ALMUSTAQBAL UNIVERSITY

DEPARTMENT OF CONSTRUCTION & BUILDING ENGINEERING TECHNOLOGY

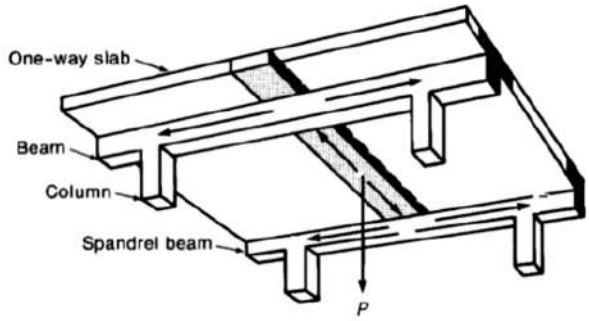
ANALYSIS & DESIGN OF REINFORCED CONCRETE STRUCTURE II

CALCULATING THE MINIMUM THICKNESS OF ONE-WAY & TWO-WAY SLABS

TYPES OF SLABS

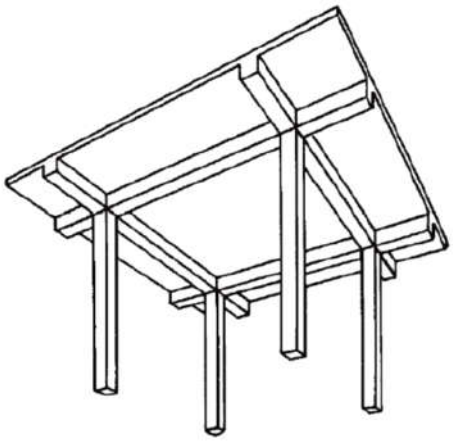
1. ONE WAY SLAB:

Slabs maybe supported on two opposite sides only, in such case, the structural action of the slab is essentially (one-way) and the loads are carried by the slab in the direction perpendicular to the supporting beams.

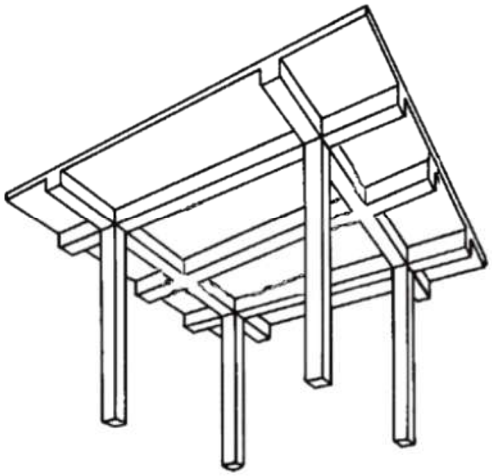


2. TWO-WAY SLABS:

Slabs have beams or supports on all four sides. The loads are carried by the slab in two perpendicular directions to the supporting beams.

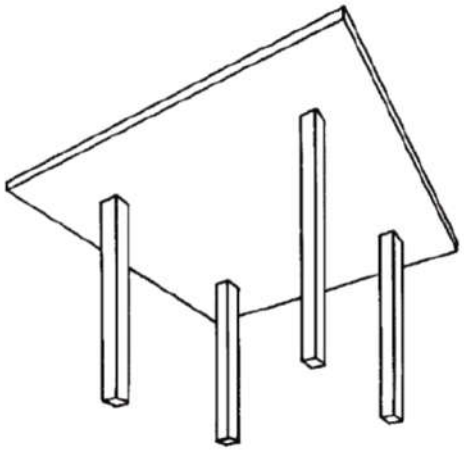


3. If the ratio of length to width of one slab panel is larger than 2, most of the load is carried by the short direction to the supporting beams and one one-way action is obtained in effect even though supports are provided on all directions.



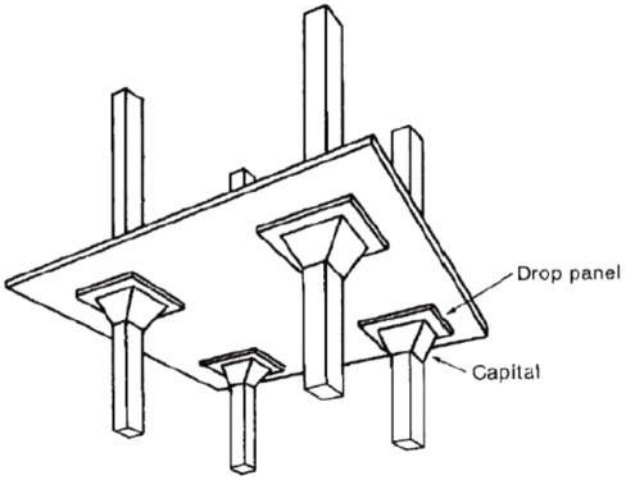
4. Concrete slab carried by columns:

These slabs are supported by columns only without beams or girders. Such slabs are called ***FLAT PLATES*** and are used where spans are not large, and loads are not heavy.



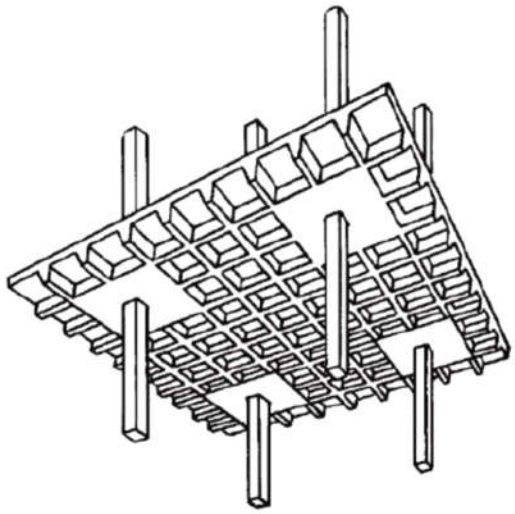
5. Flat slabs:

Flat slabs are also beamless slabs with column capitals, drop panels or a combination of both.



6. TWO-WAY JOIST SYSTEMS (GRID SLABS):

This type of slabs is used to reduce the dead load of the solid slab. Voids are formed in a rectilinear pattern by using metal or fibre glass form inserts. This results in the creation of a two-way ribbed construction (waffle slab). Usually, the inserts are omitted near the columns.

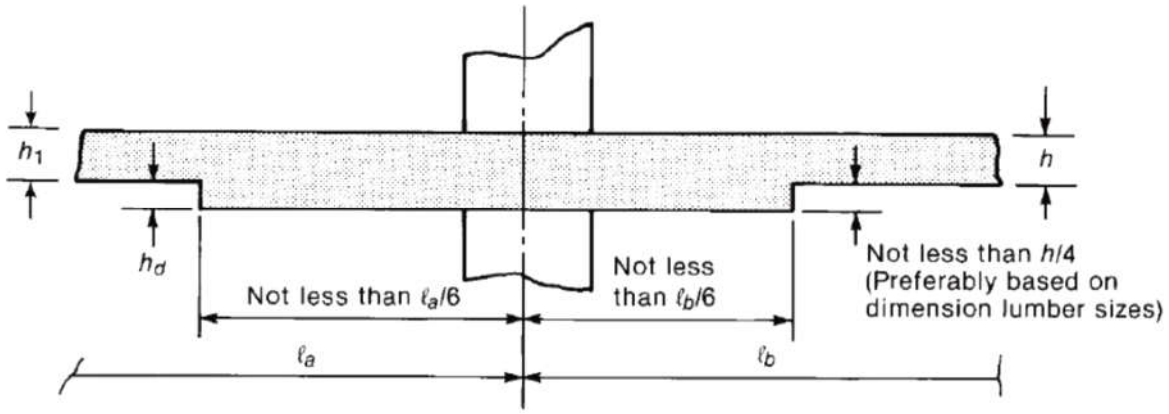


IMPORTANT DEFINITIONS

1. DROP PANEL:

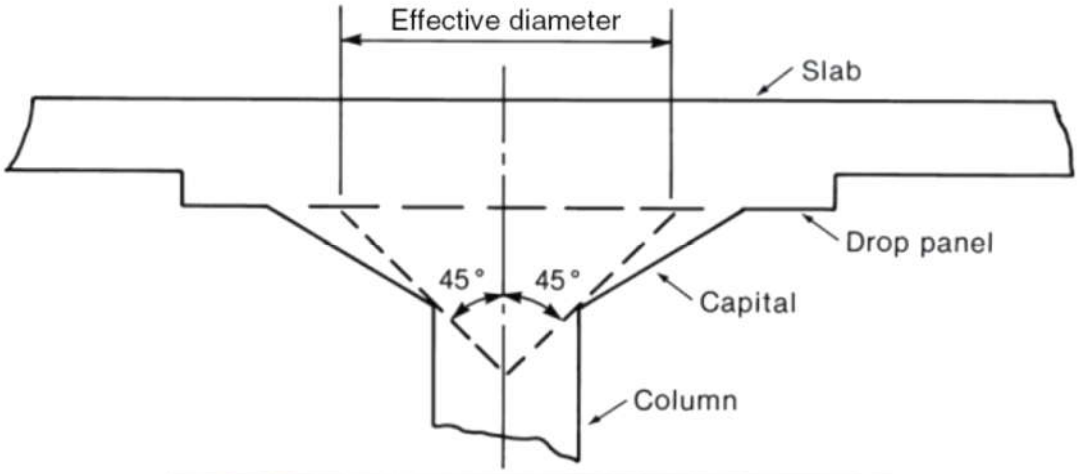
A projection below the slab used to reduce the amount of negative reinforcement over a column or the minimum required slab thickness, and to increase the slab shear strength.

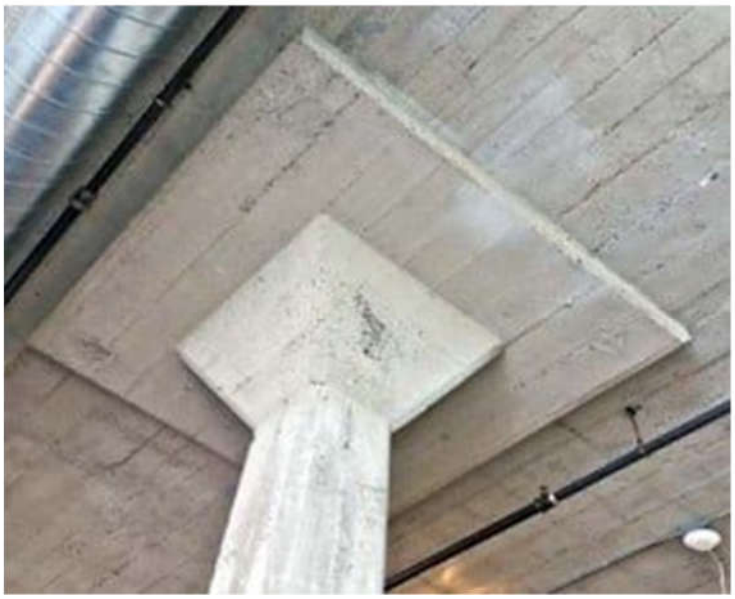
In computing the required slab reinforcement, the thickness of the drop panel below the slab shall not be assumed lesser than ¼ the slab thickness.



2. COLUMN CAPITAL:

An enlargement of the top of the concrete column located directly below the slab or drop panel. They are cast monolithically with the column. The column capital is nearly 20-25% of the average span.





MINIMUM SLAB THICKNESS

1. ONE-WAY SLABS:

For solid non-prestressed slabs not supported or attached to partitions or other construction, which is likely to be damaged by large deflection, the overall slab thickness (h) shall not be less than the limits in table 7.3.1.1 below:

Table 7.3.1.1—Minimum thickness of solid nonprestressed one-way slabs

Support condition	Minimum h[1]
Simply supported	$\ell/20$
One end continuous	$\ell/24$
Both ends continuous	$\ell/28$
Cantilever	$\ell/10$

2. TWO-WAY SLABS:

a. Without interior beams

The thickness of a two-way slab is determined using the table 8.3.11

	Without drop panels‡			With drop panels‡		
	Exterior panels		Interior panels	Exterior panels		Interior panels
f_y, MPa†	Without edge beams	With edge beams§		Without edge beams	With edge beams§	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

*For two-way construction, ℓ_n is the length of clear span in the long direction, measured face-to-face of supports in slabs without beams and face-to-face of beams or other supports in other cases.
†For f_y between the values given in the table, minimum thickness shall be determined by linear interpolation.
‡Drop panels as defined in 13.2.5.
§Slabs with beams between columns along exterior edges. The value of α_f for the edge beam shall not be less than 0.8.

IMPORTANT NOTE:
The determined thickness from the table above should be integrated to the nearest 10.

161mm≃170mm, 165mm≃170mm

The determined minimum thickness should be larger than the following values:

- For slabs without drop panels..... 125mm.
- For slabs with drop panels..... 100mm.

b. With interior beams

In the case of two-way slabs with interior beams, a value called $\boldsymbol{\alpha_f}$ should be determined. This is the ratio of flexural stiffness of the beam section to the flexural stiffness of the slab bounded laterally by centre-line of the panel on each side of the beam. This is determined by:

$$\alpha_f = \frac{I_b}{I_s}$$

The required ratio is the average ratio α_{fm} where it will be given to you in the question. In order to determine the minimum thickness for the slab, table 8.3.1.2 shall be used.

Table 8.3.1.2—Minimum thickness of nonprestressed two-way slabs with beams spanning between supports on all sides

α_{fm}[1]	Minimum h, mm		
$\alpha_{fm} \le 0.2$	8.3.1.1 applies		(a)
$0.2 < \alpha_{fm} \le 2.0$	Greater of:	$\dfrac{\ell_n\left(0.8+\frac{f_y}{1400}\right)}{36+5\beta(\alpha_{fm}-0.2)}$	(b)[2],[3]
		125	(c)
$\alpha_{fm} > 2.0$	Greater of:	$\dfrac{\ell_n\left(0.8+\frac{f_y}{1400}\right)}{36+9\beta}$	(d)[2],[3]
		90	(e)

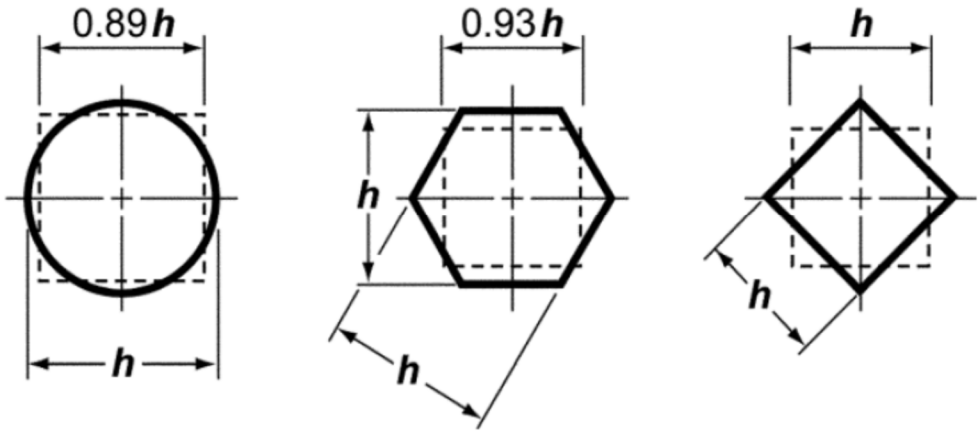
[1] α_{fm} is the average value of α_f for all beams on edges of a panel and α_f shall be calculated in accordance with 8.10.2.7.

[2] ℓ_n is the clear span in the long direction, measured face-to-face of beams (mm).

[3] β is the ratio of clear spans in long to short directions of slab.

IMPORTANT NOTE:

- Every value calculated from the above table should be integrated to the nearest 10mm.
- l_n is the clear span from the faces of opposite supports. If the support has an irregular shape (circular, polygon,..etc) it should be transferred to a square support.



Example One: *find the minimum thickness of a slab for an interior panel due to deflection control. Use* $f_y = 420MPa$.

a. *Slab with beams having the dimensions of (8.2x7.7)m clear span with an* α_{fm}*= 2.3.*
b. *Slab without drop panel having the dimensions of (5.4x4.8)m clear span with an* α_{fm}*= 0.18.*
c. *Flat plate slab with the dimensions of (4.2x4.6)m clear span.*
d. *Flat slab with drop panels with the dimensions of (6x6.2)m clear span.*
e. *Slab with beams having the dimensions of (5.8X5.8)m clear span with an* α_{fm}*= 1.5.*

Solution:

a. Since the slab is with beams, we must check table 8.3.1.2

Table 8.3.1.2—Minimum thickness of nonprestressed two-way slabs with beams spanning between supports on all sides

α_{fm}[1]	Minimum h, mm		
$\alpha_{fm} \le 0.2$	8.3.1.1 applies		(a)
$0.2 < \alpha_{fm} \le 2.0$	Greater of:	$\frac{\ell_n\left(0.8+\frac{f_y}{1400}\right)}{36+5\beta(\alpha_{fm}-0.2)}$	(b)[2],[3]
		125	(c)
$\alpha_{fm} > 2.0$	Greater of:	$\frac{\ell_n\left(0.8+\frac{f_y}{1400}\right)}{36+9\beta}$	(d)[2],[3]
		90	(e)

[1] α_{fm} is the average value of α_f for all beams on edges of a panel and α_f shall be calculated in accordance with 8.10.2.7.
[2] ℓ_n is the clear span in the long direction, measured face-to-face of beams (mm).
[3] β is the ratio of clear spans in long to short directions of slab.

$$h = \frac{l_n(0.8 + \frac{f_y}{1400})}{36 + 9\beta}$$

$$\beta = \frac{long}{short} = \frac{8200}{7700} = 1.065,$$

$$h = \frac{8200(0.8 + \frac{420}{1400})}{36 + 9(1.065)} = 197.87mm \qquad > 90mm$$

use $h = 200mm$

b. Since the slab has beams, check table 8.3.1.2.

Table 8.3.1.2—Minimum thickness of nonprestressed two-way slabs with beams spanning between supports on all sides

α_{fm}[1]	Minimum h, mm		
$\alpha_{fm} \le 0.2$	8.3.1.1 applies		(a)
$0.2 < \alpha_{fm} \le 2.0$	Greater of:	$\frac{\ell_n\left(0.8+\frac{f_y}{1400}\right)}{36+5\beta(\alpha_{fm}-0.2)}$	(b)[2],[3]
		125	(c)
$\alpha_{fm} > 2.0$	Greater of:	$\frac{\ell_n\left(0.8+\frac{f_y}{1400}\right)}{36+9\beta}$	(d)[2],[3]
		90	(e)

[1] α_{fm} is the average value of α_f for all beams on edges of a panel and α_f shall be calculated in accordance with 8.10.2.7.

[2] ℓ_n is the clear span in the long direction, measured face-to-face of beams (mm).

[3] β is the ratio of clear spans in long to short directions of slab.

$\therefore$ *we have to use table* 8.3.1.1

f_y, MPa†	Without drop panels‡			With drop panels‡		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams§		Without edge beams	With edge beams§	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

*For two-way construction, ℓ_n is the length of clear span in the long direction, measured face-to-face of supports in slabs without beams and face-to-face of beams or other supports in other cases.
†For f_y between the values given in the table, minimum thickness shall be determined by linear interpolation.
‡Drop panels as defined in 13.2.5.
§Slabs with beams between columns along exterior edges. The value of α_f for the edge beam shall not be less than 0.8.

$$\therefore h = \frac{l_n}{33} = \frac{5400}{33} = 163.65 \quad > 125mm$$

$use\ h = 170mm$

c. Since the slab is a flat plate, we use table 8.3.1.1

f_y, MPa†	Without drop panels‡			With drop panels‡		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams§		Without edge beams	With edge beams§	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

*For two-way construction, ℓ_n is the length of clear span in the long direction, measured face-to-face of supports in slabs without beams and face-to-face of beams or other supports in other cases.
†For f_y between the values given in the table, minimum thickness shall be determined by linear interpolation.
‡Drop panels as defined in 13.2.5.
§Slabs with beams between columns along exterior edges. The value of α_f for the edge beam shall not be less than 0.8.

$$\therefore h = \frac{l_n}{33} = \frac{4600}{33} = 139.4mm \quad > 125mm$$

$$use\ h = 140mm$$

d. Since the slab is a flat slab, use table 8.3.1.1

f_y, MPa†	Without drop panels‡			With drop panels‡		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams§		Without edge beams	With edge beams§	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

*For two-way construction, ℓ_n is the length of clear span in the long direction, measured face-to-face of supports in slabs without beams and face-to-face of beams or other supports in other cases.
†For f_y between the values given in the table, minimum thickness shall be determined by linear interpolation.
‡Drop panels as defined in 13.2.5.
§Slabs with beams between columns along exterior edges. The value of α_f for the edge beam shall not be less than 0.8.

$$h = \frac{l_n}{36} = \frac{6200}{36} = 172.2mm \quad > 100mm$$
$$use\ h = 180mm$$

e. Since the slab has beams, use table 8.3.1.2

Table 8.3.1.2—Minimum thickness of nonprestressed two-way slabs with beams spanning between supports on all sides

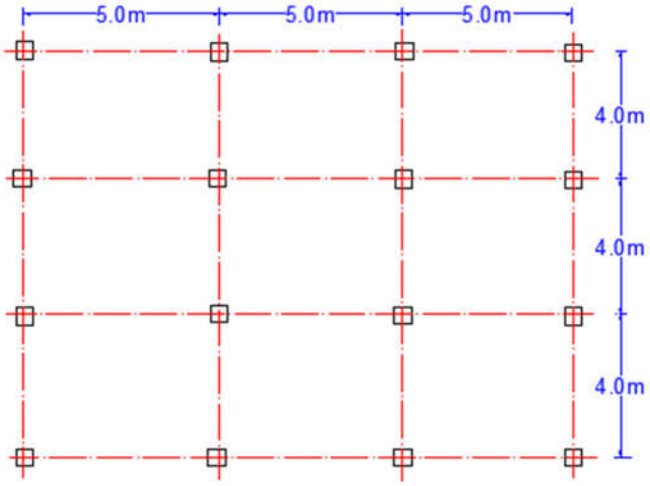
α_{fm}[1]	Minimum h, mm		
$\alpha_{fm} \le 0.2$	8.3.1.1 applies		(a)
$0.2 < \alpha_{fm} \le 2.0$	Greater of:	$\frac{\ell_n\left(0.8+\frac{f_y}{1400}\right)}{36+5\beta(\alpha_{fm}-0.2)}$	(b)[2][3]
		125	(c)
$\alpha_{fm} > 2.0$	Greater of:	$\frac{\ell_n\left(0.8+\frac{f_y}{1400}\right)}{36+9\beta}$	(d)[2][3]
		90	(e)

[1]α_{fm} is the average value of α_f for all beams on edges of a panel and α_f shall be calculated in accordance with 8.10.2.7.
[2]ℓ_n is the clear span in the long direction, measured face-to-face of beams (mm).
[3]β is the ratio of clear spans in long to short directions of slab.

$$h = \frac{l_n(0.8+\frac{f_y}{1400})}{36+5\beta(\alpha_{fm}-0.20)} = \frac{5800(0.8+\frac{420}{1400})}{36+5\times1(1.5-0.2)} = 150.12mm \quad > 125mm$$

$$\therefore use\ h = 160mm$$

Example Two: *find the minimum slab thickness required to control the deflection according to ACI code for the slab shown below. Use* $f_y = 420MPa$ *and the column size is* $300 \times 300mm$.



Solution:

$l_n = 5000 - 300 = 4700mm$

Since the slab is a flat plate, we use table 8.3.1.1

For exterior panel:

f_y, MPa[†]	Without drop panels[‡]			With drop panels[‡]		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams[§]		Without edge beams	With edge beams[§]	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

[‘]For two-way construction, ℓ_n is the length of clear span in the long direction, measured face-to-face of supports in slabs without beams and face-to-face of beams or other supports in other cases.
[†]For f_y between the values given in the table, minimum thickness shall be determined by linear interpolation.
[‡]Drop panels as defined in 13.2.5.
[§]Slabs with beams between columns along exterior edges. The value of α_f for the edge beam shall not be less than 0.8.

$h = \frac{l_n}{30} = \frac{4700}{30} = 156mm \quad > 125mm$
$\therefore h = 160mm$

For interior panel:

f_y, MPa†	Without drop panels‡			With drop panels‡		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams§		Without edge beams	With edge beams§	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

*For two-way construction, ℓ_n is the length of clear span in the long direction, measured face-to-face of supports in slabs without beams and face-to-face of beams or other supports in other cases.
†For f_y between the values given in the table, minimum thickness shall be determined by linear interpolation.
‡Drop panels as defined in 13.2.5.
§Slabs with beams between columns along exterior edges. The value of α_f for the edge beam shall not be less than 0.8.

$$h = \frac{l_n}{33} = \frac{4700}{33} = 142.42mm \quad > 125mm$$
$$\therefore use\ h = 150mm.$$

For the overall slab, use the greatest thickness.

$$\therefore \boldsymbol{use\ h = 160mm}$$

Example Three: *resolve the previous example by assuming that $f_y = 350MPa$.*

Solution:

For exterior panel:

f_y, MPa†	Without drop panels‡			With drop panels‡		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams§		Without edge beams	With edge beams§	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

*For two-way construction, ℓ_n is the length of clear span in the long direction, measured face-to-face of supports in slabs without beams and face-to-face of beams or other supports in other cases.
†For f_y between the values given in the table, minimum thickness shall be determined by linear interpolation.
‡Drop panels as defined in 13.2.5.
§Slabs with beams between columns along exterior edges. The value of α_f for the edge beam shall not be less than 0.8.

interpolation$\rightarrow \frac{33-30}{420-280} = \frac{x}{420-350} \rightarrow x = 1.5$

$$total\ factor = 30 + 1.5 = 31.5$$
$$\therefore h = \frac{l_n}{31.5} = \frac{4700}{31.5} = 149mm \quad > 125mm$$

$$use\ h = 150mm$$

For interior panel:

f_y, MPa†	Without drop panels‡			With drop panels‡		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams§		Without edge beams	With edge beams§	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

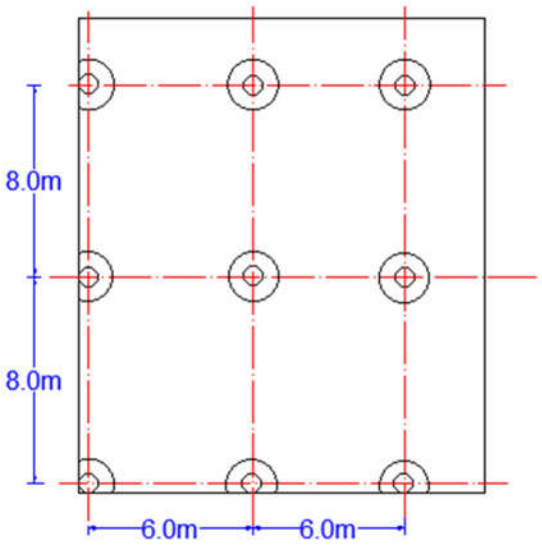
*For two-way construction, ℓ_n is the length of clear span in the long direction, measured face-to-face of supports in slabs without beams and face-to-face of beams or other supports in other cases.
†For f_y between the values given in the table, minimum thickness shall be determined by linear interpolation.
‡Drop panels as defined in 13.2.5.
§Slabs with beams between columns along exterior edges. The value of α_f for the edge beam shall not be less than 0.8.

$$\frac{36-33}{420-280}=\frac{x}{420-350}\rightarrow x=1.5$$

$$total\ factor=33+1.5=34.5$$
$$h=\frac{4700}{34.5}=136mm \quad >125mm$$
$$use\ h=140mm$$

$$\boldsymbol{for\ all\ panels\ use\ h=150mm}$$

Example Four: *determine the minimum thickness for the flat slab shown below. The column capital has a diameter of 1000mm. use* $f_y=420MPa$.



Solution:

For exterior panel:

f_y, MPa†	Without drop panels‡			With drop panels‡		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams§		Without edge beams	With edge beams§	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

*For two-way construction, ℓ_n is the length of clear span in the long direction, measured face-to-face of supports in slabs without beams and face-to-face of beams or other supports in other cases.
†For f_y between the values given in the table, minimum thickness shall be determined by linear interpolation.
‡Drop panels as defined in 13.2.5.
§Slabs with beams between columns along exterior edges. The value of α_f for the edge beam shall not be less than 0.8.

Table 8.3.1.1

$h = \frac{l_n}{30}$

$l_n = 8000 - 0.89D = 8000 - 0.89 \times 1000$
$= 7110mm$

$\therefore h = \frac{7110}{30} = 237mm \quad > 125mm$

Use $h = 240mm$

For interior panel:

Table 8.3.1.1

$h = \frac{l_n}{33} = \frac{7110}{33} = 215.4mm \quad > 125mm$

$use\ h = 220mm$

For the entire slab use $\boldsymbol{h = 240mm}$