Probability and Random Variables in Brief

Term	Discrete Random Variables (DRV)	Continues Random Variables (CRV)
Basic Prob. Definitions	X = {x_1 , x_2 , x_3 , x_4 , x_5 , …… x_N } , $\sum_{i=1}^{N} p_i$ **=1** **Prob.=** {p_1 , p_2 , p_3 , p_4 , p_5 , …… p_N } , $p_i \neq 0$ *in general*	$\int_{-\infty}^{\infty} f(x)dx = 1$, (f(x) is the pdf non-negative, less than 1) p(x=a) =0 (why) , but $p(a<x<b)= \int_a^b f(x)dx$
Axioms of Probability	P(A) = Relative frequency of A OR Percentage occurrence of A. For null event P(∅) = 0 and the sample space P(S)= 1 Basic rule P(A+B)=P(A) + P(B) – p(A.B) , and Conditional Probability P(A|B)=P(A.B)/P(B) and P(B|A)=P(A.B))/P(A) For mutual event P(A.B)=0, and for independent P(A.B)=P(A).P(B)	
Cumulative Distribution Function (CDF)	F(x=a) = p($x \leq a$) = $\sum_{x \leq a} p_i$	F(x=a) = p($x \leq a$) = $\int_{-\infty}^{a} f(x)dx$
	CDF is non-negative and increasing function with x, $F(-\infty)=0$, $F(\infty) = 1$	
Joint RV	Joint Probability = P(x_i , y_j) , $\sum_x \sum_y P(x_i, y_j) = 1$ P(x_i , y_j) = P(x_i). P(y_j) (independent RV) P(x_i , y_j) = 0 (Mutual exclusive events) P(x_i) = $\sum_y P(x_i, y_j)$, P(y_j) = $\sum_x P(x_i, y_j)$	Joint pdf = $f(x,y)$, $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dxdy = 1$ $f(x,y)= f(x).f(y)$ (independent) $f(x) = \int_{-\infty}^{\infty} f(x,y)dy$, f(y)= $\int_{-\infty}^{\infty} f(x,y)dx$,
Expectation	E[h(x)] =$\overline{h(x)}$= $\sum_x h(x_i).P(x_i)$	E[h(x)]= $\overline{h(x)}$= $\int_{-\infty}^{\infty} h(x).f(x)dx$
	E[h(x)] is linear operation. E[h(x)] may be –ve, 0, or +ve. For c=constant, E[c.h(x)] = c. E[h(x)] , and E[c] =c , also E[h_1(x) + h_2(x)] = E[h_1(x)] + E[h_2(x)] , E[h(x)] is non-negative when h(x) is positive function as in h(x)=x^2 for example.	
Moments	Moments about the origin m_k=E[x^k] , while moments about the origin is M_k=E[(x-$\overline{x}$)^k] Useful moments m_1= E[x] = $\overline{x}$ (the mean value or DC level of x), m_2= E[x^2] = $\overline{x^2}$= mean square value (average total power) M_1=0 (why), M_2 = E[(x-$\overline{x}$)^2] = σ^2 = variance, and σ $= standard\ deviation.$ Also $\sigma^2 = \overline{x^2} - \overline{x}^2$!	
Useful Distributions	Binomial Distribution, also known as Bernoulli or repeated trials: P_k= $C_k^N\ p^k(1-p)^{N-k}$ where; p=Prob. of single trial, and k=0,1,2,3,. . . .N. with $\overline{k}$=p.N and $\sigma^2 = N.p.(1-p)$	Uniform Distribution; f(x) = 1/2A for -A<x<A = 0 elsewhere In general, any distribution with f(x) constant is uniform
	Poisson Distribution: P_k= $\frac{e^{-\lambda}\lambda^k}{k!}$ for k=0,1,2,3……∞ with $\overline{k}$= $\sigma^2 = \lambda$ with λ being the number of calls or arrivals per unit time.	Gaussian or Normal distribution: f(x)= $\frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\overline{x})^2/2\sigma^2}$ For σ^2=1 and $\overline{x}$=0, it is called standard normal distribution or ***N(1,0)***, f(z)= $\frac{1}{\sqrt{2\pi}}e^{-z^2/2}$ - $also\ -\infty < z < \infty$ To convert into standard units, z=(x-$\overline{x}$)/ σ
Relations between RVs	Correlation of x and y=E[x.y]. If E[x.y]= E[y]. E[y], then x and y are said to be uncorrelated. If E[x.y]=0 then x and y are said to be orthogonal Independent ≫ $f(x,y)$= $f(x).f(y)$ and so E[h(x).h(y)]=E[h(x)].E(h(y)]. Any independent variables are also uncorrelated.	
Transformation of RV	Given the random variable x with pdf f(x), it is required to determine the pdf of y (i.e. f(y)) when y=g(x) (i.e function of x): Transformation rule:since y=g(x),then $x = g^{-1}(y)$: $f_y(y) = f_x(g^{-1}(y)) . \left\|\frac{dg^{-1}(y)}{dy}\right\|$ (e.g. y=g(x)= x^3 then x=$g^{-1}(y)$=$\sqrt[3]{y}$)	