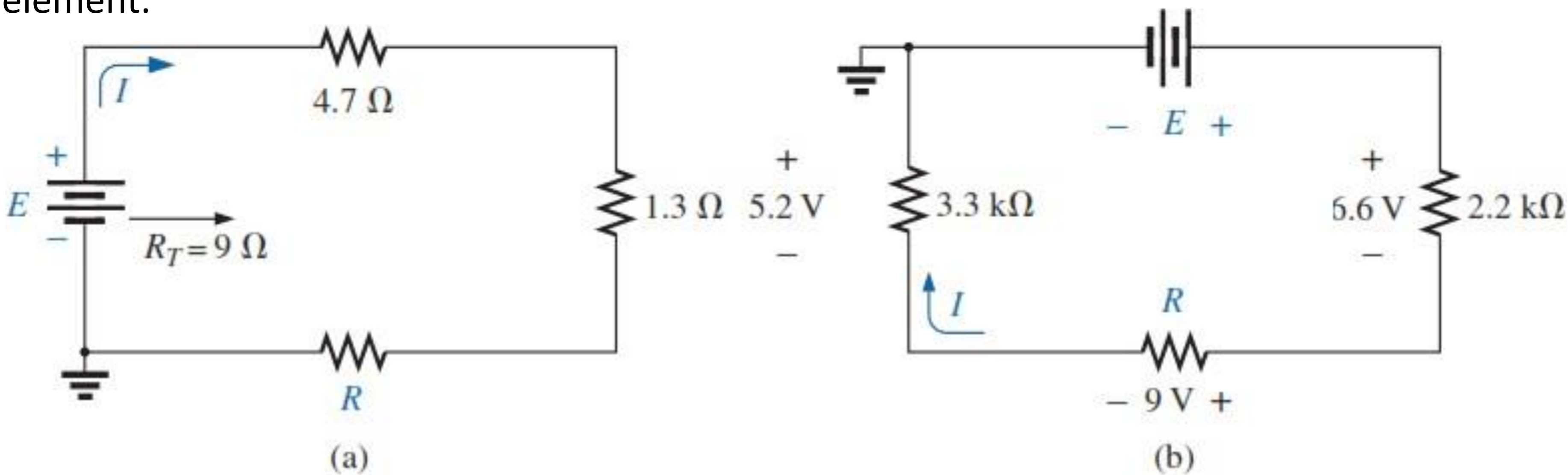
a. Find the total resistance.

b. Calculate the current.

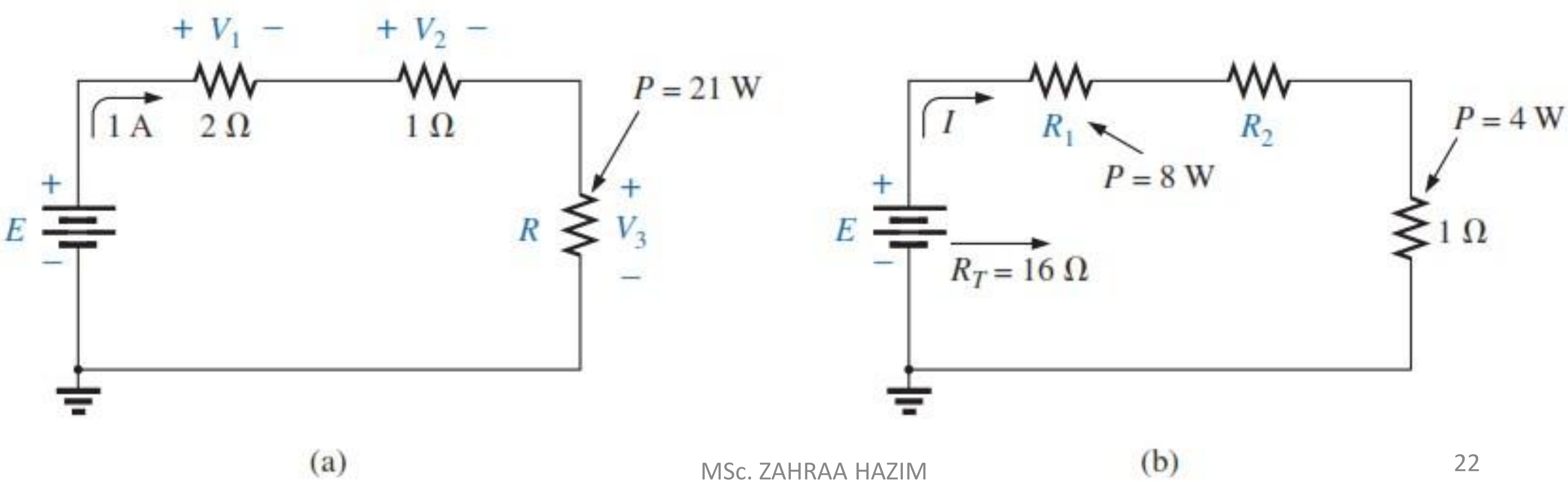
c. Find the voltage across each resistive element.



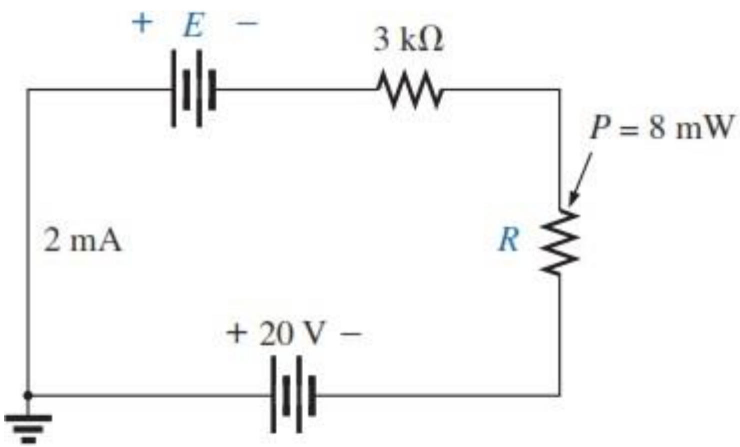
10. For each network in Fig. , constructed of standard values, determine:
a. The current *I.* b. The source voltage *E.* c. The unknown resistance. d. The voltage across each element.



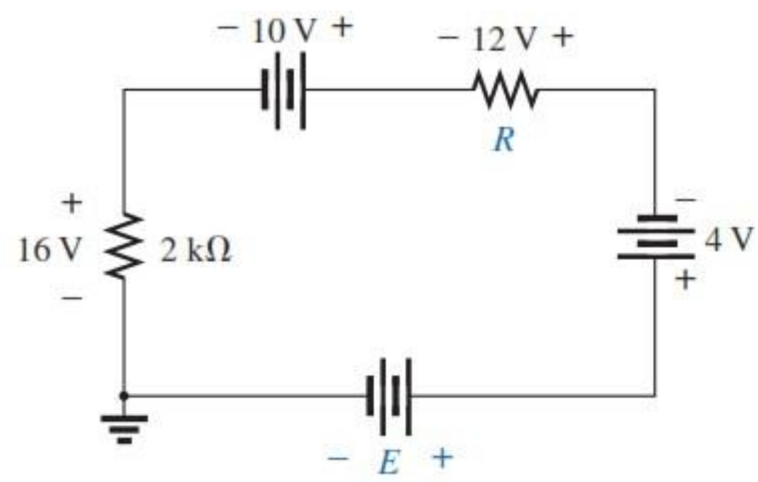
14. Find the unknown quantities for the circuits in Fig. using the information provided.



19. Find the unknown voltage source and resistor for the networks in Fig. First combine the series voltage sources into a single source. Indicate the direction of the resulting current.



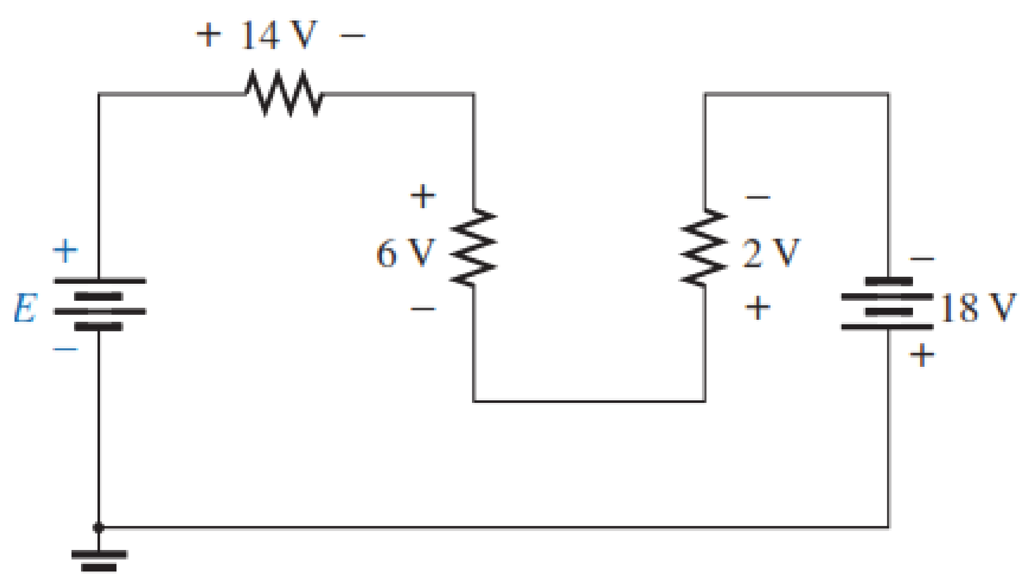
(a)



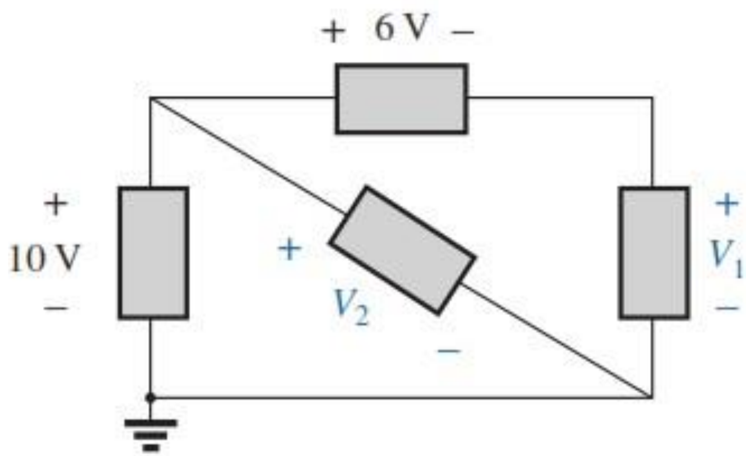
(b)

SECTION 5.6 Kirchhoff's Voltage Law

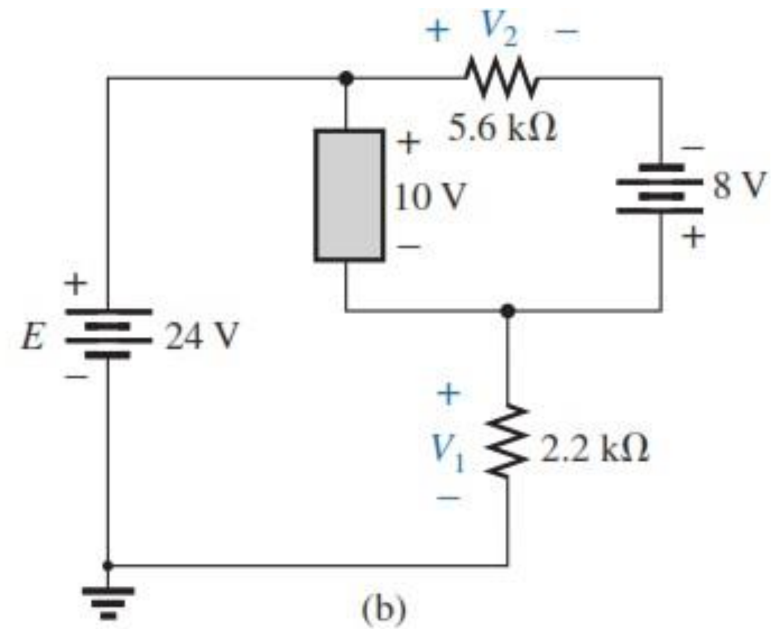
20. Using Kirchhoff's voltage law, find the unknown voltages for the circuit shown.



22. Using Kirchhoff's voltage law, determine the unknown voltages for the series circuits shown.



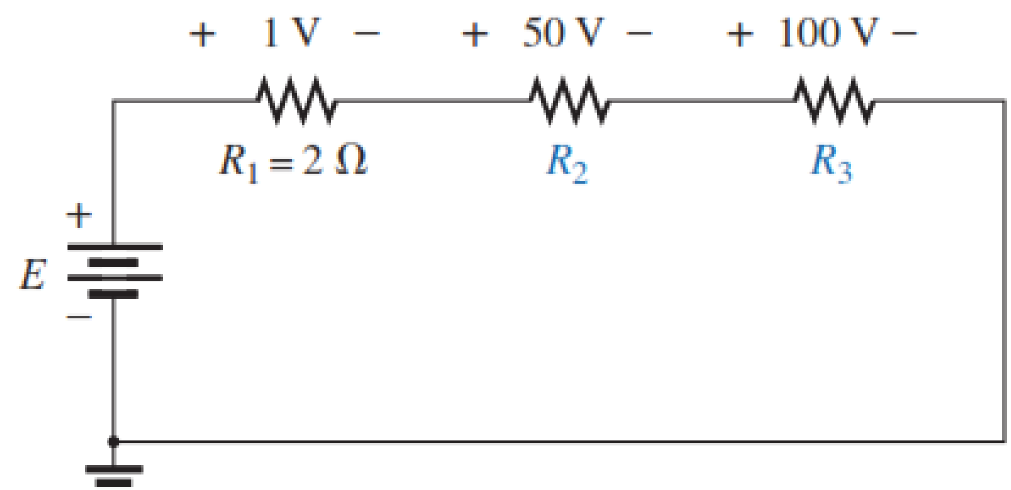
(a)



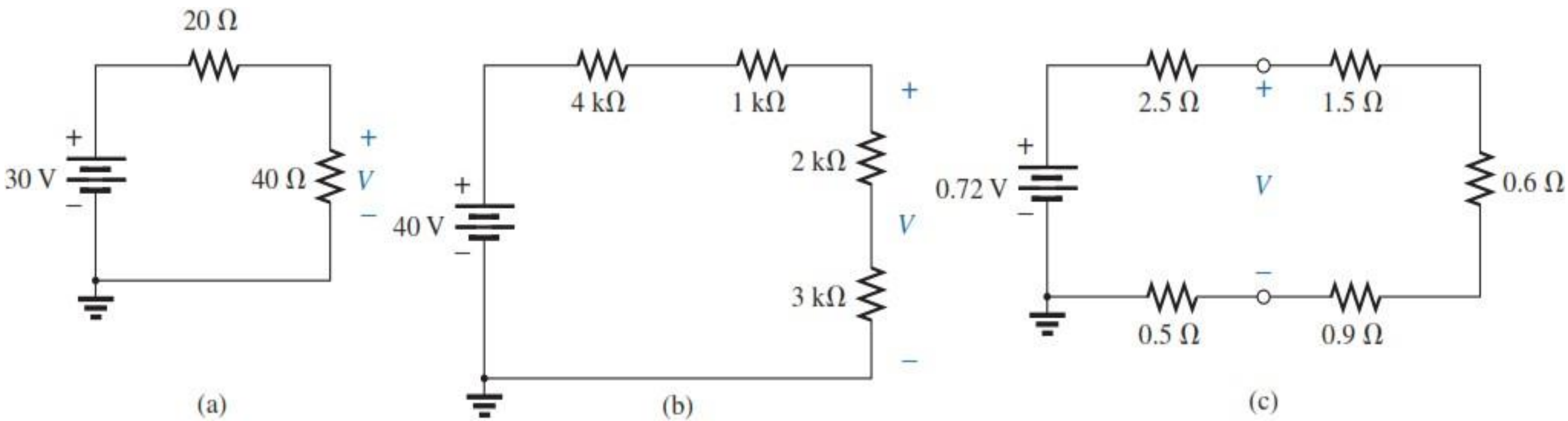
(b)

SECTION 5.7 Voltage Division in a Series Circuit

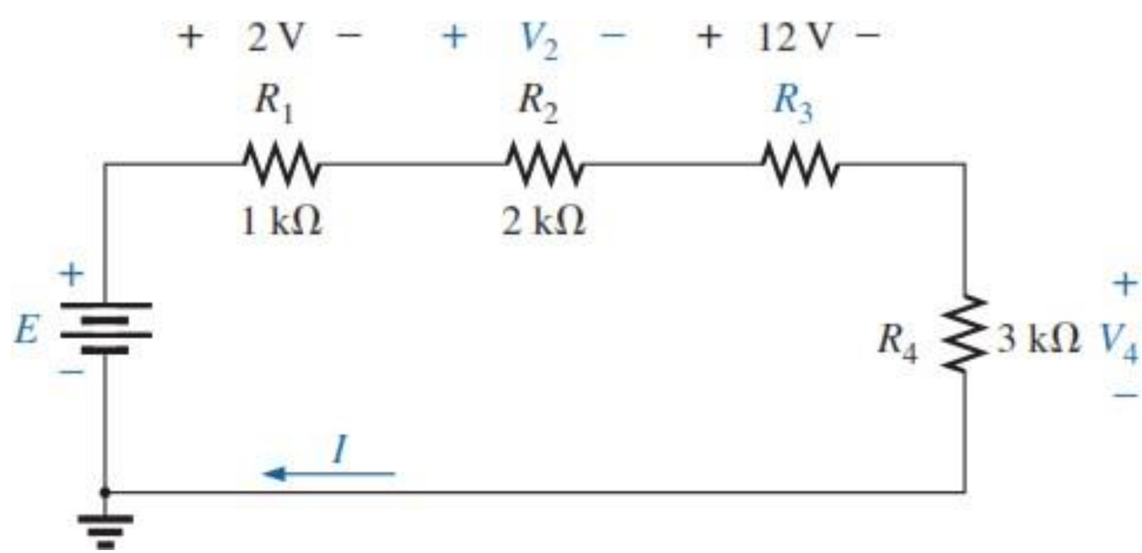
24. Determine the values of the unknown resistors in Fig. using the provided voltage levels.



26. Using the voltage divider rule, find the indicated voltages in Fig.



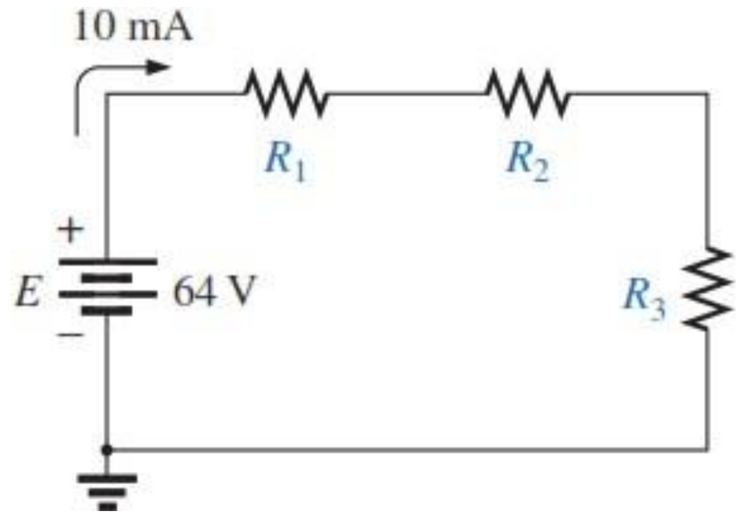
28. Using the information provided, find the unknown quantities of Fig.



31. Design a voltage divider circuit that will permit the use of an 8 V, 50 mA bulb in an automobile with a 12 V electrical system.

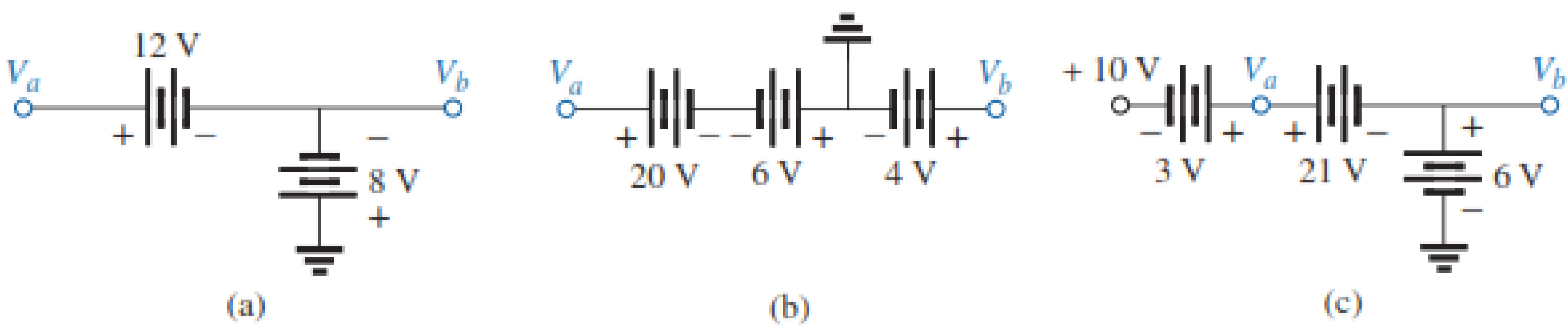
34. a. Design the circuit in Fig. such that $V_{R2} = 3V_{R1}$ and $V_{R3} = 4V_{R2}$.

b. If the current is reduced to 10 mA, what are the new values R_1, R_2, and R_3? How do they compare to the results of part (a)?



SECTION 5.9 Notation

35. Determine the voltages *Va, Vb, and Vab* for the networks shown

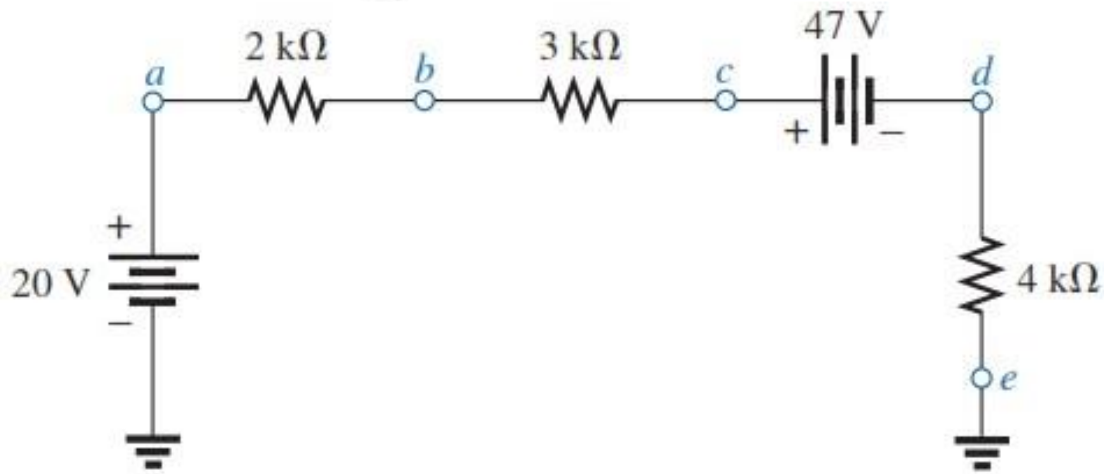


38. For the network in Fig. determine the voltages:

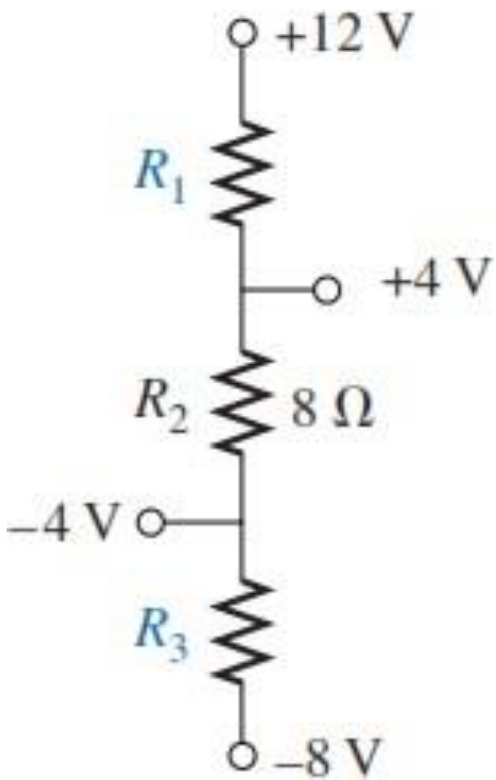
a. V_a, V_b, V_c, V_d, V_e

b. V_{ab}, V_{dc}, V_{cb}

c. V_{ac}, V_{db}



39. Given the information appearing in Fig., find the level of resistance for R_1 and R_3.



42. For the integrated circuit in Fig , determine V_0, V_4, V_7, V_{10}, V_{23}, V_{30}, V_{67}, V_{56}, and I (magnitude and direction.)