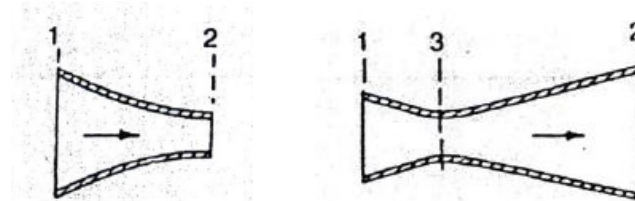


Chapter 4: Nozzle

4.1 Nozzle: is a flow passage of varying cross sectional area in which the velocity of fluid increase and pressure decrease in the direction of flow.

During flow in the nozzle the enthalpy drops and heat drop in expansion in spent of increasing the velocity of fluid. Nozzles are generally use in turbines, jet engine, injector...etc. there are **two types** of nozzle; **convergent nozzle** and **convergent-divergent nozzle**, as shown in figure (4.1). The effects of varying of cross-sectional area in the direction of flow these are; accelerate the fluid and dropping of pressure.



a. convergent nozzle b. convergent-divergent nozzle

Figure: (4.1) types of nozzles.

Let us consider the expansion through a nozzle between sections 1 and x, as shown in figure (4.2). Application of steady flow energy equation yields;

$$h_1 + \frac{1}{2}C_1^2 + gz_1 + Q = h_x + \frac{1}{2}C_x^2 + gz_x + W \quad \dots (4.1)$$

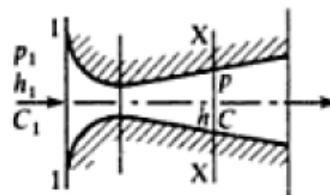


Figure (4.2) fluid flow through convergent-divergent nozzle.

In nozzle the velocity of fluid is so high that there is no time available for heat exchange with the surroundings and **the expansion may be considered adiabatic**. Also, the change in potential energy may be negligible if the elevation does not change from inlet to exit. Work done during flow is absent. Thus, the energy equation becomes:

$$h_1 + \frac{1}{2}C_1^2 = h_x + \frac{1}{2}C_x^2 \quad \dots (4.2)$$

$$h_1 - h_x = \frac{1}{2}(C_x^2 - C_1^2) \dots (4.3)$$

$$C_x^2 - C_1^2 = 2(h_1 - h_x) \rightarrow C_x = \sqrt{2(h_1 - h_x) + C_1^2} \dots (4.4)$$

If the inlet velocity of fluid C_1 is small compared with the exit velocity from the nozzle, the exhaust velocity can be re-written as:

$$C_2 = \sqrt{2(h_1 - h_2)} \dots (4.5)$$

For perfect gas the flow velocity can be stated as;

$$C_2 = \sqrt{2 \times C_p(T_1 - T_2)} \dots (4.6)$$

The mass flow rate across the nozzle is constant, and then by applying the continuity equation will get:

$$\dot{m}_1 = \dot{m}_x = \dot{m}_2 \dots (4.7)$$

$$\text{Thus ; } \rho_1 A_1 C_1 = \rho_x A_x C_x = \rho_2 A_2 C_2 \dots (4.8-a)$$

$$\text{Or, } \frac{A_1 C_1}{v_1} = \frac{A_x C_x}{v_x} = \frac{A_2 C_2}{v_2} \dots (4.8-a)$$

Where ρ : fluid density in kg/m³.

v : specific volume m³/kg

A: cross section area in m².

C: fluid velocity in m/s.

4.2 Nozzle efficiency

In nozzle design, the usual practice is to base all calculations on isentropic flow and then make an allowance for friction using a coefficient or efficiency.

The nozzle efficiency (η_N), is defined as the ratio of the actual enthalpy drop to the isentropic enthalpy drop, see figure (4.3).

$$\eta_N = \frac{\text{actual enthalpy drop}}{\text{isentropic enthalpy drop}} = \frac{h_1 - h_2}{h_1 - h_{2s}} \dots (4.9)$$

Substituting equation (4.3) into equation (4.9) yields,

$$\eta_N = \frac{(C_2^2 - C_1^2)}{(C_{2s}^2 - C_1^2)} \dots (4.10)$$

If the nozzle inlet velocity is negligibly small [$C_1^2 \approx 0$] equation (4.10) can be re-written as;

$$\eta_N = \frac{C_2^2}{C_{2s}^2} \dots (4.11)$$

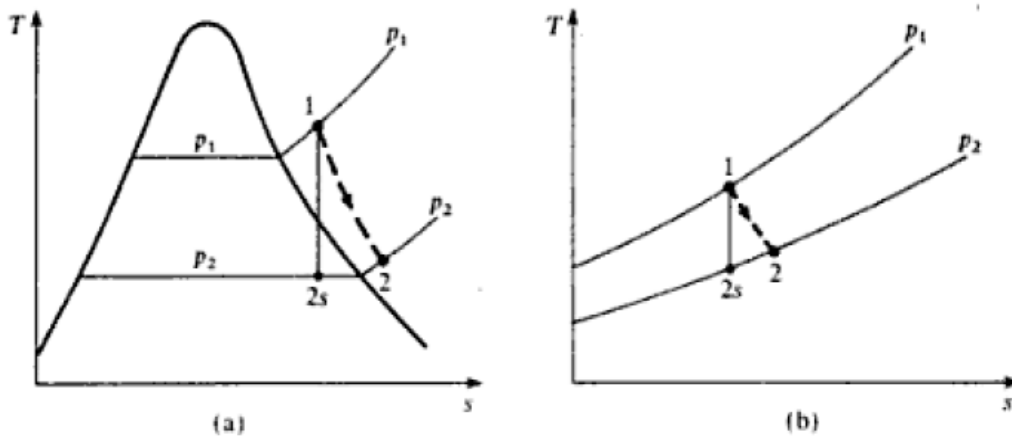


Figure (4.3): Nozzle expansion process for (a) vapor, (b) perfect gas.

For perfect gas only [$h = C_p \times T$], equation (4.9) can be written as:

$$\eta_N = \frac{C_p(T_1 - T_2)}{C_p(T_1 - T_{2s})} = \frac{T_1 - T_2}{T_1 - T_{2s}} \dots (4.12) \quad (\text{for perfect gas only})$$

Nozzles on practice are used with variety of shapes and cross sections. The cross section can be either circular or rectangular, and the axis of the nozzle can be straight or curved. A typical circular section, straight axis nozzle is shown in figure (4.4-a) and series of typical plate-type, curved-axis steam nozzles is shown in figure (4.4-b), this type used in reaction steam turbine.

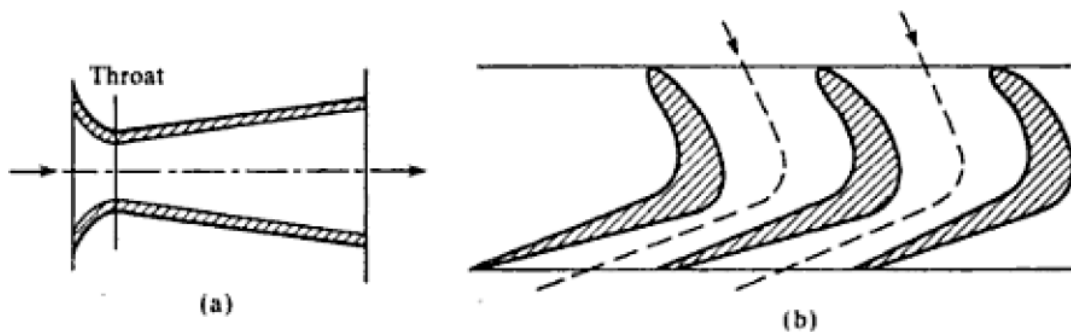


Figure (4.4) Typical (a) circular section nozzle (b) curved axis steam nozzles.

4.3 Critical Pressure Ratio

4.3.1 For perfect gas critical pressure ratio is:

$$(3/12)$$

$$\frac{P_c}{P_1} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \dots (4.13)$$

And the critical temperature ratio is

$$\frac{T_c}{T_1} = \left(\frac{P_c}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \dots (4.14)$$

and also it can be stated as:

$$\frac{T_c}{T_1} = \frac{2}{\gamma+1} \dots (4.15)$$

Substituting equation (4.15) into equation (4.6) and assuming the inlet velocity is negligibly small [$C_1 \approx 0$], after rearranging the critical velocity becomes:

$$C_c = \sqrt{\gamma R T_c} \dots (4.16)$$

The critical velocity given by equation (4.16) is the velocity at the throat of a correctly designed convergent-divergent nozzle, or the velocity at the exit of a convergent nozzle when the pressure ratio across the nozzle is the critical pressure ratio.

It can be noticed that the critical velocity is the velocity of sound at the critical conditions.

4.3.2 For Steam: an approximation can be obtained if it is assumed that steam expands isentropically in the nozzle following the law [$PV^n = \text{const.}$], where n is the isentropic (adiabatic) index and it is not a ratio of specific heats as is γ . Then, the throat pressure can be calculated as follows:

$$\frac{P_c}{P_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}} \dots (4.17)$$

If the steam entering the nozzle as **dry saturated** ($n = 1.135$)

$$\frac{P_c}{P_1} = \left(\frac{2}{1.135 + 1}\right)^{\frac{1.135}{1.135-1}}$$

$$P_c = 0.577 \times P_1 \dots (4.18)$$

If the steam entering the nozzle as **superheated** ($n = 1.3$)

$$\frac{P_c}{P_1} = \left(\frac{2}{1.3 + 1} \right)^{\frac{1.3}{1.3-1}}$$

$$P_c = 0.546 \times P_1 \dots (4.19)$$

The temperature at the throat (critical temperature) can be found from steam tables at the value of critical pressure P_c and $s_c = s_1$. The critical velocity can be calculated as:

$$C_c = \sqrt{2(h_1 - h_c)} \dots (4.20)$$

Where h_c : is the enthalpy of steam read from steam tables at P_c , and s_c

Example (4.1) Steam at **0.7 MPa** and **250 °C** expands in a nozzle to **0.1 MPa**. Determine the **throat and nozzle exit cross section areas** if the steam flow rate is **2.5 kg/s** and neglecting the loss between the entrance of the nozzle and the throat. Assuming that, the nozzle efficiency between the throat and the nozzle exit is **0.9**.

Solution:

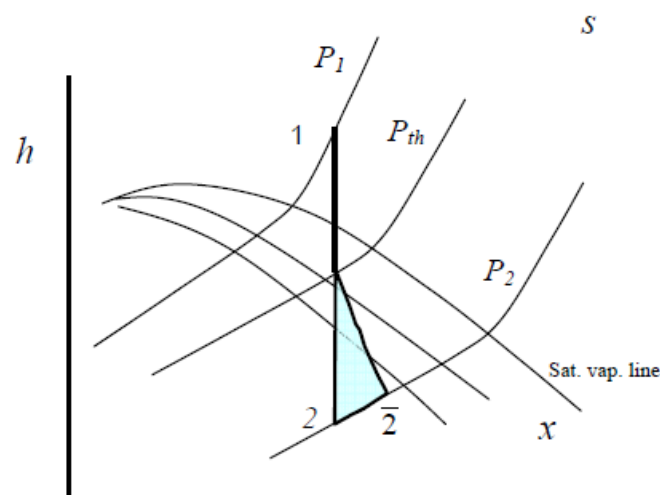


Figure (4.5)

From superheated steam tables

@ $P_1=0.7$ Mpa and $T_1=250^\circ\text{C}$ $h_1=2953.6$ kJ/kg and $s_1=7.11$ kJ/kg.K

$$P_c = 0.546 \times P_1 = 0.546 \times 0.7 = \mathbf{0.3822} \text{ Mpa}$$

If there is no losses from nozzle entrance to throat section.

$$s_{cs} = s_1 = \mathbf{7.11} \text{ kJ/kg.K}$$

@ $P_c = \mathbf{0.3822}$ Mpa and $s_{cs} = \mathbf{7.11}$ kJ/kg.K

$$\rightarrow h_{cs} = \mathbf{2806.7} \text{ kJ/kg and } v_c = \mathbf{0.50253} \text{ m}^3/\text{kg}$$

$$C_{cs} = \sqrt{2(h_1 - h_{cs})} = \sqrt{2(2953.6 - 2806.7) \times 1000} = \mathbf{542} \text{ m/s}$$

$$m^0 = \frac{A_{ts} \times C_{cs}}{v_{cs}} \rightarrow A_{ts} = \frac{m^0 \times v_{cs}}{C_{cs}} = \frac{2.5 \times 0.50253}{542} \times 10^6$$

$$A_{ts} = 2318 \text{ mm}^2$$

From saturated steam tables

$$@P_2 = 0.1 \text{ Mpa} \rightarrow v_g = 1.694 \text{ m}^3/\text{kg}, s_f = 1.3025, s_g = 7.3593 \text{ kJ/kg.K}$$

$$h_f = 417.44, h_g = 2675.5 \text{ kJ/kg.K}$$

$$@P_2 = 0.1 \text{ Mpa and } s_{2s} = s_{cs} = 7.11 \text{ kJ/kg.K}$$

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{7.11 - 1.3025}{7.3593 - 1.3025} = 0.96$$

$$h_{2s} = h_f + x_{2s}(h_g - h_f) = 417.44 + 0.96(2675.5 - 417.44) = 2585.2$$

$$\eta_N = \frac{h_{cs} - h_2}{h_{cs} - h_{2s}} \rightarrow h_{cs} - h_2 = \eta_N \times (h_{cs} - h_{2s})$$

$$h_2 = 2806.7 - 0.9(2806.7 - 2585.2) = 2607.4 \text{ kJ/kg}$$

$$h_2 = h_f + x_2(h_g - h_f) \rightarrow x_2 = \frac{h_2 - h_f}{h_g - h_f} = \frac{2607.4 - 417.44}{2675.5 - 417.44} = 0.97$$

$$v_2 = x_2 \times v_g = 0.97 \times 1.694 = 1.643 \text{ m}^3/\text{kg}$$

$$C_2 = \sqrt{2(h_1 - h_2)} = \sqrt{2(2953.6 - 2607.4) \times 1000} = 832 \text{ m/s}$$

In the same manner

$$A_2 = \frac{m^0 \times v_2}{C_2} = \frac{2.5 \times 1.643}{832} \times 10^6$$

$$A_2 = 4937 \text{ mm}^2$$

Example (4.2) A steam at a pressure of 1.47 MPa and 250°C enters a nozzle with a mass flow rate of 1 kg /s and expands to 0.395 MPa. Determine the throat section and the nozzle exit areas if:

- a- the expansion happens without a loss.
- b- the loss is 10% of the total thermal descent

Solution:

From superheated steam tables:

$$@P_1 = 1.47 \text{ Mpa} \& T_1 = 250^\circ\text{C} \rightarrow h_1 = 2924.4 \text{ kJ/kg}$$

$$\& s_1 = 6.7209 \text{ kJ/kg.K}$$

$$P_c = 0.546 \times P_1 = 0.546 \times 1.47 = 0.802 \text{ Mpa}$$

- a- the expansion happens without a loss.

There is no loss from nozzle entrance to throat section.

$$s_{cs} = s_1 = 6.7209 \text{ kJ/kg.K}$$

$$@P_c = 0.3822 \text{ Mpa and } s_{cs} = 6.7209 \text{ kJ/kg.K}$$

$$h_{cs} = 2795 \text{ kJ/kg and } v_{cs} = 0.2516 \text{ m}^3/\text{kg}$$

(6/12)

$$C_{cs} = \sqrt{2(h_1 - h_{cs})} = \sqrt{2(2924.4 - 2795) \times 1000} = 508.7 \text{ m/s}$$

$$m^o = \frac{A_t \times C_{cs}}{v_{cs}} \rightarrow A_{ts} = \frac{m^o \times v_{cs}}{C_{cs}} = \frac{1 \times 0.2516}{508.7} \times 10^6$$

$$A_{ts} = 495 \text{ mm}^2$$

In the same manner, for the exhaust section of nozzle.

From saturated steam tables

$$@P_2 = 0.395 \text{ Mpa} \rightarrow v_g = 0.4625 \text{ m}^3/\text{kg}$$

$$s_f = 1.7766, s_g = 6.8958 \text{ kj/kg.K}$$

$$h_f = 604.73, h_g = 2738.5 \text{ kj/kg.K}$$

There is no loss from nozzle entrance to exhaust section:

$$s_{2s} = s_1 = 6.7209 \text{ kj/kg.K} = s_f + x_{2s}(s_g - s_f)$$

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{6.7209 - 1.7766}{6.8958 - 1.7766} = 0.966$$

$$h_{2s} = h_f + x_{2s}(h_g - h_f) = 604.73 + 0.966(2738.5 - 604.73) = 2665.6$$

$$v_{2s} = x_{2s} \times v_g = 0.966 \times 0.4625 = 0.44677 \text{ m}^3/\text{kg}$$

$$C_{2s} = \sqrt{2(h_1 - h_{2s})} = \sqrt{2(2924.4 - 2665.6) \times 1000} = 719.44 \text{ m/s}$$

$$m^o = \frac{A_t \times C_{2s}}{v_{2s}} \rightarrow A_{2s} = \frac{m^o \times v_c}{C_{2s}} = \frac{1 \times 0.44677}{719.44} \times 10^6$$

$$A_{2s} = 621 \text{ mm}^2$$

b- The loss is 10% of the total thermal descent, as shown in figure (6.3).

There is loss from nozzle entrance to throat section.

$$\eta_N = \frac{h_1 - h_c}{h_1 - h_{cs}} \rightarrow h_1 - h_c = \eta_N \times (h_1 - h_{cs})$$

$$h_c = 2924.4 - 0.9(2924.4 - 2795) = 2808 \text{ kj/kg}$$

From superheated steam tables

$$@P_c = 0.802 \text{ Mpa} \ \& \ h_c = 2808 \text{ kj/kg} \rightarrow v_c = 0.26 \text{ m}^3/\text{kg}$$

(7/12)

$$C_c = \sqrt{2(h_1 - h_c)} = \sqrt{2(2924.4 - 2808) \times 1000} = 482.5 \text{ m/s}$$

$$m^o = \frac{A_t \times C_c}{v_c} \rightarrow A_t = \frac{m^o \times v_2}{C_2} = \frac{1 \times 0.26}{482.5} \times 10^6$$

$$A_t = 539 \text{ mm}^2$$

In the same manner, for the exhaust section of nozzle.

$$\eta_N = \frac{h_1 - h_2}{h_1 - h_{2s}} \rightarrow h_1 - h_2 = \eta_N \times (h_1 - h_{2s})$$

$$h_2 = 2924.4 - 0.9(2924.4 - 2665.6) = 2691.5 \text{ kJ/kg}$$

$$h_2 = h_f + x_2(h_g - h_f) \rightarrow x_2 = \frac{h_2 - h_f}{h_g - h_f} = \frac{2691.5 - 604.73}{2738.5 - 604.73} = 0.978$$

$$v_2 = x_2 \times v_g = 0.978 \times 0.4625 = 0.4523 \text{ m}^3/\text{kg}$$

$$C_2 = \sqrt{2(h_1 - h_2)} = \sqrt{2(2924.4 - 2691.5) \times 1000} = 682.5 \text{ m/s}$$

$$m^o = \frac{A_2 \times C_2}{v_2} \rightarrow A_2 = \frac{m^o \times v_2}{C_2} = \frac{1 \times 0.4523}{682.5} \times 10^6$$

$$A_2 = 662 \text{ mm}^2$$

Example (4.3) A steam turbine generates 1200 kW by passing a steam at 2 MPa, 300°C and a flow rate of 8.4 kg/kWh. This vapor expands to 0.02 MPa where the throat diameter of each nozzle in the turbine is 1.2 cm and the loss between the throat and the nozzle end is 12% of the thermal descent. Determine the number of nozzles and the diameter of the nozzle exit.

Solution: the expansion happens as shown in figure (4.5).

$$m^o = 8.4 \left(\frac{\text{kg}}{\text{kWh}} \right) \times \frac{1}{3600} \left(\frac{\text{h}}{\text{s}} \right) = \frac{8.4}{3600} \frac{\text{kg}}{\text{kWh.s}} = \frac{8.4}{3600} \left(\frac{\text{kg}}{\text{kWh.s}} \right) \times 1200 \text{ (kW)}$$

$$m^o = 2.8 \left(\frac{\text{kg}}{\text{s}} \right)$$

From superheated steam tables:

$$@P_1 = 2 \text{ Mpa} \ \& \ T_1 = 300^\circ\text{C} \rightarrow h_1 = 3023.5 \text{ kJ/kg}$$

$$\ \& \ s_1 = 6.7663 \text{ kJ/kg.K}$$

$$P_c = 0.546 \times P_1 = 0.546 \times 2 = 1.092 \text{ Mpa}$$

There is no loss from nozzle entrance to throat section, thus:

$$s_c = s_1 = 6.7663 \text{ kJ/kg.K}$$

From steam tables

$$@P_c = 1.092 \text{ Mpa} \ \& \ s_c = 6.7663 \text{ kJ/kg.K} \rightarrow h_c = 2827.9 \text{ kJ/kg}$$

(8/12)

$$\& v_c = 0.20596 \text{ m}^3/\text{kg}$$

$$C_c = \sqrt{2(h_1 - h_c)} = \sqrt{2(3023.5 - 2827.9) \times 1000} = 625.5 \text{ m/s}$$

$$m^o = \frac{A_t \times C_c}{v_c} \rightarrow A_t = \frac{m^o \times v_c}{C_c} = \frac{2.8 \times 0.20596}{625.5} = 0.000922 \text{ m}^2$$

$$A_t = n \times \left(\frac{\pi}{4} \times d_t^2\right) \rightarrow n = \frac{A_t}{\left(\frac{\pi}{4} \times d_t^2\right)} = \frac{0.000922}{\left[\frac{\pi}{4} \times \left(\frac{1.2}{100}\right)^2\right]} = 8.152 \approx 9$$

Number of nozzles is $n = 9$

From steam tables:

$$@P_2 = 20 \text{ kpa} \& s_{2s} = s_c = 6.7663 \text{ kj/kg.K} \rightarrow v_g = 7.6491 \text{ m}^3/\text{kg}$$

$$h_f = 251.38 \& h_g = 2609.7 \text{ kj/kg}$$

$$s_f = 0.8319 \& s_g = 7.9085 \text{ kj/kg.K}$$

$$s_{2s} = s_f + x_{2s}(s_g - s_f) \rightarrow x_{2s} = \frac{s_2 - s_f}{s_g - s_f} = \frac{6.7663 - 0.8319}{7.9085 - 0.8319} = 0.84$$

$$h_{2s} = h_f + x_{2s}(h_g - h_f) = 251.38 + 0.84(2609.7 - 251.38)$$

$$h_{2s} = 2232.4 \text{ kj/kg}$$

There is a loss from throat section and the end of the nozzle by 12%:

$$\eta_N = \frac{h_c - h_2}{h_c - h_{2s}} \rightarrow \eta_N \times (h_c - h_{2s}) = h_c - h_2 \rightarrow h_2 = h_c - \eta_N \times (h_c - h_{2s})$$

$$h_2 = 2827.9 - 0.88(2827.9 - 2232.4) = 2304 \text{ kj/kg}$$

$$h_2 = h_f + x_2(h_g - h_f) \rightarrow x_2 = \frac{2304 - 251.38}{2609.7 - 251.38} = 0.87$$

$$v_2 = x_2 \times v_g = 0.87 \times 7.6491 = 6.654 \text{ m}^3/\text{kg}$$

$$C_2 = \sqrt{2(h_1 - h_2)} = \sqrt{2(3023.5 - 2304) \times 1000} = 1200 \text{ m/s}$$

$$m^o = \frac{A_2 \times C_2}{v_2} \rightarrow A_2 = \frac{m^o \times v_2}{C_2} = \frac{2.8 \times 6.654}{1200}$$

$$A_2 = 0.015526 \text{ m}^2$$

$$A_2 = n \times \left(\frac{\pi}{4} \times d_2^2\right) \rightarrow d_2 = \sqrt{\frac{A_2 \times 4}{n \times \pi}} = \sqrt{\frac{0.015526 \times 4}{9 \times \pi}}$$

$$d_2 = 0.0468 \text{ m} = 4.68 \text{ cm}$$

Example (4.4) A steam at a flow rate of 4 kg/s expands in a nozzle from 0.8 MPa and $200 \text{ }^\circ\text{C}$ to 0.2 MPa , and the expansion from the beginning of the nozzle to the throat was without a loss and the loss was 10% from the throat to the end of the nozzle. Calculate the throat section area and the area of the nozzle exit if the velocity of the steam at the entrance of the nozzle is 70 m/s .

Solution: the expansion in the nozzle looks like figure (4.5).

From superheated steam tables:

$$@P_1 = 0.8 \text{ Mpa} \& T_1 = 200^\circ\text{C} \rightarrow h_1 = 2839.2 \text{ kj/kg}$$

$$\& s_1 = 6.8158 \text{ kJ/kg.K}$$

$$P_c = 0.546 \times P_1 = 0.546 \times 0.8 = 0.4368 \text{ Mpa}$$

a- the expansion happens without a loss.

There is no loss from nozzle entrance to throat section.

$$s_{cs} = s_1 = 6.8158 \text{ kJ/kg.K}$$

$$@P_c = 0.4368 \text{ Mpa and } s_{cs} = 6.8158 \text{ kJ/kg.K}$$

$$h_{cs} = 2745 \text{ kJ/kg and } v_{cs} = 0.4073 \text{ m}^3/\text{kg}$$

$$C_{cs} = \sqrt{2(h_1 - h_{cs}) + C_1^2} = \sqrt{2(2839.2 - 2745) \times 1000 + 70^2}$$

$$C_{cs} = 439.7 \text{ m/s}$$

$$m^o = \frac{A_t \times C_{cs}}{v_{cs}} \rightarrow A_{ts} = \frac{m^o \times v_{cs}}{C_{cs}} = \frac{4 \times 0.4073}{439.7} \times 10^6$$

$$A_{ts} = 3705 \text{ mm}^2$$

In the same manner, for the exhaust section of nozzle.

From saturated steam tables

$$@P_2 = 0.2 \text{ Mpa} \rightarrow v_g = 0.8857 \text{ m}^3/\text{kg}$$

$$s_f = 1.53, s_g = 7.1271 \text{ kJ/kg.K}$$

$$h_f = 504.68, h_g = 2706.6 \text{ kJ/kg.K}$$

There is 10% loss from nozzle throat to exhaust section:

$$s_{2s} = s_1 = 6.8158 \text{ kJ/kg.K} = s_f + x_{2s}(s_g - s_f)$$

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f} = \frac{6.8158 - 1.53}{7.1271 - 1.53} = 0.944$$

$$h_{2s} = h_f + x_{2s}(h_g - h_f) = 504.68 + 0.944(2706.6 - 504.68) = 2583.3$$

$$\eta_N = \frac{h_{cs} - h_2}{h_{cs} - h_{2s}} \rightarrow h_{cs} - h_2 = \eta_N \times (h_c - h_{2s})$$

$$h_2 = 2745 - 0.9(2745 - 2583.3) = 2600 \text{ kJ/kg}$$

$$h_2 = h_f + x_2(h_g - h_f) \rightarrow x_2 = \frac{h_2 - h_f}{h_g - h_f} = \frac{2600 - 504.68}{2706.6 - 504.68} = 0.951$$

$$v_2 = x_2 \times v_g = 0.951 \times 0.8857 = 0.842 \text{ m}^3/\text{kg}$$

$$C_2 = \sqrt{2(h_1 - h_2) + C_1^2} = \sqrt{2(2839.2 - 2600) \times 1000 + 70^2}$$

$$C_2 = 695 \text{ m/s}$$

In the same manner

$$A_2 = \frac{m^o \times v_2}{C_2} = \frac{4 \times 0.842}{695} \times 10^6$$

$$A_2 = 4846 \text{ mm}^2$$

Example (4.5) A vapor enters a nozzle at 0.7 MPa, 200 °C and with a velocity of 150 m/s, and exits at 0.07 MPa and a quality of 94%. Determine the exit steam velocity and also the area of the inlet and exit nozzle areas when the mass flow rate is 4.5 kg/s

Solution:

For entrance section: From superheated steam tables:

$$@P_1 = 0.7 \text{ Mpa} \ \& \ T_1 = 200^\circ\text{C} \ \rightarrow \ h_1 = 2844.65 \text{ kj/kg}$$

$$v_1 = 0.30641 \text{ m}^3/\text{kg} \ \& \ s_1 = 6.89115 \text{ kj/kg.K}$$

$$m^o = \frac{A_1 \times C_1}{v_1} \ \rightarrow \ A_1 = \frac{m^o \times v_1}{C_1} = \frac{4.5 \times 0.30641}{150} \times 10^6$$

$$A_1 = 9192 \text{ mm}^2$$

For the exit section: from saturated steam tables:

$$@P_2 = 0.07 \text{ Mpa} \ \rightarrow \ v_g = 2.4216 \text{ m}^3/\text{kg},$$

$$h_f = 375.6, \ h_g = 2656 \text{ kj/kg}$$

$$v_2 = x_2 \times v_g = 0.94 \times 2.4216 = 2.2763 \text{ m}^3/\text{kg}$$

$$h_2 = h_f + x_2(h_g - h_f) = 375.6 + 0.94(2656 - 375.6)$$

$$h_2 = 2519 \text{ kj/kg}$$

$$C_2 = \sqrt{2(h_1 - h_2) + C_1^2} = \sqrt{2(2844.65 - 2519) \times 1000 + 150^2}$$

$$C_2 = 821 \text{ m/s}$$

$$m^o = \frac{A_2 \times C_2}{v_2} \ \rightarrow \ A_2 = \frac{m^o \times v_2}{C_2} = \frac{4.5 \times 2.2763}{821} \times 10^6$$

$$A_2 = 12482 \text{ mm}^2$$

Exercises

Problem (4.1) Steam at 1.8 MPa and 275 °C expands through a convergent-divergent nozzle to 0.1 MPa. If there is 10 % loss in energy between the throat and the nozzle exit and the vapor flow rate is 0.5 kg/s, determine the cross-sectional area of the throat and the nozzle exit.

Problem (4.2) A vapor at a flow rate of 3 kg/s enters a group of eight (8) nozzles at 1.5 MPa, 240 °C and a velocity of 480 m/s, and then expands to 0.07 MPa. If the efficiency of each nozzle is 0.95 and the cross-sectional area of each nozzle is rectangular with sides' length ratio is 1:2.5. Determine:

a- The steam exit velocity for each nozzle and the dimensions of the nozzle section at its entrance and its exit.

b- The steam exit velocity in case of neglecting the steam entrance velocity.

Ans. [1011.16 m/s, 1.75cm, 4.38cm, 1.65cm, 4.14cm, 1007.96 m/s]

Problem (4.3) A vapor enters a group of convergent-divergent nozzles at 3 MPa and 300 °C and expands to 0.5 MPa. If the exit velocity and the steam circulation rate are 800 m/s and 14 kg/s respectively, determine:

a- The nozzle efficiency.

b- The total cross-sectional area of the nozzles.

c- The velocity at the throat.

Ans. [80%, 6120 mm², 530 m/s]