

ALMUSTAQBAL UNIVERSITY



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**DEPARTMENT OF BUILDING & CONSTRUCTION
ENGINEERING TECHNOLOGY**

ANALYSIS AND DESIGN OF REINFORCED CONCRETE STRUCTURES II

EXAMPLES IN PUNCHING SHEAR

EXAMPLE ONE: the flat slab of 200mm total thickness and 160mm effective depth is carried by 300mm square columns with a span of 4.5m from centre to centre of columns in each direction. Determine if shear reinforcement is required for the slab for the interior column. $W_u = 29.94 \text{ kPa}$, $f_y = 414 \text{ MPa}$ and $f_c' = 30 \text{ MPa}$.

SOLUTION:

$$b_o = (300 + 160) \times 4 = 1840 \text{ mm}$$

$$V_u = 29.94 \times (4.5 \times 4.5 - 0.46 \times 0.46) = 580 \text{ kN}$$

$$v_u = \frac{V_u}{b_o d} = \frac{580 \times 10^3}{1840 \times 160} = 1.97 \text{ MPa}$$

$$v_c = \min. \left\{ \begin{array}{l} 0.33 \times 1 \times \sqrt{f_c'} = 0.33 \sqrt{30} = \mathbf{1.807 \text{ MPa}} \\ 0.17 \left(1 + \frac{2}{\beta}\right) \sqrt{f_c'} = 0.17 \left(1 + \frac{2}{1}\right) \times \sqrt{30} = 2.973 \text{ MPa} \\ 0.083 \left(2 + \frac{\alpha_s d}{b_o}\right) \sqrt{f_c'} = 0.083 \left(2 + \frac{40 \times 160}{1840}\right) \times \sqrt{30} = 2.49 \text{ MPa} \end{array} \right\}$$

$$\beta = \frac{300}{300} = 1$$

$$\therefore v_c = \mathbf{1.807 \text{ MPa}}$$

$$\phi v_c = 0.75 \times 1.807 = 1.355 \text{ MPa} < v_u = 1.97 \text{ MPa} \quad \text{not o.k.}$$

\therefore shear reinforcement is required.

$$v_u = 1.97 < \phi 0.5 \sqrt{f_c'} = 0.75 \times 0.5 \times \sqrt{30}$$

$$v_u = 1.97 < 2.054 \text{ MPa} \quad \text{ok}$$

Find ϕv_s :

$$v_s = \frac{v_u}{\phi} - v_c, \text{ where } v_c = 0.17 \sqrt{f_c'} = 0.17 \sqrt{30} = 0.931 \text{ MPa.}$$

$$v_s = \frac{v_u}{\phi} - v_c = \frac{1.97}{0.75} - 0.931 = 1.696 \text{ MPa.}$$

$$v_s = \frac{A_v f_y}{b_o s} \rightarrow A_v = \frac{v_s b_o s}{f_y} = \frac{1.696 \times 1840 \times 80}{414} \quad s = \frac{d}{2} = 80 \text{ mm}$$

$$\therefore A_v = 603 \text{ mm}^2$$

The required area of vertical shear reinforcement = 603 mm^2

EXAMPLE TWO: Check the two-way shear (punching shear) only around an interior column (400x500) mm in a flat plate floor of a span (5.6x5.6) m. Find the area of vertical shear reinforcement if required. Assume $d=170\text{mm}$. Total $W_u=18\text{kPa}$ (including slab weight). $f_c'=30\text{MPa}$, $f_y=420\text{MPa}$.

SOLUTION:

$$b_o = (500 + 170 + 400 + 170) \times 2 = 2480\text{mm}$$

$$V_u = 18 \times (5.6 \times 5.6 - 0.67 \times 0.57) = 557.6\text{kN}.$$

$$v_u = \frac{V_u}{b_o d} = \frac{557.6 \times 10^3}{2480 \times 170} = 1.322\text{MPa}.$$

$$v_c = \left\{ \begin{array}{l} 0.33 \sqrt{f_c'} = 0.33 \sqrt{30} = \mathbf{1.807\text{MPa}} \\ 0.17 \left(1 + \frac{2}{\beta}\right) \sqrt{f_c'} = 0.17 \left(1 + \frac{2}{1.25}\right) \sqrt{30} = 2.42\text{MPa} \\ 0.083 \left(2 + \frac{\alpha_s d}{b_o}\right) \sqrt{f_c'} = 0.083 \left(2 + \frac{40 \times 170}{2480}\right) \sqrt{30} = 2.155\text{MPa} \end{array} \right\}$$

$$\therefore v_c = 1.807\text{MPa}, \beta = \frac{500}{400} = 1.25$$

$$\phi v_c = 0.75 \times 1.807 = 1.355\text{MPa} > v_u = 1.322\text{MPa} \quad \text{ok}$$

\therefore No shear reinforcement is required.

EXAMPLE THREE: Check the two-way shear (punching shear) only around the edge column (400x400) mm in a flat plate floor of a span (6.0x6.0) m. Find the area of vertical shear reinforcement if required. Assume $d=158\text{mm}$. Total $W_u=16\text{kPa}$ (including slab weight). $f_c'=25\text{MPa}$, $f_y=400\text{MPa}$.

SOLUTION:

$$b_o = (400 + 79) \times 2 + (400 + 158) = 1516\text{mm}.$$

$$V_u = 16 \times (6 \times 3.2 - 0.558 \times 0.479) = 302.923\text{kN}.$$

$$v_u = \frac{V_u}{b_o d} = \frac{302.923 \times 10^3}{1516 \times 158} = 1.265\text{MPa}.$$

$$\beta = \frac{400}{400} = 1$$

$$v_c = \left\{ \begin{array}{l} 0.33 \sqrt{f_c'} = 0.33\sqrt{25} = \mathbf{1.65MPa} \\ 0.17 \left(1 + \frac{2}{\beta}\right) \sqrt{f_c'} = 0.17 \left(1 + \frac{2}{1}\right) \sqrt{25} = 2.55MPa \\ 0.083 \left(2 + \frac{\alpha_s d}{b_o}\right) \sqrt{f_c'} = 0.083 \left(2 + \frac{30 \times 158}{1516}\right) \sqrt{25} = 2.128MPa \end{array} \right\}$$

$$\therefore v_c = 1.65MPa$$

$$\phi v_c = 0.75 \times 1.65 = 1.238MPa < v_u = 1.265MPa \dots \text{Not ok}$$

$$v_u \leq 0.5 \sqrt{f_c'} \rightarrow 1.265 < 1.875 \dots \text{Ok.}$$

$$v_c = 0.17 \sqrt{f_c'} = 0.17\sqrt{25} = 0.85MPa$$

$$v_s = \frac{v_u}{\phi} - v_c = \frac{1.265}{0.75} - 0.85 = 0.837 MPa$$

$$s = \frac{d}{2} = \frac{158}{2} = 79mm.$$

$$v_s = \frac{A_v f_y}{b_o s} \rightarrow A_v = \frac{0.837 \times 1516 \times 79}{400} = 250.6mm^2$$

\therefore The required area of vertical shear reinforcement is **250.6mm²**.

EXAMPLE FOUR: Check the two-way shear (punching shear) only around the corner column (400x400) mm in a flat plate floor of a span (6.0x 6.0) m. Find the area of vertical shear reinforcement if required. Assume d=158mm. Total Wu=19kPa (including slab weight). fc'= 25MPa, fy=400MPa.

SOLUTION:

$$b_o = (400 + 79) \times 2 = 958mm.$$

$$V_u = 19 \times (3.2 \times 3.2 - 0.479 \times 0.479) = 190.201 kN.$$

$$v_u = \frac{V_u}{b_o d} = \frac{190.201 \times 10^3}{958 \times 158} = 1.257MPa.$$

$$\beta = \frac{400}{400} = 1$$

$$v_c = \left\{ \begin{array}{l} 0.33 \sqrt{f'_c} = 0.33\sqrt{25} = \mathbf{1.65MPa} \\ 0.17 \left(1 + \frac{2}{\beta}\right) \sqrt{f'_c} = 0.17 \left(1 + \frac{2}{1}\right) \sqrt{25} = 2.55MPa \\ 0.083 \left(2 + \frac{\alpha_s d}{b_o}\right) \sqrt{f'_c} = 0.083 \left(2 + \frac{20 \times 158}{958}\right) \sqrt{25} = 2.199MPa \end{array} \right\}$$

$$\therefore v_c = 1.65MPa$$

$$\phi v_c = 0.75 \times 1.65 = 1.238MPa < v_u = 1.257MPa \dots \text{Not ok}$$

\therefore Shear reinforcement is required.

$$v_u \leq 0.5 \sqrt{f'_c} \rightarrow 1.257 < 1.875 \dots \text{Ok.}$$

$$v_c = 0.17 \sqrt{f'_c} = 0.17\sqrt{25} = 0.85MPa$$

$$v_s = \frac{v_u}{\phi} - v_c = \frac{1.257}{0.75} - 0.85 = 0.826 MPa$$

$$s = \frac{d}{2} = \frac{158}{2} = 79mm.$$

$$v_s = \frac{A_v f_y}{b_o s} \rightarrow A_v = \frac{0.826 \times 957 \times 79}{400} = 156.12mm^2$$

\therefore The required area of vertical shear reinforcement is **156.12mm²**.

EXAMPLE FIVE: Check the two-way shear (punching shear) only around the interior column (450x450) mm in a flat plate floor of a span (5.8x5.8) m. Find the area of vertical shear reinforcement if required. Assume d=150mm. Total Wu=17.5kPa (including slab weight). fc'= 25MPa, fy=420MPa.

SOLUTION:

$$b_o = (450 + 150) \times 4 = 2400mm.$$

$$V_u = 17.5 \times (5.8 \times 5.8 - 0.6 \times 0.6) = 562.1 kN.$$

$$v_u = \frac{V_u}{b_o d} = \frac{562.1 \times 10^3}{2400 \times 150} = 1.561MPa.$$

$$\beta = \frac{450}{450} = 1$$

$$v_c = \left\{ \begin{array}{l} 0.33 \sqrt{f_c'} = 0.33 \sqrt{25} = \mathbf{1.867 MPa} \\ 0.17 \left(1 + \frac{2}{\beta}\right) \sqrt{f_c'} = 0.17 \left(1 + \frac{2}{1}\right) \sqrt{25} = 2.885 MPa \\ 0.083 \left(2 + \frac{\alpha_s d}{b_o}\right) \sqrt{f_c'} = 0.083 \left(2 + \frac{30 \times 150}{2400}\right) \sqrt{25} = 2.133 MPa \end{array} \right\}$$

$$\therefore v_c = 1.867 MPa$$

$$\phi v_c = 0.75 \times 1.867 = 1.4 MPa < v_u = 1.561 MPa \dots \text{Not ok}$$

\therefore Shear reinforcement is required.

$$v_u \leq 0.5 \sqrt{f_c'} \rightarrow 1.561 < 2.12 MPa \dots \text{Ok.}$$

$$v_c = 0.17 \sqrt{f_c'} = 0.17 \sqrt{32} = 0.962 MPa$$

$$v_s = \frac{v_u}{\phi} - v_c = \frac{1.561}{0.75} - 0.962 = 1.119 MPa$$

$$s = \frac{d}{2} = \frac{150}{2} = 75 mm.$$

$$v_s = \frac{A_v f_y}{b_o s} \rightarrow A_v = \frac{1.119 \times 2400 \times 75}{420} = 479.6 mm^2$$

\therefore The required area of vertical shear reinforcement is **479.6 mm²**.