Thermodynamics

مرحلة ثانية

الدكتور عباس فاضل محمود

CONVERSION FACTORS

1. **Force:**

$$1 \text{ newton} = \text{kg-m/sec}^2 = 0.012 \text{ kgf}$$

$$1 \text{ kgf} = 9.81 \text{ N}$$

2. **Pressure:**

$$1 \text{ bar} = 750.06 \text{ mm Hg} = 0.9869 \text{ atm} = 10^5 \text{ N/m}^2 = 10^3 \text{ kg/m-sec}^2$$

$$1 \text{ N/m}^2 = 1 \text{ pascal} = 10^{-5} \text{ bar} = 10^{-2} \text{ kg/m-sec}^2$$

$$1 \text{ atm} = 760 \text{ mm Hg} = 1.03 \text{ kgf/cm}^2 = 1.01325 \text{ bar}$$

$$= 1.01325 \times 10^5 \text{ N/m}^2$$

3. **Work, Energy or Heat:**

$$1 \text{ joule} = 1 \text{ newton metre} = 1 \text{ watt-sec}$$

$$= 2.7778 \times 10^{-7} \text{ kWh} = 0.239 \text{ cal}$$

$$= 0.239 \times 10^{-3} \text{ kcal}$$

$$1 \text{ cal} = 4.184 \text{ joule} = 1.1622 \times 10^{-6} \text{ kWh}$$

$$1 \text{ kcal} = 4.184 \times 10^3 \text{ joule} = 427 \text{ kgf-m}$$

$$= 1.1622 \times 10^{-3} \text{ kWh}$$

$$1 \text{ kWh} = 8.6042 \times 10^5 \text{ cal} = 860 \text{ kcal} = 3.6 \times 10^6 \text{ joule}$$

4. **Power:**

$$1 \text{ watt} = 1 \text{ joule/sec} = 0.860 \text{ kcal/h}$$

$$1 \text{ h.p.} = 75 \text{ m kgf/sec} = 0.1757 \text{ kcal/sec} = 735.3 \text{ watt}$$

$$1 \text{ kW} = 1000 \text{ watts} = 860 \text{ kcal/h}$$

5. **Specific heat:**

$$1 \text{ kcal/kg-}°\text{K} = 0.4184 \text{ joules/kg-K}$$

6. **Thermal conductivity:**

$$1 \text{ watt/m-K} = 0.8598 \text{ kcal/h-m-}°C$$

$$1 \text{ kcal/h-m-}°C = 1.16123 \text{ watt/m-K} = 1.16123 \text{ joules/s-m-K}.$$

7. Heat transfer coefficient:

$$1 \text{ watt/m}^2\text{-K} = 0.86 \text{ kcal/m}^2\text{-h-}°C$$

$$1 \text{ kcal/m}^2\text{-h-}°C = 1.163 \text{ watt/m}^2\text{-K}.$$

Nomenclature

Table 1.1 *SI system of units*

Quantity	Unit	Symbol
Basic Units		
Length (L)	Metre	m
Mass (M)	Kilogram	kg
Time (t)	Second	s
Temperature (T)	Kelvin	K
Plane angle	Radian	rad
Solid angle	Steradian	sr
Luminous intensity	Candela	cd
Molecular substance	Mole	mol.
Electric Current	Ampere	A
Derived Units		
Force (F)	Newton	N $\{kg.m/s^2\}$
Energy (E)	Joule	J $\{N.m = kg.\ m^2/s^2\}$
Power	Watt	W $\{J/s = kg.\ m^2/s^3\}$
Pressure	Pascal	Pa $\{N/m^2 = kg/(ms^2)\}$

1. Steam

1.1 Introduction

What is steam? The English language is a little fuzzy here. Some definitions say that any water vapor (H2O in gaseous form instead of liquid) is steam. By that definition, the atmosphere itself is a very low-temperature steam engine. Other definitions say that steam is water vapor if the water vapor happens to be boiling. (Which brings up another question: what does "boiling," mean?)

Finally, a third definition says that steam is "pressurized water vapor", i.e. water vapor at a pressure higher than 1 atmosphere. Those are incompatible definitions, which makes clear thinking difficult.

Steam is the technical term for the gaseous phase of water, which is formed when water boils. Technically speaking, in terms of the chemistry and physics, steam is invisible and cannot be seen; however, in common language it is often used to refer to the visible mist or aerosol of water droplets formed as this water vapor condenses in the presence of (cooler) air. At lower pressures, such as in the upper atmosphere or at the top of high mountains water boils at a lower temperature than the nominal 100 °C (212 °F) at standard temperature and pressure. If heated further it becomes superheated steam.

The enthalpy of vaporization is the energy required to turn water into the gaseous form when it increases in volume by 1,700 times at standard temperature and pressure; this change in volume can be converted into mechanical work by steam engines such as reciprocating piston type engines and steam turbines, which are a sub-group of steam engines. Piston type steam engines played a central role to the Industrial Revolution and modern steam turbines are used to generate more than 80% of the world's electricity. If liquid water comes in contact with a very hot substance (such as lava, or molten metal) it can create a steam explosion. Steam explosions have been responsible for many foundry accidents, and may also have been responsible for much of the damage to the plant in the Chernobyl disaster.

Dryness Fraction

The figure below shows a typical P-v diagram for steam, which you will all be familiar with by now. On the diagram are shown isothermals (lines of constant temperature). Within the Liquid-Vapor mixture region, these lines are horizontal. Any horizontal line on a P-v diagram is an isobar (line of constant pressure). It is therefore clear that if the temperature and pressure of wet steam are stated then it is not enough information to

completely specify the state of the steam. For we will not know the fraction of it which is liquid and what fraction of it is vapor.

The ratio of the mass of vapor to the mass of mixture is called the dryness fraction, x

$$x = \frac{\text{mass of vapour}}{\text{mass of liquid\& vapour mixture}}$$

How is the dryness fraction of the wet steam actually determined in practice? Well, we use a calorimeter.

1.2 Calorimeters

Calorimeters, in general, are to do with measuring heat. Many, such as the bomb calorimeter, are used to determine the amount of heat generated by burning a certain mass of a certain substance. Here we are concerned with those calorimeters which we might use to determine the dryness fraction of wet steam.

1.2 DETERMINATION OF DRYNESS FRACTION OF STEAM

The dryness fraction of steam can be measured by using the following *calorimeters:*
1. Tank or bucket calorimeter
2. Throttling calorimeter

3. Separating and throttling calorimeter.

Tank or Bucket Calorimeter

The dryness fraction of steam can be found with the help of tank calorimeter as follows:

A known mass of steam is passed through a known mass of water and steam is completely

Condensed. The heat lost by steam is equated to heat gained by the water.

Fig. (1) Shows the arrangement of this calorimeter.

The steam is passed through the sampling tube into the bucket calorimeter containing *a known* mass of water.

The weights of calorimeter with water before mixing with steam and after mixing the steam are obtained by weighing.

The temperature of water before and after mixing the steam are measured by *mercury thermometer*.

The pressure of steam passed through the sampling tube is measured with the help of *pressure gauge*.
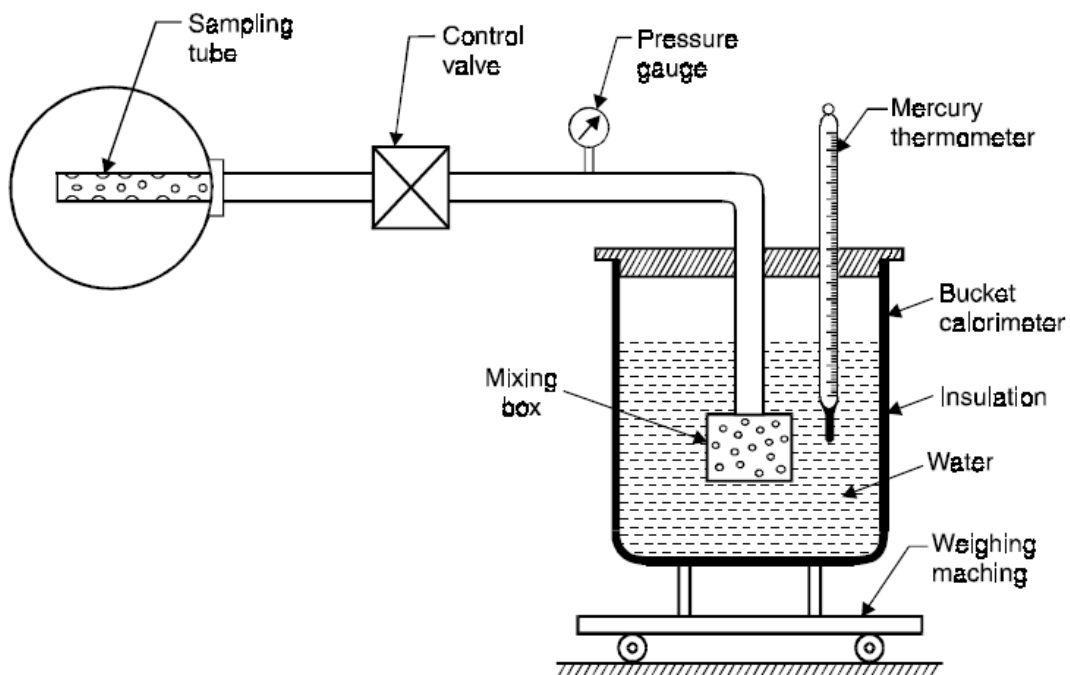
Fig. (1). Tank or bucket calorimeter.

Let,

p_s = Gauge pressure of steam (bar),

p_a = Atmospheric pressure (bar),

t_s = Daturation temperature of steam known from steam table at pressure $(p_s + p_a)$,

h_{fg} = Latent heat of steam,

x = Dryness fraction of steam,

c_{pw} = Specific heat of water,

c_{pc} = Specific heat of calorimeter,

m_c = Mass of calorimeter, kg,

m_{cw} = Mass of calorimeter and water, kg,

$m_w = (m_{cw} - m_c)$ = Mass of water in calorimeter, kg,

m_{cws} = Mass of calorimeter, water and condensed steam, kg,

$m_s = (m_{cws} - m_{cw})$ = Mass of steam condensed in calorimeter, kg,

t_{cw} = Temperature of water and calorimeter before mixing the steam, °C, and

t_{cws} = Temperature of water and calorimeter after mixing the steam, °C.

Neglecting the losses and *assuming* that the *heat lost by steam is gained by water and calorimeter*, we have

$$(m_{cws} - m_{cw}) \, [xh_{fg} + c_{pw} \, (t_s - t_{cws})]$$
$$= (m_{cw} - m_c)c_{pw} \, (t_{cws} - t_{cw}) + m_c \, c_{pc} \, (t_{cws} - t_{cw})$$

$$\therefore \quad m_s[xh_{fg} + c_{pw} \, (t_s - t_{cws})] = (t_{cws} - t_{cw}) \, [m_{cw} - m_c)(c_{pw} + m_c c_{pc}]$$

or
$$m_s[xh_{fg} + c_{pw} \, (t_s - t_{cws})] = (t_{cws} - t_{cw})(m_w c_{pw} + m_c c_{pc})$$

The $m_{cc}pc$ is known as *water equivalent of calorimeter*.

The value of dryness fraction '*x*' can be found by solving the above equation.

The value of dryness fraction found by this method involves some *inaccuracy* since losses due to convection and radiation are *not* taken into account. The calculated value of dryness fraction neglecting losses is *always less* than the actual value of the dryness.

Example (1).

Steam at a pressure of 5 bar passes into a tank containing water where it gets condensed. The mass and temperature in the tank before the admission of steam are 50 kg and 20°C respectively. Calculate the dryness fraction of steam as it enters the tank if 3 kg of steam gets condensed and resulting temperature of the mixture becomes40°C. Take water equivalent of tank as 1.5 kg.

Solution. Pressure of steam, $\qquad p = 5$ bar
Mass of water in the tank $\qquad = 50$ kg
Initial temperature of water $\qquad = 20°C$
Amount of steam condensed, $\qquad m_s = 3$ kg
Final temperature after condensation of steam $= 40°C$
Water equivalent of tank $\qquad = 1.5$ kg
Dryness fraction of steam, x = ?

At 5 bar. From steam tables,

$$h_f = 640.1 \text{ kJ/kg} ; \dot{h}_{fg} = 2107.4 \text{ kJ/kg}$$

Total mass of water, $m_w =$ mass of water in the tank + water equivalent of tank

$$= 50 + 1.5 = 51.5 \text{ kg}$$

Also, heat lost by steam = heat gained by water

$$m_s [(h_f + x h_{fg}) - 1 \times 4.18 (40 - 0)] = m_w [1 \times 4.18 (40 - 20)]$$

or $\qquad 3[(640.1 + x \times 2107.4) - 4.18 \times 40] = 51.5 \times 4.18 \times 20$

or $\qquad 3(472.9 + 2107.4x) = 4305.4$

or $\qquad 472.9 + 2107.4x = 1435.13$

∴ $\qquad x = \dfrac{1435.13 - 472.9}{2107.4} = 0.456.$

Hence **dryness fraction of steam = 0.456. Ans.**

Example (2).

Steam at a pressure of 1.1 bar and 0.95 dry is passed into a tank containing 90 kg of water at 25°C. The mass of tank is 12.5 kg and specific heat of metal is 0.42 kJ/kg K. If the temperature of water rises to 40°C after the passage of the steam, determine the mass of steam condensed. Neglect radiation and other losses.

Solution. Pressure of steam, $\qquad p = 1.1$ bar
Dryness fraction of steam, $\qquad x = 0.95$
Mass of water in the tank $\qquad = 90$ kg
Initial temperature of water in the tank $= 25°C.$
Mass of tank $\qquad = 12.5$ kg
Specific heat of metal $\qquad = 0.42$ kJ/kg K.
Final temperature of water $\qquad = 40°C.$
Mass of steam condensed, $\qquad \mathbf{m_s} = ?$
Since the radiation losses are neglected

∴ Heat lost by steam = heat gained by water

or $\qquad m_s[(h_f + x h_{fg}) - 1 \times 4.18 (40 - 0)] = m[1 \times 4.18(40 - 25)]$

But $\qquad m = m_1 + m_2$

where $\qquad m_1 =$ mass of cold water in the vessel before steam supply

$\qquad m_2 =$ water equivalent of vessel $= 0.42 \times 12.5 = 5.25$ kg

At 1.1 bar. From steam tables,

$$h_f = 428.8 \text{ kJ/kg} ; h_{fg} = 2250.8 \text{ kJ/kg}$$

∴ $\qquad m_s [(428.8 + 0.95 \times 2250.8) - 1 \times 4.18 \times 40]$

$$= (90 + 5.25) [1 \times 4.18 \times (40 - 25)]$$

$$m_s [2567.06 - 167.20] = 95.25 \times 62.7$$

i.e., $\qquad 2399.86 m_s = 5972.17$

∴ $\qquad m_s = 2.488$ kg

Hence, **mass of steam condensed = 2.488 kg. Ans.**

Throttling Calorimeter

The dryness fraction of wet steam can be determined by using a throttling calorimeter, which is illustrated diagrammatically in Fig. (2).
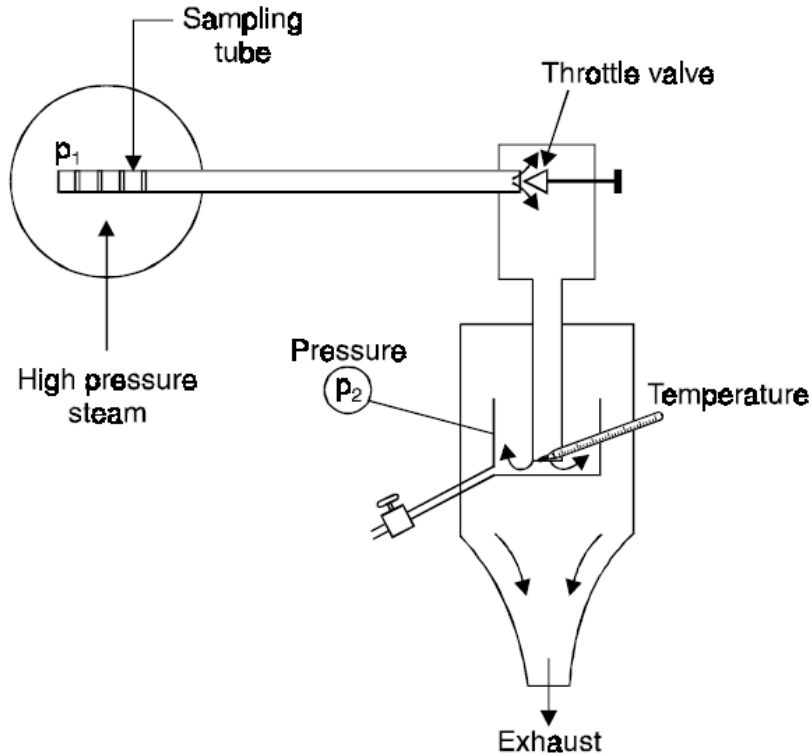

Fig. (2). Throttling calorimeter.

The steam to be sampled is taken from the pipe by means of suitable positioned and dimensioned sampling tube. It passes into an insulated container and is throttled through an orifice to atmospheric pressure. Here the temperature is taken and the steam ideally should have about 5.5 K of superheat.

The throttling process is shown on *h-s* diagram in Fig. (3) by the line 1-2. If steam initially wet is throttled through a sufficiently large pressure drop, then the steam at state 2 will become superheated. State 2 can then be defined by the *measured pressure and temperature*. The enthalpy, *h2* can then be found and hence

$$h_2 = h_1 = (h_{f1} + x_1 h_{fg1}) \qquad \text{at } p_1$$

$$where \ h_2 = h_{f2} + h_{fg2} + c_{ps}(T_{sup2} - T_{s2})$$

$$x_1 = \frac{h_2 - h_{f1}}{h_{fg1}}$$
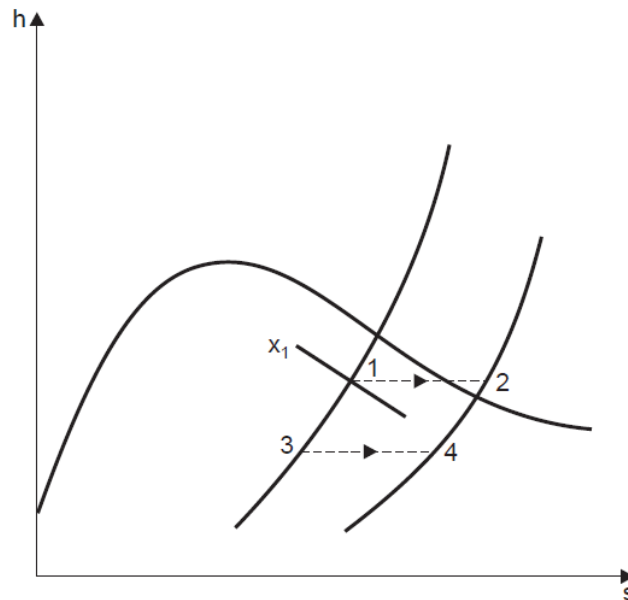
Fig. (3). Throttling process.

Hence, the dryness fraction is determined and state 1 is defined.

Example (3).

A throttling calorimeter is used to measure the dryness fraction of the steam in the steam main, which has steam flowing at a pressure of 8 bar. The steam after passing through the calorimeter is at 1 bar pressure and 115°C.

Solution. Condition of steam *before throttling* :
$$p_1 = 8 \text{ bar}, \ x_1 = ?$$
Condition of steam *after throttling* :
$$p_2 = 1 \text{ bar}, \ t_2 = t_{sup_2} = 115°C$$
As throttling is a *constant enthalpy* process
$$\therefore \qquad h_1 = h_2$$
i.e. $\quad h_{f_1} + x_1 h_{gf_1} = h_{f_2} + h_{fg_2} + c_{ps}(T_{sup_2} - T_{s_2})$ $\qquad [\because \ T_{sup_2} = 115 + 273 = 388 \text{ K}$
$$T_{s_2} = 99.6 + 273 = 372.6 \text{ K (at 1 bar)}]$$

$$720.9 + x_1 \times 2046.5 = 417.5 + 2257.9 + 2.1(388 - 372.6)$$
$$720.9 + 2046.5 \, x_1 = 2707.7$$
$$\therefore \qquad x_1 = \frac{2707.7 - 720.9}{2046.5} = 0.97$$

Separating and Throttling Calorimeter

If the steam whose dryness fraction is to be determined is *very wet* then throttling to atmospheric pressure *may not be sufficient to ensure superheated steam at exit*. In this case, it is *necessary to dry the steam partially, before throttling*. This is done by *passing the steam sample from the main through a separating calorimeter* as shown in Fig. (4).

the steam is made to change direction suddenly, and the water, being denser than the dry steam is separated out. The quantity of water, which is separated out (mw), is measured at the separator, the steam remaining, which *now has a higher dryness fraction*, is passed through the *throttling calorimeter*. With the combined separating and throttling calorimeter it is *necessary* to condense the steam after throttling and measure the amount of condensate (m_s). If a throttling calorimeter only is sufficient, there is no need to measure condensate, the pressure and temperature measurements at exit being sufficient.

Dryness fraction at 2 is x_2, therefore, the mass of dry steam leaving the separating calorimeter is equal to $x_2 m_s$ and this must be the mass of dry vapour in the sample drawn from the main at state 1.
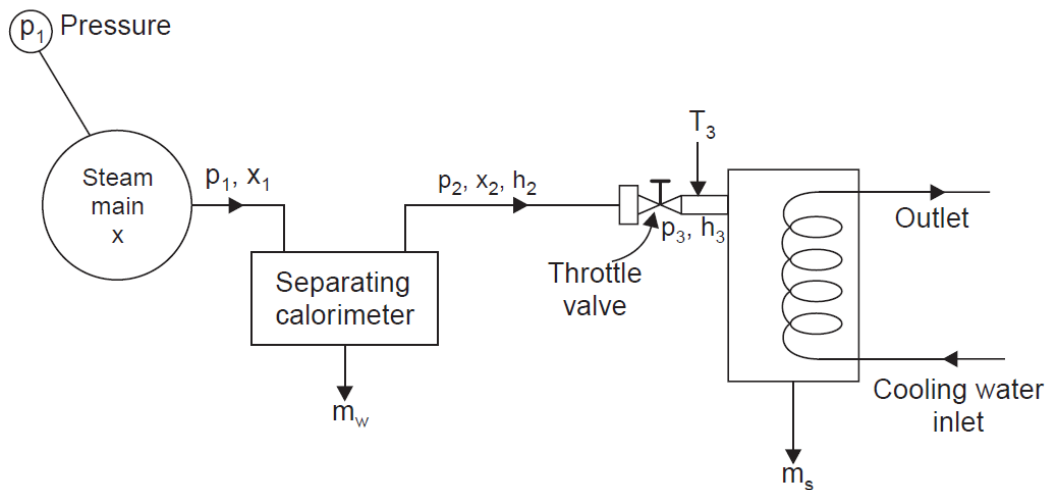
Fig. (4). Separating and throttling calorimeter.

$$x_1 = \frac{\text{Mass of dry vapour}}{\text{Total mass}} = \frac{x_2\, m_s}{m_w + m_s}$$

The dryness fraction, x_2, can be determined as follows:

$$h_3 = h_2 = h_{f2} + x_2 h_{fgs} \quad at \ p_2 [h_3 = h_{fs} + h_{fgs} +$$
$$c_{ps}\left(T_{sup3} - T_{s3}\right) \text{ at pressure } p_3]$$

Or
$$\boldsymbol{x_2 = \frac{h_3 - h_{f2}}{h_{fg2}}}$$

The values of h_{f2} and hf_{g2} are read from steam tables at pressure p_2. The pressure in the separator is small so that p_1 is approximately equal to p_2.

Example (4).

The following observations were taken with a separating and a throttling calorimeter arranged in series:

Water separated = 2 kg, steam discharged from the throttling calorimeter = 20.5 kg, temperature of steam after throttling = 110°C, initial pressure = 12 bar abs., barometer = 760 mm of Hg, final pressure = 5 mm of Hg

Estimate the quality of steam supplied.

Solution. Quantity of water separated out, $m_w = 2$ kg

Steam (condensate) discharged from the throttling calorimeter, $m_s = 20.5$ kg

Temperature of steam after throttling, $t_{sup} = 110$°C

Initial pressure of steam, $p_1 = 12$ bar abs.

Final pressure of steam, $p_3 = 760 + 5 = 765$ mm

$$= \frac{765}{1000} \times 1.3366 \qquad (\because \ 1 \text{ m Hg} = 1.3366 \text{ bar})$$

$$\simeq 1 \text{ bar}$$

From steam tables :

At $p_1 = p_2 = 12$ **bar.** $h_f = 798.4$ kJ/kg, $h_{fg} = 1984.3$ kJ/kg

At $p_3 = 1$ **bar.** $t_s = 99.6$°C, $h_f = 417.5$ kJ/kg, $h_{fg} = 2257.9$ kJ/kg

$t_{sup} = 110$°C (given)

Also $\qquad h_3 = h_2$

$$(h_{f_3} + h_{fg_3}) + c_{ps}(T_{sup_3} - T_{s_3}) = h_{f_2} + x_2 h_{fg_2}.$$

Taking $\qquad c_{ps} = 2$ **kJ/kg K,** we get

$$417.5 + 2257.9 + 2[(110 + 273) - (99.6 + 273)] = 798.4 + x_2 \times 1984.3$$

$$2696.2 = 798.4 + 1984.3 \, x_2$$

$$\therefore \qquad x_2 = \frac{2696.2 - 798.4}{1984.3} = 0.956$$

Now, quality of steam supplied,

$$x_1 = \frac{x_2 m_s}{m_w + m_s} = \frac{0.956 \times 20.5}{2 + 20.5} = 0.87. \quad \textbf{Ans.}$$

2. Vapor Power Cycles

RANKINE CYCLE: THE IDEAL CYCLE FOR VAPOR POWER CYCLES

Many of the impracticalities associated with the Carnot cycle can be eliminated by superheating the steam in the boiler and condensing it completely in the condenser, as shown schematically on a *T-s* diagram in Fig. …. The cycle that results is the **Rankine cycle**, which is the ideal cycle for vapor power plants. The ideal Rankine cycle does not involve any internal irreversibilities and consists of the following four processes:

1-2 Isentropic compression in a pump

2-3 Constant-pressure heat addition in a boiler

3-4 Isentropic expansion in a turbine

4-1 Constant-pressure heat rejection in a condenser
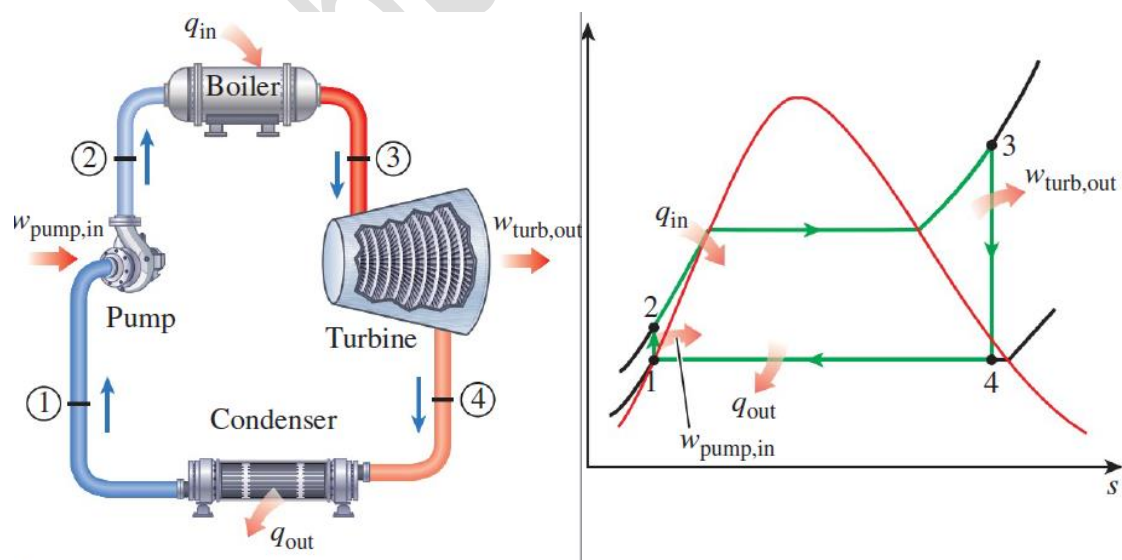
FIGURE 5
The simple ideal Rankine cycle.

$$(q_{in} - q_{out}) + (w_{in} - w_{out}) = h_e - h_i \ (kJ/kg)$$

Pump $(q = 0)$: $w_{pump,in} = h_2 - h_1$
or,
$w_{pump,in} = v (P_2 - P_1)$
where

$h_1 = h_{f\,@\,P1}$ and $v \cong v_1 = v_{f\,@\,P1}$

Boiler $(w = 0)$: $q_{in} = h_3 - h_2$

Turbine $(q = 0)$: $w_{turb,out} = h_3 - h_4$

Condenser $(w = 0)$: $q_{out} = h_4 - h_1$

The *thermal efficiency* of the Rankine cycle is determined from

$$\eta_{th} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}}$$

where

$$w_{net} = q_{in} - q_{out} = w_{turb,out} - w_{pump,in}$$

HOW CAN WE INCREASE THE EFFICIENCY OF THE RANKINE CYCLE?

Steam power plants are responsible for the production of most electric power in the world, and even small increases in thermal efficiency can mean large savings from the fuel requirements. Therefore, every effort is made to improve the efficiency of the cycle on which steam power plants operate.

The basic idea behind all the modifications to increase the thermal efficiency of a power cycle is the same: *Increase the average temperature at which heat is transferred to the working fluid in the boiler, or decrease the average temperature at which heat is rejected from the working fluid in the condenser.* That is, the average fluid temperature should be as high as possible during heat addition and as low as possible during heat rejection.

Next we discuss three ways of accomplishing this for the simple ideal Rankine cycle.

LOWERING THE CONDENSER PRESSURE (LOWERS TLOW,AVG)

Steam exists as a saturated mixture in the condenser at the saturation temperature corresponding to the pressure inside the condenser. Therefore, lowering the operating pressure of the condenser automatically lowers the temperature of the steam, and thus the temperature at which heat is rejected.
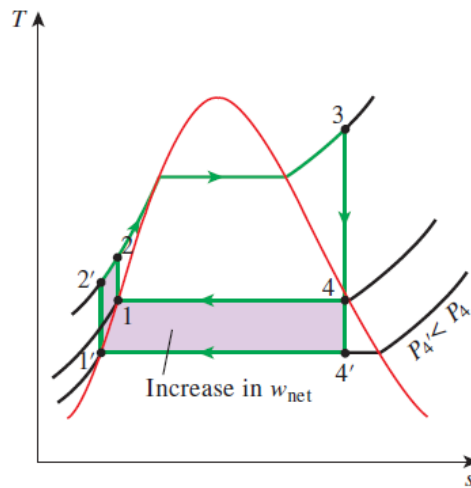
FIGURE 6
effect of lowering the condenser pressure on the ideal Rankine cycle.

SUPERHEATING THE STEAM TO HIGH TEMPERATURES (INCREASES THIGH,AVG)

The average temperature at which heat is transferred to steam can be increased without increasing the boiler pressure by superheating the steam to high temperatures. The effect of superheating on the performance of vapor power cycles is illustrated on a *T-s* diagram in Fig. …. The colored area on this diagram represents the increase in the network. The total area under the process curve 3-3′ represents the increase in the heat input. Thus both the network and heat input increase as a result of superheating the steam to a

higher temperature. The overall effect is an increase in thermal efficiency, however, since the average temperature at which heat is added increases.
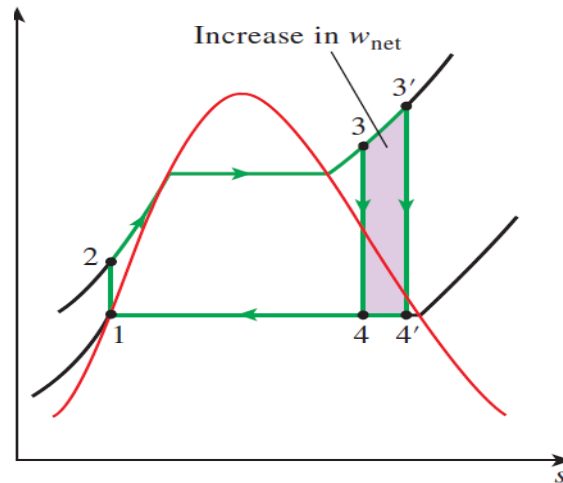
FIGURE 7
The effect of superheating the steam to higher temperatures on the ideal Rankine cycle.

THE IDEAL REHEAT RANKINE CYCLE

We noted in Sec. 10–4 that increasing the boiler pressure increases the thermal efficiency of the Rankine cycle, but it also increases the moisture content of the steam to unacceptable levels. Then it is natural to ask the following question:

How can we take advantage of the increased efficiencies at higher boiler pressures without facing the problem of excessive moisture at the final stages of the turbine?

Two possibilities come to mind:

1. Superheat the steam to very high temperatures before it enters the turbine. This would be the desirable solution since the average temperature at which heat is added would also increase, thus increasing the cycle efficiency.

This is not a viable solution, however, since it requires raising the steam temperature to metallurgically unsafe levels.

2. Expand the steam in the turbine in two stages, and reheat it in between. In other words, modify the simple ideal Rankine cycle with a **reheat** process.

Reheating is a practical solution to the excessive moisture problem in turbines, and it is commonly used in modern steam power plants. The *T-s* diagram of the ideal reheat Rankine cycle and the schematic of the power plant operating on this cycle are shown in Fig..... The ideal reheat Rankine cycle differs from the simple ideal Rankine cycle in that the expansion process takes place in two stages. In the first stage (the high-pressure turbine), steam is expanded isentropically to an intermediate pressure and sent back to the boiler, where it is reheated at constant pressure, usually to the inlet temperature of the first turbine stage. Steam then expands isentropically in the second stage (low-pressure turbine) to the condenser pressure. Thus the total heat input and the total turbine work output for a reheat cycle become

$$q_{in} = q_{primary} + q_{reheat} = (h_3 - h_2) + (h_5 - h_4)$$
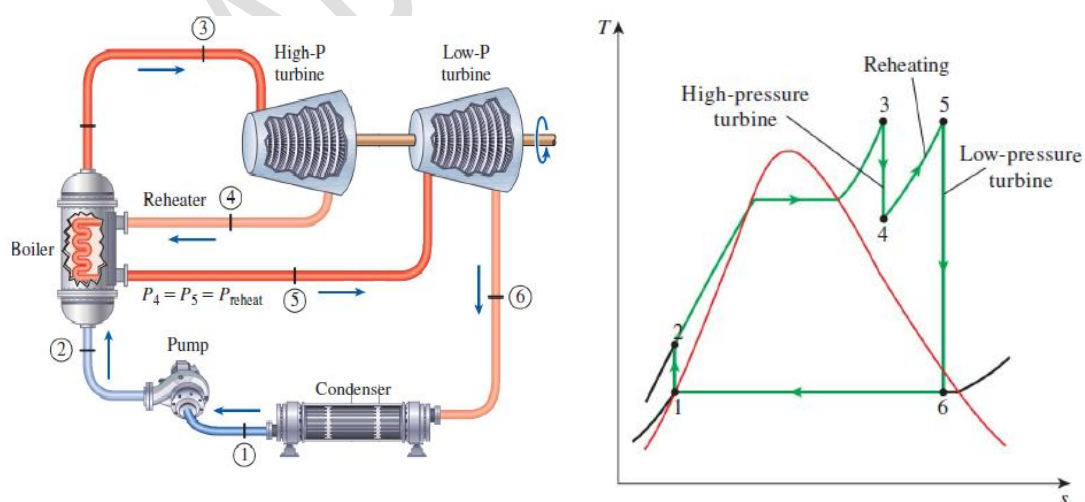
FIGURE 8
The ideal reheat Rankine cycle.

$$W_{\text{turb,out}} = W_{\text{turb,I}} + W_{\text{turb,II}} = (h_3 - h_4) + (h_5 - h_6)$$

The incorporation of the single reheat in a modern power plant improves the cycle efficiency by 4 to 5 percent by increasing the average temperature at which heat is transferred to the steam.

THE IDEAL REGENERATIVE RANKINE CYCLE

A careful examination of the *T-s* diagram of the Rankine cycle redrawn in Fig. 10–15 reveals that heat is transferred to the working fluid during process 2-2′ at a relatively low temperature. This lowers the average heat-addition temperature and thus the cycle efficiency.

To remedy this shortcoming, we look for ways to raise the temperature of the liquid leaving the pump (called the *feedwater*) before it enters the boiler. One such possibility is to transfer heat to the feedwater from the expanding steam in a counterflow heat exchanger built into the turbine, that is, to use **regeneration**. This solution is also impractical because it is difficult to design such a heat exchanger and because it would increase the moisture content of the steam at the final stages of the turbine.
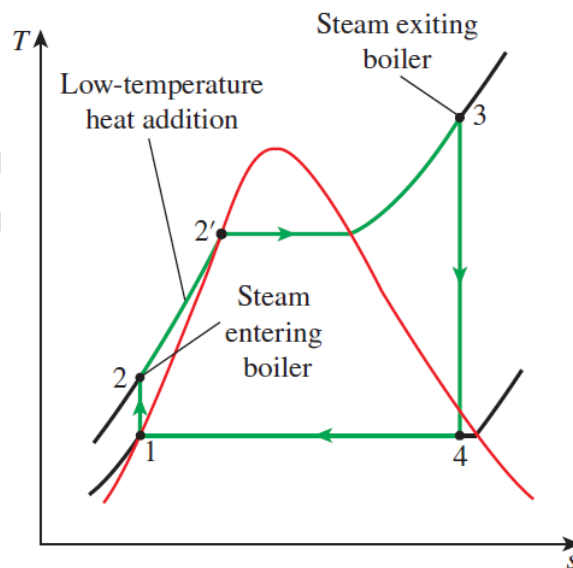
FIGURE 9
The first part of the heat-addition process in the boiler takes place at relatively low temperatures.

OPEN FEEDWATER HEATERS

An **open** (or **direct-contact**) **feedwater heater** is basically a *mixing chamber,* where the steam extracted from the turbine mixes with the feedwater exiting the pump. Ideally, the mixture leaves the heater as a saturated liquid at the heater pressure. The schematic of a steam power plant with one open feedwater heater (also called *single-stage regenerative cycle*) and the *T-s* diagram of the cycle are shown in Fig. 10–16. In an ideal regenerative Rankine cycle, steam enters the turbine at the boiler pressure (state 5) and expands isentropically to an intermediate pressure (state 6). Some steam is extracted at this state and routed to the feedwater heater, while the remaining steam continues to expand isentropically to the condenser pressure (state 7). This steam leaves the condenser as a saturated liquid at the condenser pressure (state 1). The condensed water, which is also called the *feedwater,* then enters an isentropic pump, where it is compressed to the feedwater heater pressure (state 2) and is routed to the feedwater heater, where it mixes with the steam extracted from the turbine. The fraction of the steam extracted is such that the mixture leaves the heater as a saturated liquid at the heater pressure (state 3). A second pump raises the pressure of the water to the boiler pressure (state 4). The cycle is completed by heating the water in the boiler to the turbine inlet state (state 5).

In the analysis of steam power plants, it is more convenient to work with quantities expressed per unit mass of the steam flowing through the boiler. For each 1 kg of steam leaving the boiler, y kg expands partially in the turbine and is extracted at state 6. The remaining $(1 - y)$ kg expands completely to the condenser pressure. Therefore, the mass flow rates are different in different components. If the mass flow rate through the boiler is $\dot{m}$, for example, it is $(1 - y)\dot{m}$ through the condenser. This aspect of the regenerative Rankine cycle should be considered in the analysis of the cycle as well as in the interpretation of the areas on the *T−s* diagram. In light of Fig. 10–16, the heat

and work interactions of a regenerative Rankine cycle with one feedwater heater can be expressed per unit mass of steam flowing through the boiler as follows:
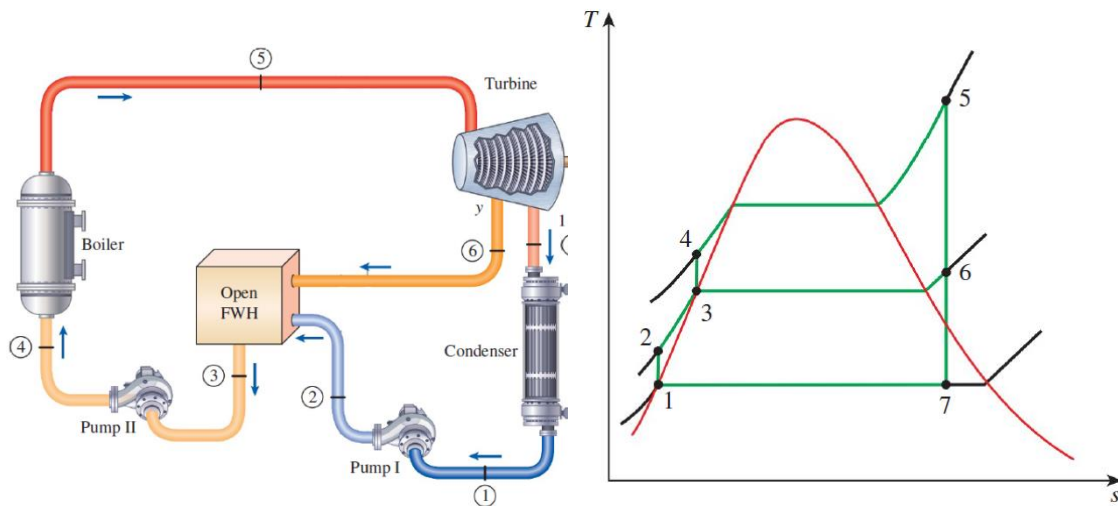
The ideal regenerative Rankine cycle with an open feedwater heater.

$\dot{m}_2 + \dot{m}_6 = \dot{m}_3$

Satisfied with the *extraction fraction* as

$y = \dot{m}_6/\dot{m}_5$

so

$\dot{m}_7 = (1 - y)\,\dot{m}_5 = \dot{m}_1 = \dot{m}_2$

The energy equation with no external heat transfer and no work becomes

$\dot{m}_2 h_2 + \dot{m}_6 h_6 = \dot{m}_3 h_3$

into which we substitute the mass flow rates ($\dot{m}_3 = \dot{m}_5$) as

$(1 - y)\,\dot{m}_5 h_2 + y\dot{m}_5 h_6 = \dot{m}_5 h_3$

We take state 3 as the limit of saturated liquid (we do not want to heat it further, as it would

move into the two-phase region and damage the pump *P*2) and then solve for

y:

$$y = \frac{h3 - h2}{h6 - h2}$$

$q_{in} = h_5 - h_4$

$q_{out} = (1 - y)(h_7 - h_1)$

$W_{turb,out} = (h_5 - h_6) + (1 - y)(h_6 - h_7)$

$W_{pump,in} = (1 - y)W_{pump\ I,in} + W_{pump\ II,in}$

where

$y = \dot{m}_6 / \dot{m}_5$ **(fraction of steam extracted)**

$W_{pump\ I,in} = v_1(P_2 - P_1)$

$W_{pump\ II,in} = v_3(P_4 - P_3)$

The thermal efficiency of the Rankine cycle increases as a result of regeneration. This is because regeneration raises the average temperature at which heat is transferred to the steam in the boiler by raising the temperature of the water before it enters the boiler. The cycle efficiency increases further as the number of feedwater heaters is increased. Many large plants in operation today use as many as eight feedwater heaters. The optimum number of feedwater heaters is determined from economic considerations. The use of an additional feedwater heater cannot be justified unless it saves more in fuel costs than its own cost.

CLOSED FEEDWATER HEATERS

Another type of feedwater heater often used in steam power plants is the **closed feedwater heater**, in which heat is transferred from the extracted steam to the feedwater without any mixing taking place. The two streams now can be at different pressures, since they do not mix. The schematic of a steam power plant with one closed feedwater heater and the *T-s* diagram of the cycle are shown in Fig. 10–17. In an ideal closed feedwater heater, the feedwater is heated to the exit temperature of the extracted steam, which ideally leaves the heater as a saturated liquid at the extraction pressure. In actual power plants, the feedwater leaves the heater below the exit temperature of the extracted

steam because a temperature difference of at least a few degrees is required for any effective heat transfer to take place.
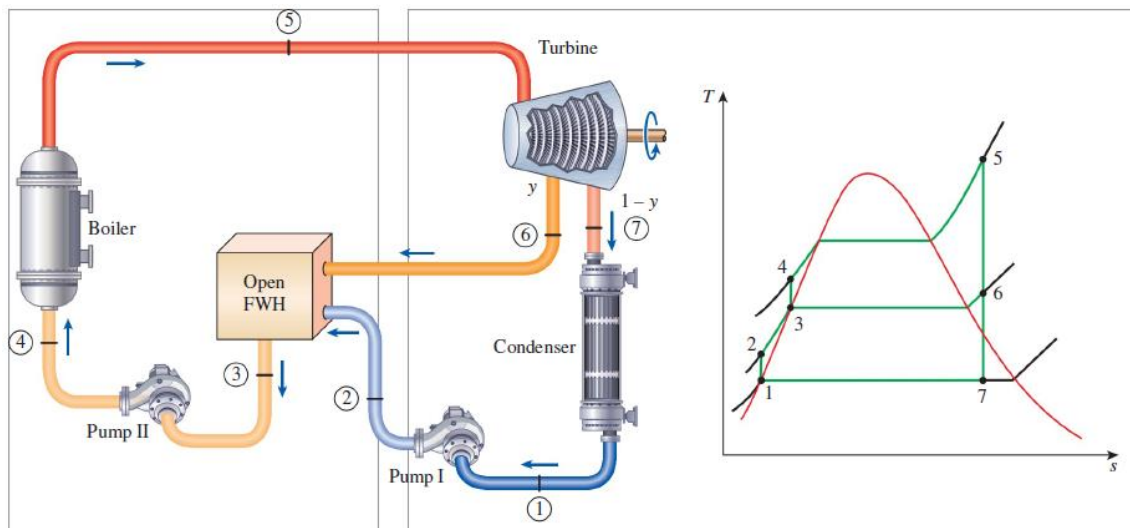
FIGURE 11
The ideal regenerative Rankine cycle with a closed feedwater heater.

The condensed steam is then either pumped to the feedwater line or routed to another heater or to the condenser through a device called a **trap**. A trap allows the liquid to be throttled to a lower-pressure region but *traps* the vapor. The enthalpy of steam remains constant during this throttling process. The open and closed feedwater heaters can be compared as follows: Open feedwater heaters are simple and inexpensive and have good heat transfer characteristics. They also bring the feedwater to the saturation state. For each heater, however, a pump is required to handle the feedwater. The closed feedwater heaters are more complex because of the internal tubing network, and thus they are more expensive. Heat transfer in closed feedwater heaters is also less effective since the two streams are not allowed to be in direct contact. However, closed feedwater heaters do not require a separate pump for each heater since the extracted steam and the feedwater can be at different pressures. Most steam power plants use a combination of open and closed feedwater heaters, as shown in Fig. ….
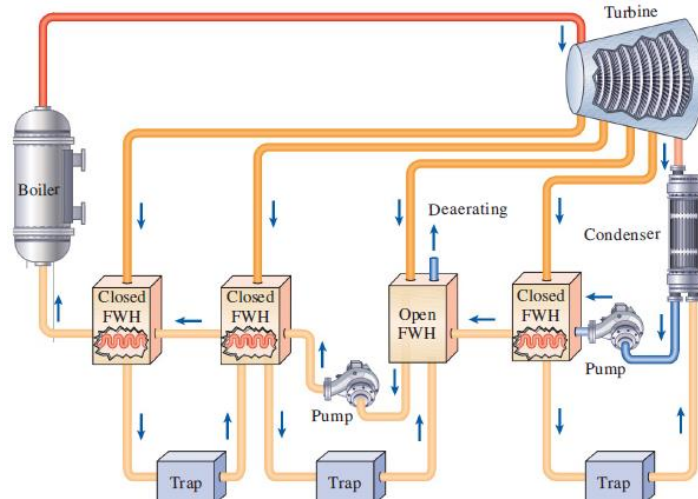
FIGURE 12
A steam power plant with one open and three closed feedwater heaters.

EXAMPLE

The Ideal Regenerative Rankine Cycle Consider a steam power plant operating on the ideal regenerative Rankine cycle with one open feedwater heater. Steam enters the turbine at 15 MPa and 600°C and is condensed in the condenser at a pressure of 10 kPa. Some steam leaves the turbine at a pressure of 1.2 MPa and enters the open feedwater heater. Determine the fraction of steam extracted from the turbine and the thermal efficiency of the cycle.
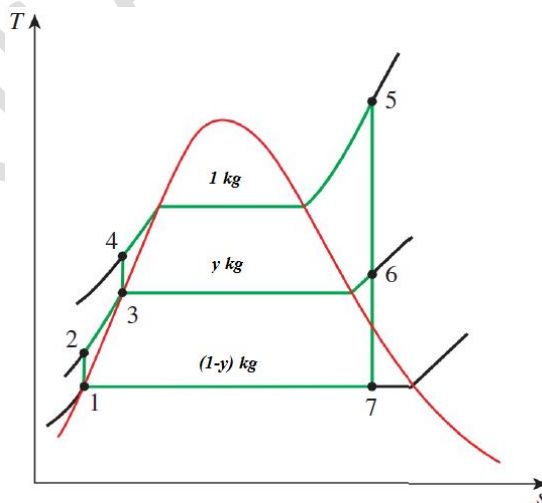
EXAMPLE

Consider a steam power plant that operates on an ideal reheat–regenerative Rankine cycle with one open feedwater heater, one closed feedwater heater, and one reheater.

Steam enters the turbine at 15 MPa and 600°C and is condensed in the condenser at a pressure of 10 kPa. Some steam is extracted from the turbine at 4 MPa for the closed feedwater heater, and the remaining steam is reheated at the same pressure to600°C. The extracted steam is completely condensed in the heater and is pumped to15 MPa before it mixes with the feedwater at the same pressure. Steam for the open feedwater heater is extracted from the low-pressure turbine at a pressure of 0.5 MPa.

Determine the fractions of steam extracted from the turbine as well as the thermal efficiency of the cycle.

COGENERATION
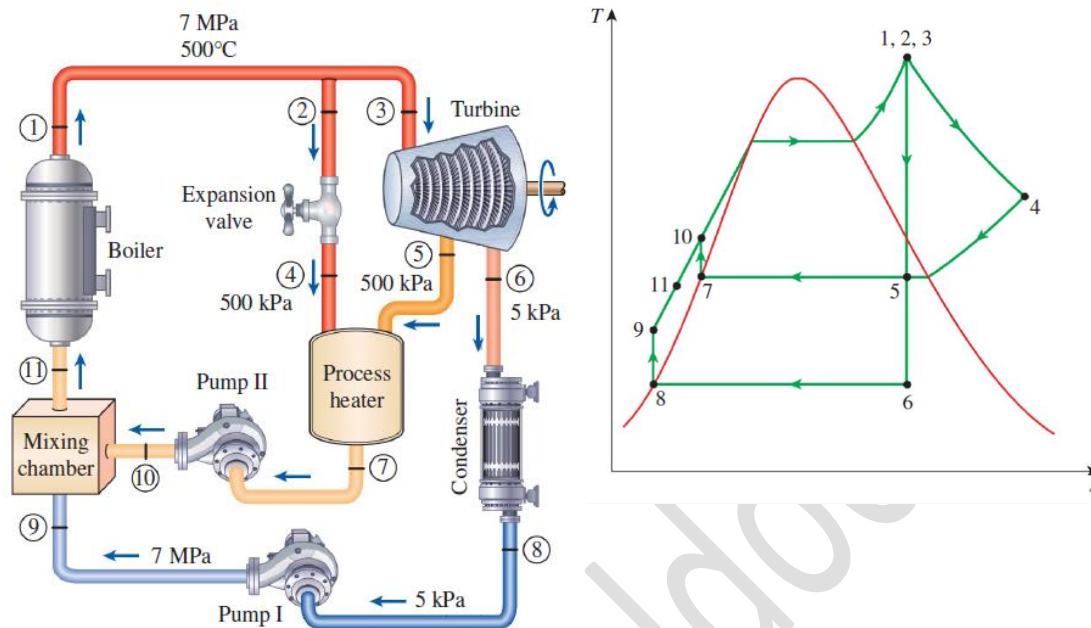
FIGURE 13
Schematic and *T-s* diagram.

There are many occasions in industrial settings where the need arises for a specific source or supply of energy within the environment in which a steam power plant is being used to generate electricity. In such cases, it is appropriate to consider supplying this source of energy in the form of steam that has already been expanded through the high-pressure section of the turbine in the power plant cycle, thereby eliminating the construction and use of a second boiler or other energy source. Such an arrangement is shown in Fig. ...., in which the turbine is tapped at some intermediate pressure to furnish the necessary amount of process steam required for the particular energy need— perhaps to operate a special process in the plant, or in many cases simply for the purpose of space heating the facilities.

This type of application is termed *cogeneration*. If the system is designed as a package with both the electrical and the process steam requirements in mind, it

is possible to achieve substantial savings in the capital cost of equipment and in the operating cost through careful consideration of all the requirements and optimization of the various parameters involved.

Specific examples of cogeneration systems are considered in the problems at the end of the chapter.

3. High speed flow

A compressible flow is that flow in which the density of the fluid changes during flow. All real fluids are compressible to some extent and therefore their density will change with change in pressure or temperature. If the relative change in density Δρ/ρ is small, the fluid can be treated as incompressible. A compressible fluid, such as air, can be considered as incompressible with constant ρ if changes in elevation are small, acceleration is small, and/or temperature changes are negligible. In other words, if Mach's number U/C, where C is the sonic velocity, is small, compressible fluid can be treated as incompressible.

• The gases are treated as compressible fluids and study of this type of flow is often referred to as 'Gas dynamics'.
• Some important problems where compressibility effect has to be considered are:
(i) Flow of gases through nozzles, orifices;
(ii) Compressors;
(iii) Flight of aero planes and projectiles moving at higher altitudes;
(iv) Water hammer and acoustics.
• Compressibility' affects the drag coefficients of bodies by formation of shock waves, discharge coefficients of measuring devices such as orificemeters, venturimeters and pitot tubes, stagnation pressure and flows in converging-diverging sections.

The Mach number

The theory of high-speed flow is concerned with flows of fluid at speeds high enough that account must be taken of the fluid's compressibility. The theory finds application in many branches of science and technology, from which we may single out, as being of unrivalled importance in the modern world, the applications to high-speed flight.
The dimensionless parameter that measures the importance of a fluid's compressibility in high-speed flow is the *Mach number*. Suppose that, at a given point in space and time, the speed of the fluid is *u* and the speed of sound is *c*. The Mach number *M* is defined as the ratio *u* /*c*. Thus

$$M = \frac{u}{c}$$

 In general, the value of the Mach number varies with position and time. However, in many problems, we may choose a representative flow speed, say *U*, and a representative sound speed, say *c*. Then the quantity *U/c* is a single number measuring the importance of compressibility in the flow, and we may say that the flow is taking place at a Mach number *M* = *U/c*.

Flow Regimes

Flows corresponding to different ranges of the Mach number *M* have very different properties. We shall distinguish five regimes, namely (a) incompressible flow, (b) subsonic flow, (c) transonic flow, (d) supersonic flow, and (e) hypersonic flow.

(a) Incompressible Flow

A flow is said to be incompressible if the *density* of a fluid element does not change during its motion. It is a property of the flow and not of the fluid. The rate of change of density of a material fluid element is given by the material derivative

$$\frac{D\rho}{Dt} = 0.$$

Hence, the flow is incompressible if the divergence of the velocity field is identically zero. Note that the density field need not be uniform in an incompressible flow. All that is required is that the density of a fluid element should not change in time as it moves through space. For example, flow in the ocean can be considered to be incompressible even though the density of water is not uniform due to stratification.

Compressible flow can with good accuracy be approximated as incompressible for steady flow if the *Mach number* is below 0.3 .

$$\text{Ma} = \frac{V}{a} < 0.3$$

(b) Subsonic Flow

This regime is defined by $0 < M < 1$, subject to the restriction that *M* is not too close to 0 or 1. The flow speed is high enough for the fluid's compressibility to be important, but low enough for the speed to be comfortably clear of the speed of sound. Since most aircraft fly well below the speed of sound, the regimes of incompressible flow and subsonic flow include most of standard aeronautics.
The subsonic regime includes much of acoustics.

(c) Transonic Flow

This regime is defined by *M* being close to 1 in some important part of the flow. The regime raises difficult and interesting mathematical questions, because the governing partial differential equations are then of mixed type. That is, in some regions, the equations are elliptic, in other regions, they are hyperbolic, and on the separating lines or surfaces, they are parabolic. The transonic regime is of vital importance to an aircraft or a land vehicle that "breaks the sound barrier."

(d) Supersonic Flow

This regime is defined by $M > 1$, subject to the restriction that *M* is not too close to

1 nor too large. Parts of the mathematical theory of supersonic flow can be obtained from that for subsonic flow by replacing $(1 \gg M^2)^{1/2}$ and an elliptic equation by $(M^2 c\ 1)^{1/2}$ and a hyperbolic equation. The theory of the supersonic regime was of importance to the design of the civil supersonic airplane *Concorde.*

(e)Hypersonic Flow

This regime is defined by $M \gg 1$ the flow speed is so high that compressibility is all-important, particularly in producing very high temperatures and ionization. The hypersonic regime is of importance for rockets and for the civil hypersonic aircraft "Orient Express."

Ernst Mach (1838–1916) was a physicist and philosopher who established the basic principles of modern scientific thought. In 1864, Mach became professor of mathematics at the University of Graz in Austria. He spent his most productive years as a professor of physics at Charles University in Prague, from 1867 to 1895. In1881, Mach began a study of the flight of artillery shells using the new technology of photography. In this research, he discovered that the angle θ of the shock cone radiating from the leading edge of a supersonic object was related to the speed of sound c and the velocity of the object V by $\sin \theta = c/V$ (see Figure), and θ was later called the Mach angle.

The ratio of the local fluid velocity V to the speed of sound in the fluid c came to be of fundamental value in the study of high-speed aerodynamics, and after1930, it was called the Mach number ($M = V/c$).
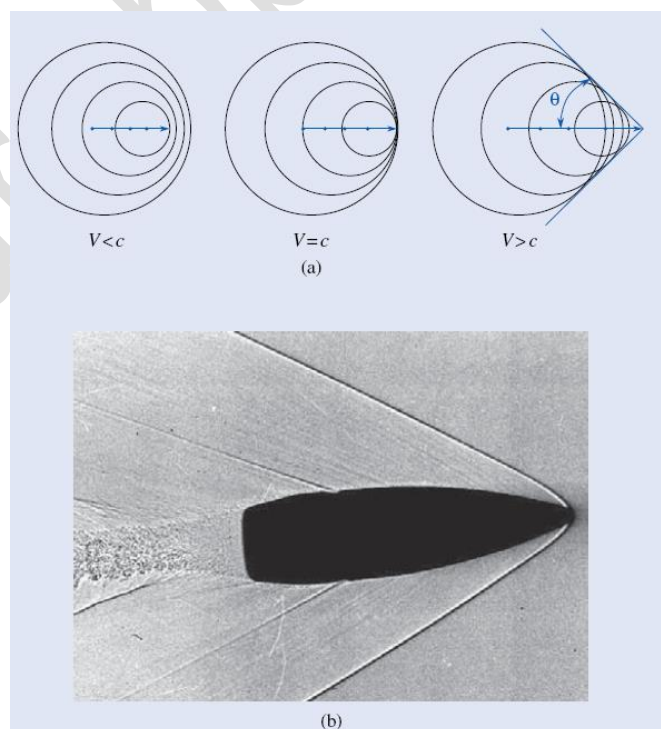
(a) The representation of a sound waves produced when an object moves from subsonic ($V < c$) to sonic ($V = c$) to supersonic ($V > c$) velocity. Generally, a conical shock wave sweeps back from the leading edge of the object with the cone angle proportional to the ratio of the sonic velocity of the fluid to the supersonic velocity of the object, c/V. (b) The shadowgraph of a supersonic bullet showing shock waves.

(f) Stagnation properties

The stagnation state of a moving fluid is the state it would achieve if it underwent an adiabatic, aergonic deceleration to zero velocity. The energy rate balance (ERB) for an adiabatic, aergonic, steady state, steady flow, singleinlet, single-outlet open system with negligible change in flow stream potential energy reduces to

$$h_{in} + V_{in}^2/(2g_c) = h_{out} + V_{out}^2/(2g_c)$$

$$V^2/(2g_c) = h_o - h = c_p(T_o - T)$$

If we let the subscript o refer to the stagnation (or zero velocity) state, then $V_0 = 0$, and the preceding equation can be used to define the stagnation specific enthalpy ho as

$$h_o = h + V^2/(2g_c)$$

For an ideal gas or a low-pressure vapor with constant specific heats, this equation can be written as

or
$$\frac{T_o}{T} = 1 + \frac{V^2}{2g_c c_p T}$$

Where T_0 is the stagnation temperature (the temperature at zero velocity).

EXAMPLE

While driving in your new sports car at 90.0 km/h in still air at 20.0°C, you put your hand out the window with your palm toward the front of the car. What is the air temperature on the center of your palm?

Solution

First, draw a sketch of the system (Figure 16.1).

When your hand is placed perpendicular to the air flow, you should feel the stagnation pressure and temperature of the air flow. The stagnation temperature is given by Eq. (16.2) as

$$T_o = T\left(1 + \frac{V^2}{2g_c c_p T}\right)$$

$$= (20.0 + 273.15)\left(1 + \frac{(90.0\,\text{km/h})(1000\,\text{m/km})(1\,\text{h/3600 s})(1\,\text{kJ/kg}/1000\,\text{m}^2/\text{s}^2)}{2(1)(1.004\,\text{kJ/kg}\cdot\text{K})(20.0 + 273.15\,\text{K})}\right)$$

$$= 294\,\text{K} = 20.3°\text{C}$$

So the stagnation temperature rise is not very much at this speed.

Exercises

1. How fast would the sports car in Example 16.1 have to travel to produce a 1.00°C stagnation temperature rise at the center of your hand? **Answer:** $V = 161$ km/h.
2. Suppose it is winter and the temperature of the air in Example 16.1 is 0.00°C. What would be the stagnation temperature rise at the center of your hand if all the other variables remain the same? **Answer:** $T_o - T = 0.310°$C (independent of the value of T).
3. Now, you are in an aircraft traveling at 800. km/h in air at 20.0°C. If you put your hand out the window now, what is the air temperature at the center of your hand? **Answer:** $T_o = 318$ K $= 44.6°$C.

20.0°C air at 90.0 km/h
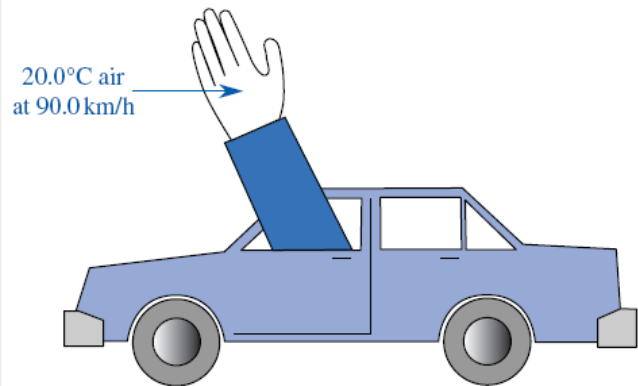
Isentropic flow

Constant entropy flow is called *isentropic* flow.

From a consideration of the second law of thermodynamics, a reversible flow maintains a constant value of entropy.

In the theory of stream tubes, isentropic flow is the basis for compressible flow.

The well-known incompressible flow does not apply if $M \ll 1$ is violated. Theron is called the Mach-number.

$$M = \frac{u}{a}$$

It is the relation between the **flow-velocity u** (fluid or object) and the **speed of sound a** in the surrounding medium.

In a **caloric ideal** gas the speed of sound depends on the **absolute temperature T** (K), the **adiabatic exponent** γ and the **specific gas constant R** (J/(kg K))

$$a = \sqrt{\gamma R T}$$.

In this case, all thermodynamic properties (temperature, pressure, density and speed of sound) can be expressed with explicit formulas, which are functions of the Mach number!

In the following all thermodynamic properties are related to their **properties at rest** (u=0). From the energy equation for a frictionless, adiabatic flow you will get that the enthalpy at rest h_t is always the same regardless of an isentropic or non-isentropic state change.

$$\frac{u^2}{2} + h = h_t$$

In fact - $h = c_p T$ - the temperature is it as well. In contrary, the pressure depends on how the gas is brought to rest. The pressure at rest is only obtained if the state change is isentropic. If the entropy changes the pressure at rest changes as well, e.g. when passing a shock.

Isentropic flow through nozzles

The final point to be made regarding the isentropic flow of an ideal gas through a nozzle involves the effect of varying the *back pressure* (the pressure outside the nozzle exit) on the mass rate of flow.
Consider first a convergent nozzle as shown in Fig. 3.1, which also shows the pressure ratio P/P_0 along the length of the nozzle. The conditions upstream are the stagnation conditions, which are assumed to be constant. The pressure at the exit plane of the nozzle is designated P_E, and the back pressure P_B. Let us consider how the mass rate of flow and the exit plane pressure P_E/P_0 vary as the back pressure P_B is decreased.
These quantities are plotted in Fig. 16.12. When P_B/P_0 _ 1, there is of course no flow, and P_E/P_0 _ 1 as designated by point *a*. Next let the back pressure *PB* be lowered to that designated by point *b*, so that P_B/P_0 is greater than the critical-pressure ratio. The mass rate of flow has a certain value and *PE _ PB*. The exit Mach number is less than 1. Next let the back pressure be lowered to the critical pressure, designated by point *c*. The Mach number at the exit is now unity, and P_E is equal to P_B. When *PB* is

decreased below the critical pressure, designated by point *d*, there is no further increase in the mass rate of flow, and *PE* remains constant at a value equal to the *Mass* critical pressure, and the exit Mach number is unity.
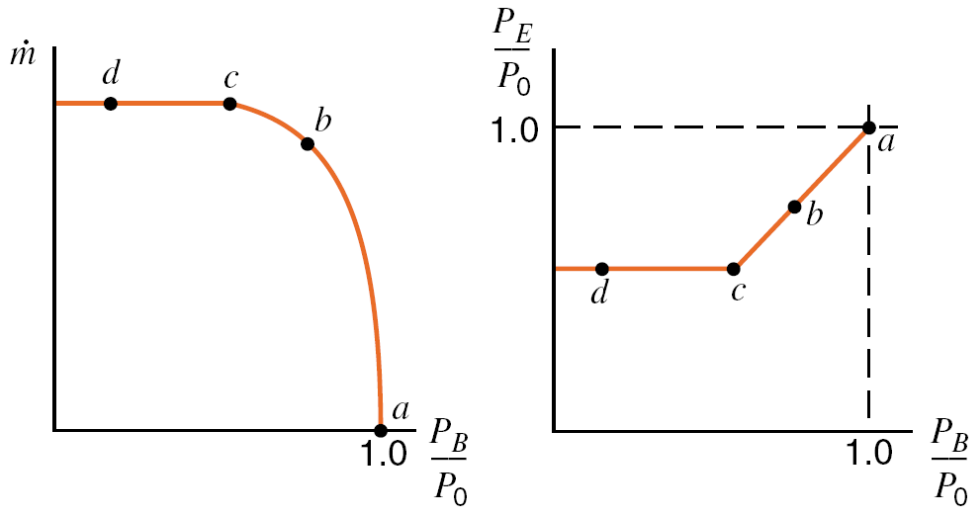
The drop in pressure from *PE* to P_B takes place outside the nozzle exit. Under these conditions the nozzle is said to be choked, which means that for given stagnation conditions the nozzle is passing the maximum possible mass flow.

Consider next a convergent-divergent nozzle in a similar arrangement, Fig. 3.2. Point *a* designates the conditions when P_B _ P_0 and there is no flow. When P_B is decreased to the pressure indicated by point *b*, so that P_B/P_0 is less than 1 but considerably greater than the critical-pressure ratio, the velocity increases in the convergent section, but *M* < 1 at the throat. Therefore, the diverging section acts as a subsonic diffuser in which the pressure increases and velocity decreases. Point *c* designates the back pressure at which *M* = 1 at the throat, but the diverging section acts as a subsonic diffuser (with *M* = 1 at the inlet) in which the pressure increases and velocity decreases. Point *d* designates one other back pressure that permits isentropic flow, and in this case the diverging section acts as a supersonic nozzle, with a decrease in pressure and an increase in velocity.

Between the back pressures designated by points *c* and *d*, an isentropic solution is not possible, and shock waves will be present. This matter is discussed in the section that follows.

When the back pressure is decreased below that designated by point *d*, the exit-plane pressure P_E remains constant, and the drop in pressure from P_E to P_B takes place outside the nozzle. This is designated by point *e*.
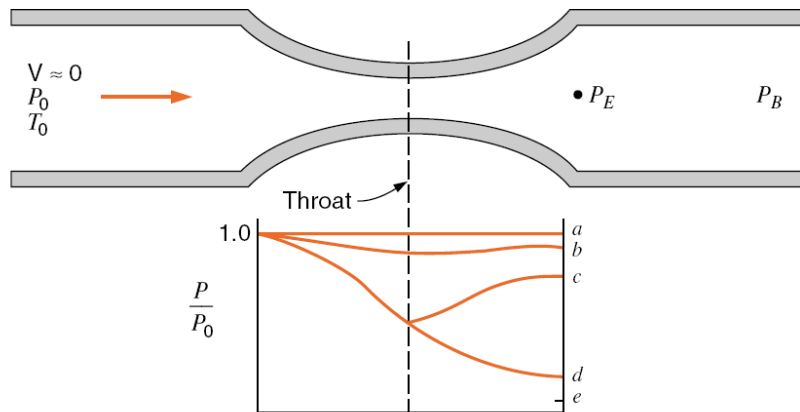
*Fig. 15 Nozzle pressure ratio as a function of back pressure for a reversible, Convergent-divergent nozzle.*

In the light of this example, we can conclude the discussion concerning the flow through a convergent-divergent nozzle. Figure 3.2 is repeated here as Fig. 3.3 for convenience, except that points *f*, *g*, and *h* have been added. Consider point *d*. We have already noted that with this back pressure the exit plane pressure *PE* is just equal to the back pressure *PB*, and isentropic flow is maintained in the nozzle. Let the back pressure be raised to that designated by point f.
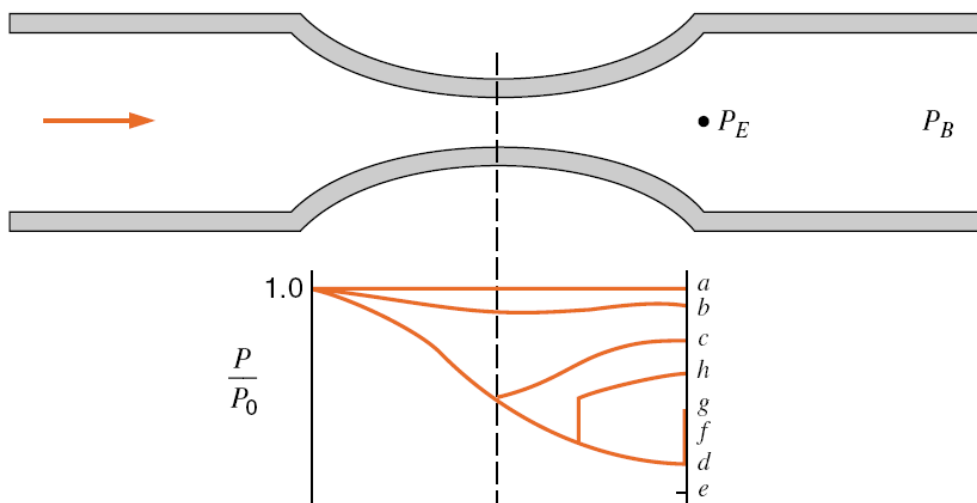
Fig. 16 Nozzle pressure ratio as a function of back pressure for a convergent– divergent nozzle.

The exit-plane pressure *PE* is not influenced by this increase in back pressure, and the increase in pressure from P_E to P_B takes place outside the nozzle. Let the back pressure be raised to that designated by point *g*, which is just sufficient to cause a normal shock to stand in the exit plane of the nozzle. The exit-plane pressure *PE* (downstream of the shock) is equal to the back pressure P_B, and *M* <1 leaving the nozzle. Now let the back pressure be raised to that corresponding to point *h*. As the back pressure is raised from *g* to *h* the normal shock moves into the nozzle as indicated. Since *M* < 1 downstream of the normal shock, the diverging part of the nozzle that is downstream of the shock acts as a subsonic diffuser. As the back pressure is increased from *h* to *c* the shock moves further upstream and disappears at the nozzle throat where the back pressure corresponds to *c*. This is reasonable since there are no supersonic velocities involved when the back pressure corresponds to *c*, and hence no shock waves are possible.

Shock wave

A **shock wave** is a type of propagating disturbance. Like an ordinary wave, it carries energy and can propagate through a medium (solid, liquid, gas or plasma) or in some cases in the absence of a material medium, through a field such as an electromagnetic field. Shock waves are characterized by an abrupt, nearly discontinuous change in the characteristics of the medium. Across a shock there is always an extremely rapid rise in pressure, temperature and density of the flow. In supersonic flows, expansion is achieved through an expansion fan. A shock wave travels through most media at a higher speed than an ordinary wave.

Normal Shocks

In some range of back pressure, the fluid that achieved a sonic velocity at the throat of a converging-diverging nozzle and is accelerating to supersonic velocities in the diverging section experiences a *normal shock,* which causes a sudden rise in pressure and temperature and a sudden drop in velocity to subsonic levels. Flow through the shock is highly irreversible, and thus it cannot be approximated as isentropic. The properties of an ideal gas with constant specific heats before (subscript *x*) and after (subscript *y)* a shock are related by

Shock wave
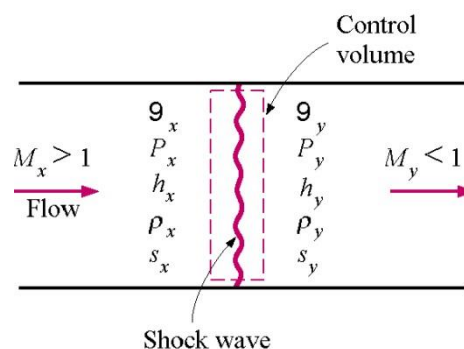
We assume steady flow with no heat and work interactions, and no potential energy changes. We have the following

Conservation of mass:

$$\rho_x A \vec{V}_x = \rho_y A \vec{V}_y$$
$$\rho_x \vec{V}_x = \rho_y \vec{V}_y$$

$$h_x + \frac{\vec{V}_x^2}{2} = h_y + \frac{\vec{V}_y^2}{2}$$

$$h_{ox} = h_{oy}$$

Conservation of energy:
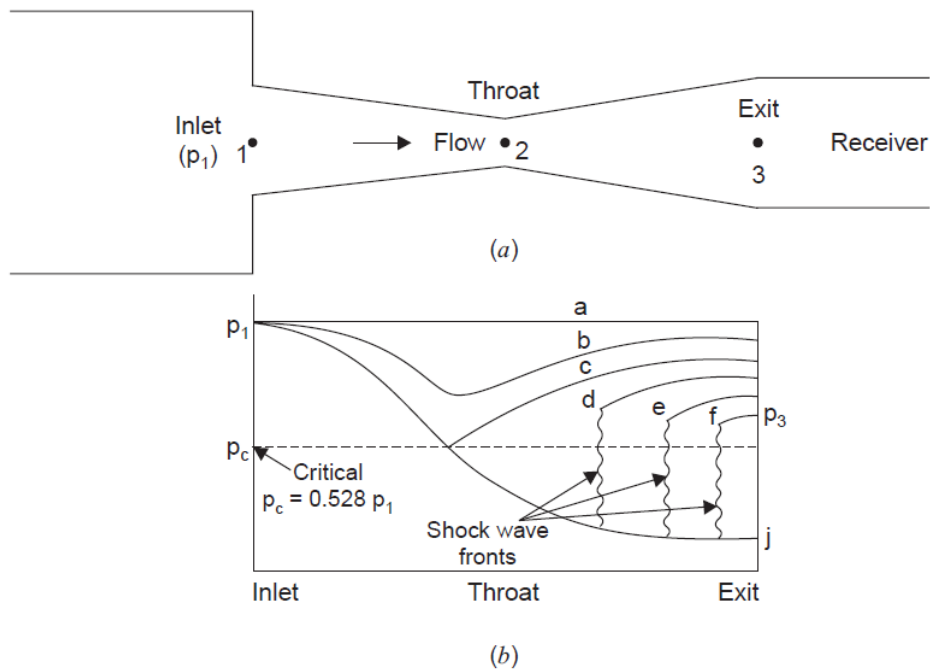
$$for\ ideal\ gases: T_{ox} = T_{oy}$$

Fig.17 (a) Laval nozzle (convergent-divergent nozzle); (b) Pressure distribution through a convergent-divergent nozzle with flow of compressible fluid.

EXAMPLE

Steam is usually accelerated in the nozzle of a turbine before it strikes the turbine blades. Steam enters an adiabatic nozzle at 7 MPa and 500°C with a velocity of 70 m/s and exits at 5 MPa and 450°C. Assuming the surroundings to be at 25°C, determine *the exit velocity of the steam*

$$P_1 = 7\,\text{MPa} \left.\right\} h_1 = 3411.4\,\text{kJ/kg}$$
$$T_1 = 500°C \quad s_1 = 6.8000\,\text{kJ/kg}\cdot\text{K}$$

$$P_2 = 5\,\text{MPa} \left.\right\} h_2 = 3317.2\,\text{kJ/kg}$$
$$T_2 = 450°C \quad s_2 = 6.8210\,\text{kJ/kg}\cdot\text{K}$$

$$P_{2s} = 5\,\text{MPa} \left.\right\} h_{2s} = 3302.0\,\text{kJ/kg}$$
$$s_{2s} = s_1$$

7 MPa
500°C
70 m/s → STEAM →
5 MPa
450°C
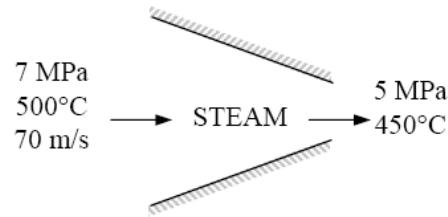
Analysis (*a*) We take the nozzle to be the system, which is a control volume. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta\dot{E}_{system}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}}^{\nearrow 0\ \text{(steady)}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}(h_1 + V_1^2/2) = \dot{m}(h_2 + V_2^2/2) \quad (\text{since } \dot{W} = \dot{Q} \cong \Delta pe \cong 0)$$

$$0 = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}$$

Then the exit velocity becomes

$$V_2 = \sqrt{2(h_1 - h_2) + V_1^2} = \sqrt{2(3411.4 - 3317.2)\,\text{kJ/kg}\left(\frac{1000\,\text{m}^2/\text{s}^2}{1\,\text{kJ/kg}}\right) + (70\,\text{m/s})^2} = \mathbf{439.6\,m/s}$$

EXAMPLE

Steam enters a nozzle at 400°C and 800 kPa with a velocity of 10 m/s, and leaves at 300°C and 200 kPa while losing heat at a rate of 25 kW. For an inlet area of 800 cm2, determine the velocity and the volume flow rate of the steam at the nozzle exit.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Potential energy change is negligible. **3** There are no work interactions.

Analysis We take the steam as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

400°C   STEAM   300°C
800 kPa →   200 kPa
10 m/s

Q
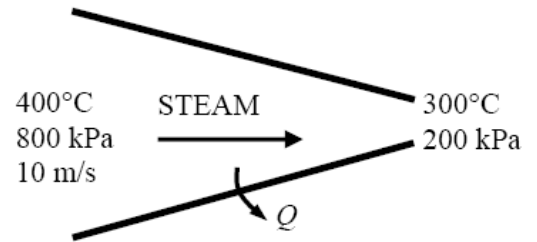
Energy balance:

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta\dot{E}_{system}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}}^{\nearrow 0 \text{ (steady)}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right) + \dot{Q}_{out} \quad \text{since } \dot{W} \cong \Delta pe \cong 0)$$

or

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} + \frac{\dot{Q}_{out}}{\dot{m}}$$

The properties of steam at the inlet and exit are (Table A-6)

$$P_1 = 800 \text{ kPa} \left.\right| v_1 = 0.38429 \text{ m}^3/\text{kg}$$
$$T_1 = 400°C \left.\right\} h_1 = 3267.7 \text{ kJ/kg}$$

$$P_2 = 200 \text{ kPa} \left.\right| v_2 = 1.31623 \text{ m}^3/\text{kg}$$
$$T_1 = 300°C \left.\right\} h_2 = 3072.1 \text{ kJ/kg}$$

The mass flow rate of the steam is

$$\dot{m} = \frac{1}{v_1} A_1 V_1 = \frac{1}{0.38429 \text{ m}^3/\text{s}} (0.08 \text{ m}^2)(10 \text{ m/s}) = 2.082 \text{ kg/s}$$

Substituting,

$$3267.7 \text{ kJ/kg} + \frac{(10 \text{ m/s})^2}{2}\left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right) = 3072.1 \text{ kJ/kg} + \frac{V_2^2}{2}\left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right) + \frac{25 \text{ kJ/s}}{2.082 \text{ kg/s}}$$

$$\longrightarrow V_2 = \textbf{606 m/s}$$

The volume flow rate at the exit of the nozzle is

$$\dot{V}_2 = \dot{m} v_2 = (2.082 \text{ kg/s})(1.31623 \text{ m}^3/\text{kg}) = \textbf{2.74 m}^3\textbf{/s}$$

4. Reciprocating compressors

1. *Introduction*

A typical refrigeration system consists of several basic components such as compressors, condensers, expansion devices, evaporators, in addition to several accessories such as controls, filters, driers, oil separators etc. For efficient operation of the refrigeration system, it is essential that there be a proper matching between various components. Before analyzing the balanced performance of the complete system, it is essential to study the design and performance characteristics of individual components. Except in special applications, the refrigeration system components are standard components manufactured by industries specializing in individual components. Generally, for large systems, depending upon the design specifications, components are selected from the manufacturers' catalogs and are assembled at site. Even though most of the components are standard off-the-shelf items, sometimes components such as evaporator may be made to order. Small capacity refrigeration systems such as refrigerators, room and package air conditioners, and water coolers are available as complete systems. In this case the manufacturer himself designs or selects the system components, assembles them at the factory, tests them for performance and then sells the complete system as a unit.

2. *Compressors*

A compressor is the most important and often the costliest component (typically 30 to 40 percent of total cost) of any vapor compression refrigeration system (VCRS). The function of a compressor in a VCRS is to continuously draw the refrigerant vapor from the evaporator, so that a low pressure and low temperature can be maintained in the evaporator at which the refrigerant can boil extracting heat from the refrigerated space. The compressor then has to raise the pressure of the refrigerant to a level at which it can condense by rejecting heat to the cooling medium in the condenser.

a. *Classification of compressors*

Compressors used in refrigeration systems can be classified in several ways:

a) Based on the working principle:

i. Positive displacement type

ii. Roto-dynamic type
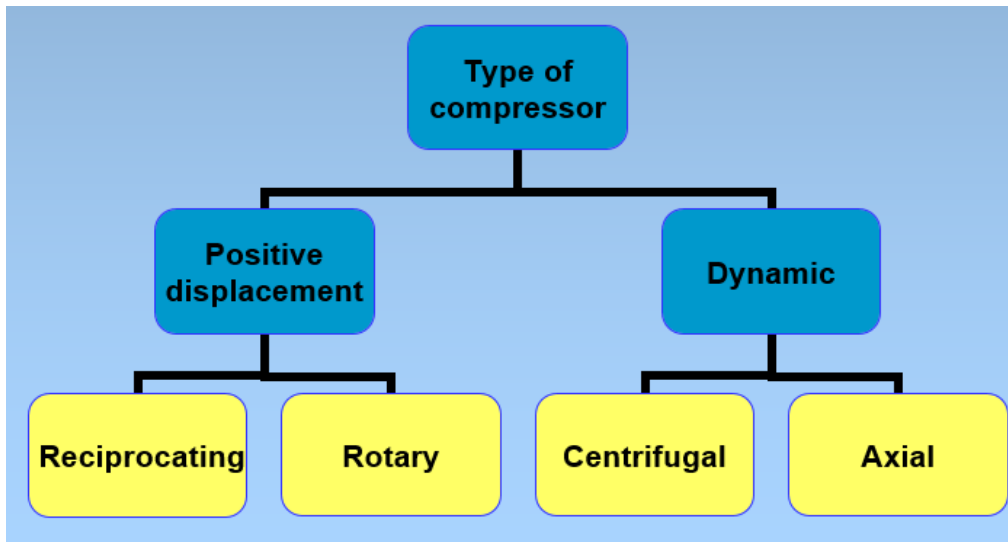
Reciprocating Compressor

• Used for air and refrigerant compression

• Works like a bicycle pump: cylinder volume reduces while pressure increases, with pulsating output

• Many configurations available

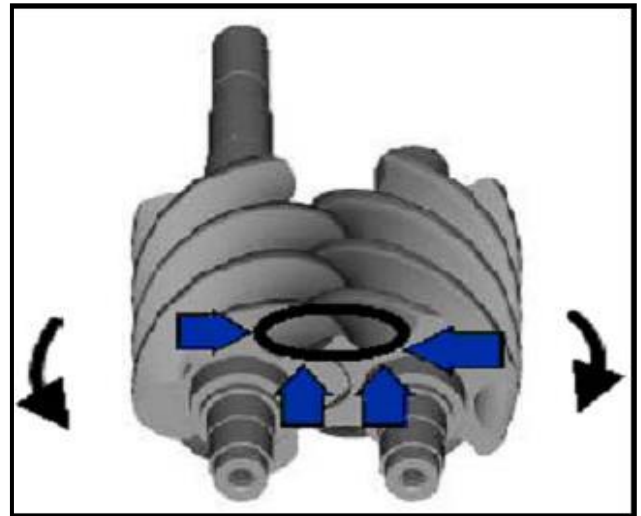• Single acting when using one side of the piston, and double acting when using both **sides**
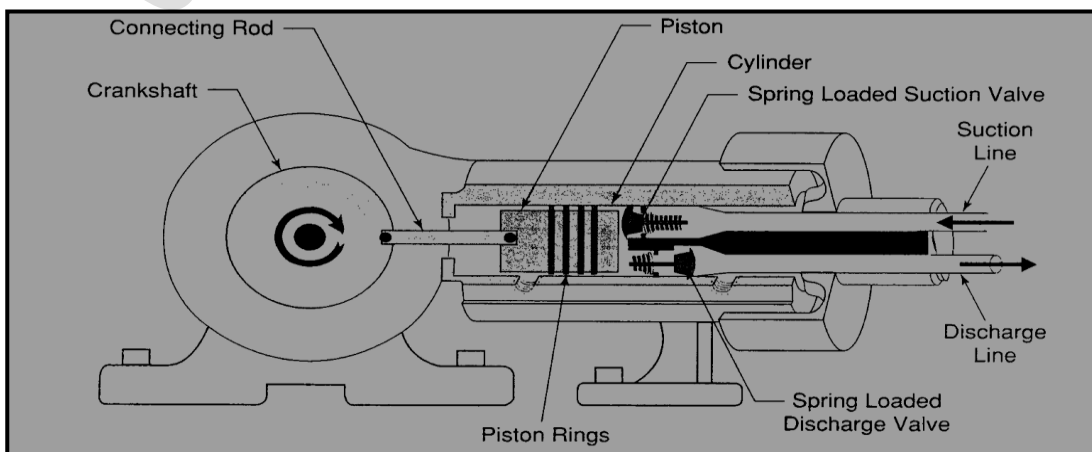
Rotary Compressor

- Rotors instead of pistons: continuous discharge

- Benefits: low cost, compact, low weight, easy to maintain

- Sizes between 30 – 200 hp

    Types

✓ Lobe compressor

✓ Screw compressor

✓ Rotary vane / Slide vane

Centrifugal Compressor

- Rotating impeller transfers energy to move air

- Continuous duty

- Designed oil free

- High volume applications > 12,000 cfm
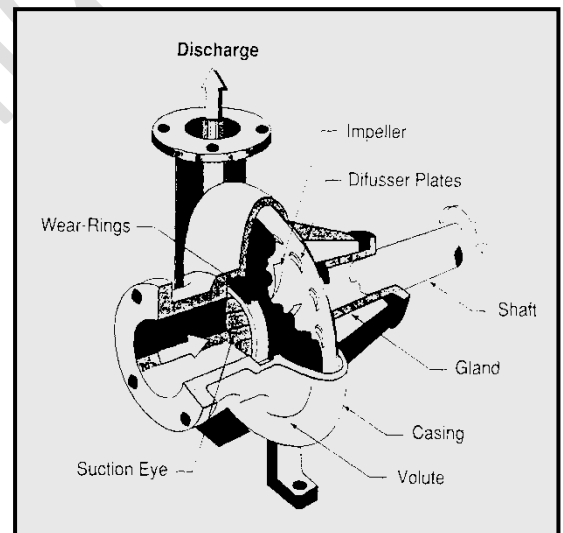
  In positive displacement type compressors, compression is achieved by trapping a refrigerant vapor into an enclosed space and then reducing its volume. Since a fixed amount of refrigerant is trapped each time, its pressure rises as its volume is reduced. When the pressure rises to a level that is slightly higher than the condensing pressure, then it is expelled from the enclosed space and a fresh charge of low-pressure refrigerant is drawn in and the cycle continues. Since the flow of refrigerant to the compressor is not steady, the positive displacement type compressor is a *pulsating flow device*. However, since the operating speeds are normally very high the flow appears to be almost steady on macroscopic time scale. Since the

flow is pulsating on a microscopic time scale, positive displacement type compressors are prone to high wear, vibration and noise level. Depending upon the construction, positive displacement type compressors used in refrigeration and air conditioning can be classified into:

i. Reciprocating type

ii. Rotary type with sliding vanes (rolling piston type or multiple vane type)

iii. Rotary screw type (single screw or twin-screw type)

iv. Orbital compressors, and

v. Acoustic compressors

In roto-dynamic compressors, the pressure rise of refrigerant is achieved by imparting kinetic energy to a steadily flowing stream of refrigerant by a rotating mechanical element and then converting into pressure as the refrigerant flows through a diverging passage. Unlike positive displacement type, the roto-dynamic type compressors are steady flow devices, hence are subjected to less wear and vibration. Depending upon the construction, roto-dynamic type compressors can be classified into:

i. Radial flow type, or

ii. Axial flow type

Centrifugal compressors (also known as turbo-compressors) are radial flow type, roto-dynamic compressors. These compressors are widely used in large capacity refrigeration and air conditioning systems. Axial flow compressors are normally used in gas liquefaction applications.

b) Based on arrangement of compressor motor or external drive:

i. Open type

ii. Hermetic (or sealed) type

iii. Semi-hermetic (or semi-sealed) type

In open type compressors the rotating shaft of the compressor extends through a seal in the crankcase for an external drive. The external drive may be an electrical motor or an engine (e.g. diesel engine). The compressor may be belt driven or gear driven. Open type compressors are normally used in medium to large capacity refrigeration system for all refrigerants and for ammonia (due to its incompatibility with hermetic motor materials). Open type compressors are characterized by high efficiency, flexibility, better compressor cooling and serviceability.

However, since the shaft has to extend through the seal, refrigerant leakage from the system cannot be eliminated completely. Hence refrigeration systems using open type compressors require a refrigerant reservoir to take care of the refrigerant leakage for some time, and then regular maintenance for charging the system with refrigerant, changing of seals, gaskets etc.

In hermetic compressors, the motor and the compressor are enclosed in the same housing to prevent refrigerant leakage. The housing has welded connections for refrigerant inlet and outlet and for power input socket. As a result of this, there is virtually no possibility of refrigerant leakage from the compressor. All motors reject a part of the power supplied to it due to eddy currents and friction, that is, inefficiencies. Similarly the compressor also gets heated-up due to friction and also due to temperature rise of the vapor during compression. In Open type, both the compressor and the motor normally reject heat to the surrounding air for efficient operation. In hermetic compressors heat cannot be rejected to the surrounding air since both are enclosed in a shell. Hence, the cold suction gas is made to flow over the motor and the compressor before entering the compressor. This keeps the motor cool. The motor winding is in direct contact with the refrigerant hence only those refrigerants, which have high dielectric strength, can be used in hermetic compressors. The cooling rate depends upon the flow rate of the refrigerant, its temperature and the thermal properties of the refrigerant. If flow rate is not sufficient and/or if the temperature is not low enough the insulation on the winding of the motor can burn out and short-circuiting may occur. Hence, hermetically sealed compressors give satisfactory and safe performance over a very narrow range of design temperature and should not be used for off-design conditions.

The COP of the hermetic compressor based systems is lower than that of the open compressor based systems since a part of the refrigeration effect is lost in cooling the motor and the compressor. However, hermetic compressors are almost universally used in small systems such as domestic refrigerators, water coolers, air conditioners etc, where efficiency is not as important as customer convenience (due to absence of continuous maintenance). In addition to this, the use of hermetic compressors is ideal in systems, which use capillary tubes as expansion devices and are critically charged systems. Hermetic compressors are normally not serviceable. They are not very flexible as it is difficult to vary their speed to control the cooling capacity.

In some (usually larger) hermetic units, the cylinder head is usually removable so that the valves and the piston can be serviced. This type of unit is called a semi-hermetic (or semi-sealed) compressor.

Reciprocating compressors

Reciprocating compressor is the workhorse of the refrigeration and air conditioning industry. It is the most widely used compressor with cooling capacities ranging from a few Watts to hundreds of kilowatts. Modern day reciprocating compressors are high speed ($\approx$ 3000 to 3600 rpm), single acting, single or multi-cylinder (upto 16 cylinders) type.

Figure 1 shows the schematic of a reciprocating compressor. Reciprocating compressors consist of a piston moving back and forth in a cylinder, with suction and discharge valves to achieve suction and compression of the refrigerant vapor. Its construction and working are somewhat similar to a two-stroke engine, as suction and compression of the refrigerant vapor are completed in one revolution of the crank. The suction side of the compressor is connected to the exit of the evaporator, while the discharge side of the compressor is connected to the condenser inlet.
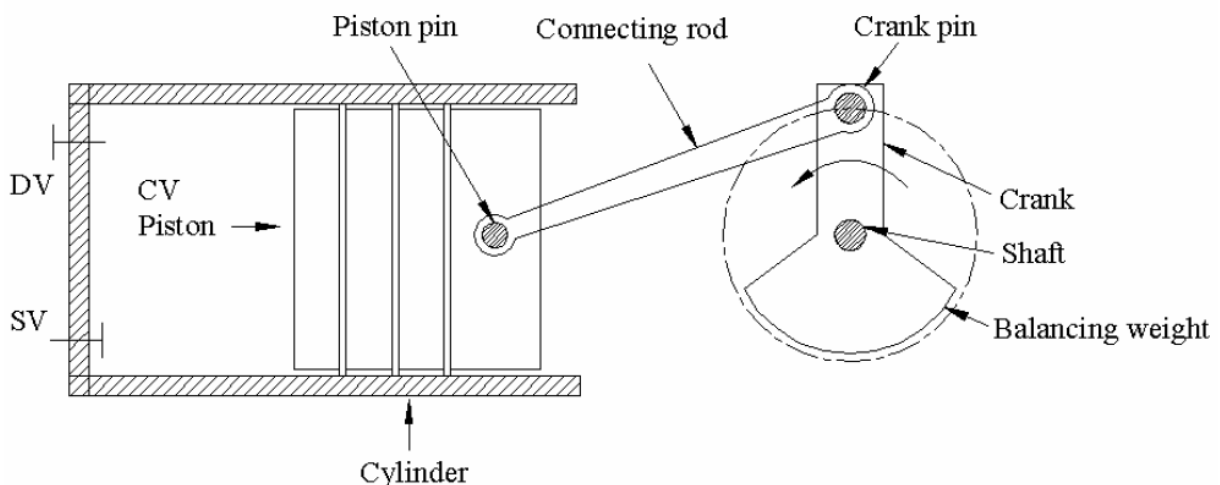
Figure 18 Schematic of a reciprocating compressor

The suction (inlet) and the discharge (outlet) valves open and close due to pressure differences between the cylinder and inlet or outlet manifolds respectively. The pressure in the inlet manifold is equal to or slightly less than the evaporator pressure. Similarly the pressure in the outlet manifold is equal to or slightly greater than the condenser pressure. The purpose of the manifolds is to provide stable inlet and outlet pressures for the smooth operation of the valves and also provide a space for mounting the valves.

The valves used are of reed or plate type, which are either floating or clamped. Usually, backstops are provided to limit the valve displacement and springs may be provided for smooth return after opening or closing. The piston speed is decided by valve type. Too high a speed will give excessive vapor velocities that will decrease the volumetric efficiency and the throttling loss will decrease the compression efficiency.

3.1 Performance of reciprocating compressors

For a given evaporator and condenser pressures, the important performance parameters of a refrigerant compressor are:

a) The mass flow rate ($\dot{m}$) of the compressor for a given displacement rate

b) Power consumption of the compressor (W_c)

c) Temperature of the refrigerant at compressor exit, T_d, and

d) Performance under part load conditions

The mass flow rate decides the refrigeration capacity of the system and for a given compressor inlet condition, it depends on the volumetric efficiency of the compressor. The volumetric efficiency, η_V is defined as the ratio of volumetric flow rate of refrigerant to the maximum possible volumetric flow rate, which is equal to the compressor displacement rate, i.e.,

$$\eta = \frac{volumetric\ flow\ rate}{compressor\ Displacement\ rate} = \frac{\dot{m}.V_e}{\dot{V}_{sw}}$$

Where $\dot{m}$ $\dot{V}_{sw}$ are the mass flow rate of refrigerant (kg/s) and compressor displacement rate (m³/s) respectively, and V_e is the specific volume (m³/kg) of the refrigerant at compressor inlet.

For a given evaporator and condenser temperatures, one can also use the volumetric refrigeration capacity (kW/m³) to indicate the volumetric efficiency of the compressor. The actual volumetric efficiency (or volumetric capacity) of the compressor depends on the operating conditions and the design of the compressor.

The power consumption (kW) or alternately the power input per unit refrigeration capacity *(kW/kW)* depends on the compressor efficiency (η_C), efficiency of the mechanical drive (η_{mech}) and the motor efficiency (η_{motor}). For a refrigerant compressor, the power input (W_c) is given by:

$$W_C = \frac{W_{ideal}}{\eta_C \eta_{mech} \eta_{motor}}$$

Where W_{ideal} is the power input to an ideal compressor.

The temperature at the exit of the compressor (discharge compressor) depends on the type of refrigerant used and the type of compressor cooling. This parameter has a bearing on the life of the compressor.

The performance of the compressor under part load conditions depends on the type and design of the compressor.

<u>a) Ideal reciprocating compressor:</u>

An ideal reciprocating compressor is one in which:

i. The clearance volume is zero, i.e., at the end of discharge process, the volume of refrigerant inside the cylinder is zero.

ii. No pressure drops during suction and compression

iii. Suction, compression and discharge are reversible and adiabatic

Figure 2 shows the schematic of an ideal compression process on pressure-volume and pressure-crank angle (θ) diagrams. As shown in the figures, the cycle of operations consists of:

Process D-A: This is an isobaric suction process, during which the piston moves from the Inner Dead Centre (IDC) to the Outer Dead Centre (ODC). The suction valve remains open during this process and refrigerant at a constant pressure P_e flows into the cylinder.

Process A-B: This is an isentropic compression process. During this process, the piston moves from ODC towards IDC. Both the suction and discharge valves remain closed during the process and the pressure of refrigerant increases from P_e to P_c.

Process B-C: This is an isobaric discharge process. During this process, the suction valve remains closed and the discharge valve opens. Refrigerant at a constant P is expelled from the compressor as the piston moves to IDC.
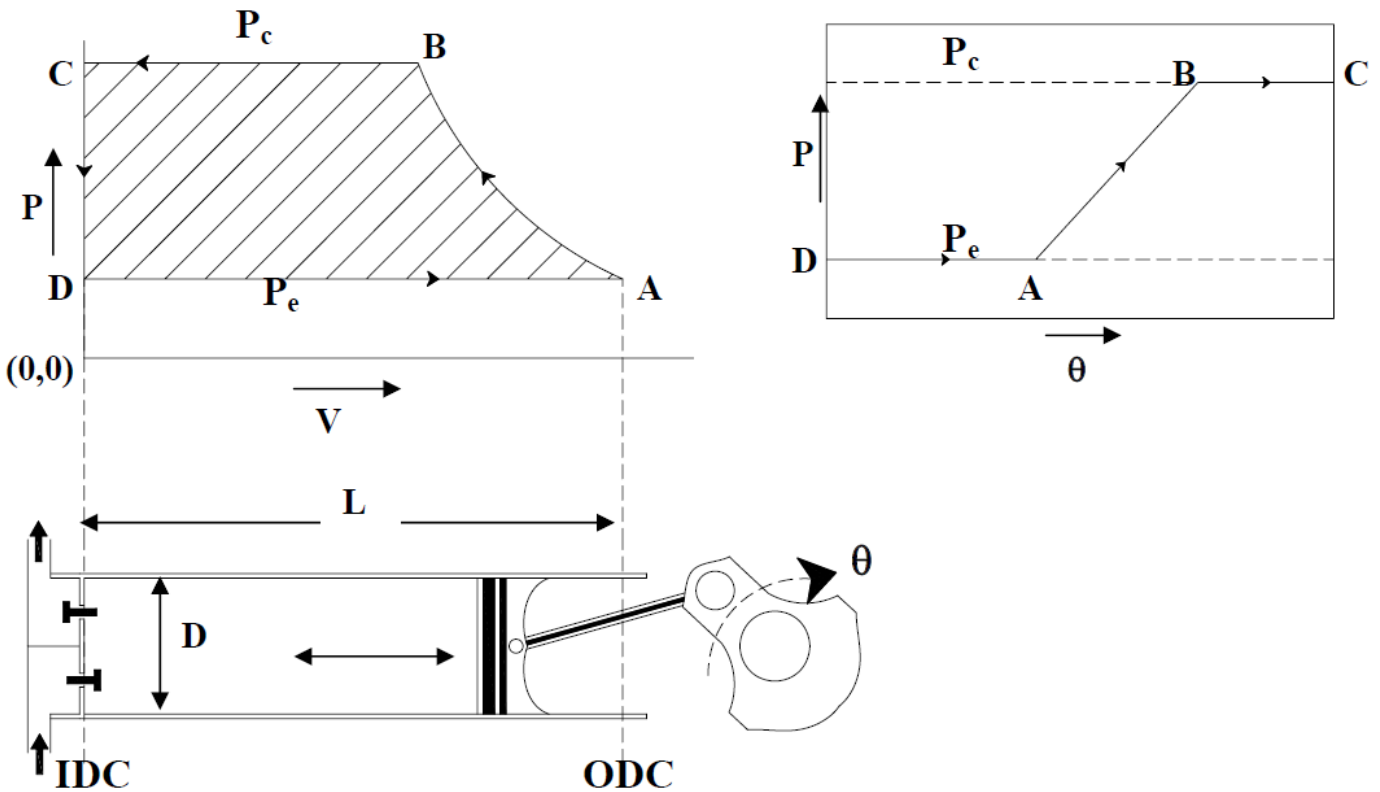
Figure 19 Ideal reciprocating compressor on P-V and P-θ diagrams

Since the clearance volume is zero for an ideal compressor, no gas is left in the compressor at the end of the discharge stroke, as a result the suction process D-A starts as soon as the piston starts moving again towards ODC. The volumetric flow rate of refrigerant at suction conditions is equal to the compressor displacement rate hence; the volumetric efficiency of the ideal compressor is 100 percent. The mass flow rate of refrigerant of an ideal compressor is given by:

$$\dot{m} = \frac{\dot{V}_{SW}}{V_e}$$

Thus for a given refrigeration capacity, the required size of the compressor will be minimum if the compressor behaves as an ideal compressor.

The swept volume $\dot{V}_{SW}$ of the compressor is given by:

$$\dot{V}_{SW} = nN\frac{\pi D^2}{4}L$$

Where

n = Number of cylinders

N = Rotational speed of compressor, revolutions per second

D = Bore of the cylinder, m

L = Stroke length, m

Work input to the ideal compressor

The total work input to the compressor in one cycle is given by:

$W_{id} = W_{D-A} + W_{A-B} + W_{B-C}$

Where,

W_{D-A} = Work done by the refrigerant on the piston during process D-A

= Area under line D-A on P-V diagram = $-P_e.V_A$

W_{A-B} = Work done by the piston on refrigerant during compression A-B

= Area under the curve A-B on P-V diagram = $\int_{V_A}^{V_B} P.dV$

W$_{B\text{-}C}$ = Work done by the piston on the refrigerant during discharge B-C

= Area under line B-C = $P_c.V_B$

$$\therefore W_{id} = -P_e.V_A + \int_{V_A}^{V_B} P.dV + P_C V_B \quad = \text{Area A-B-C-D on P-V diagram}$$

$$= \int_{P_e}^{P_c} V.dP$$

Thus, the work input to the ideal compressor per cycle is equal to the area of the cycle on P-V diagram. The specific work input, w$_{id}$ (kJ/kg) to the ideal compressor is given by:

$$W_{id} = \frac{W_{id}}{M_r} = \int_{P_e}^{P_c} V.dP$$

Where M$_r$ is the mass of refrigerant compressed in one cycle and v is the specific volume of the refrigerant.

The power input to the compressor W$_c$ is given by:

$$W_c = \dot{m}\, w_{id} = \frac{\dot{V}_{sw}}{V_e} \int_{P_e}^{P_c} V.dP$$

The mean effective pressure (mep) for the ideal compressor is given by:

$$mep = \frac{W_c}{\dot{V}_{sw}} = \frac{1}{V_e} \int_{P_e}^{P_c} V.dP$$

The concept of *mean effective pressure* is useful for real compressors as the power input to the compressor is a product of *mep* and the swept volume rate.

Thus, the power input to the compressor and its mean effective pressure can be obtained from the above equation if the relation between v and P during the compression process A-B is known. The above equation is valid for both isentropic and non-isentropic

compression processes; however, the compression process must be reversible, as the path of the process should be known for the integration to be performed.

For the isentropic process, $PV^\gamma = $ constant, hence the specific work of compression w_{id} can be obtained by integration, and it can be shown to be equal to:

$$W_{id} = \int_{P_e}^{P_c} V.dP = P_e V_e \left(\frac{\gamma}{\gamma - 1}\right)\left[\left(\frac{P_c}{P_e}\right)^{\frac{\gamma - 1}{\gamma}} - 1\right]$$

The work of compression for the ideal compressor can also be obtained by applying energy balance across the compressor, Fig. 3. Since the process is assumed to be reversible and adiabatic and if we assume changes in potential and kinetic energy to be negligible, then from energy balance across the compressor:

$$W_{id} = \frac{W_c}{\dot{m}} = (h_c - h_e)$$

The above expression can also be obtained from the thermodynamic relation:

Tds=dh-vdP $\Rightarrow$ dh=vdP $(\because ds = 0 \; for \; isntropic \; proces)$

$$\therefore W_{id} = \int_{P_e}^{P_c} vdP = \int_{P_e}^{P_c} dh = (h_d - h_e)$$

The above expression is valid only for reversible, adiabatic compression.
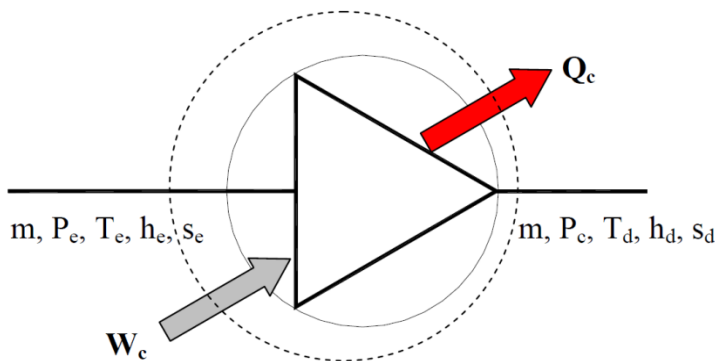
Fig. 20 Energy balance across a steady flow compressor

b) Ideal compressor with clearance:

In actual compressors, a small clearance is left between the cylinder head and piston to accommodate the valves and to take care of thermal expansion and machining tolerances. As a thumb rule, the clearance C in millimeters is given by:

C = (0.005L + 0.5) mm, where L is stroke length in mm

This space along with all other spaces between the closed valves and the piston at the inner dead center (IDC) is called as Clearance volume, Vc. The ratio of the clearance volume to the swept volume is called as Clearance ratio, ε, and i.e.

$$\varepsilon = \frac{V_c}{V_{sw}}$$

The clearance ratio ε depends on the arrangement of the valves in the cylinder and the mean piston velocity. Normally (ε) is less than 5 percent for well-designed compressors with moderate piston velocities (≈ 3 m/s); however, it can be higher for higher piston speeds.

Due to the presence of the clearance volume, at the end of the discharge stroke, some amount of refrigerant at the discharge pressure Pc will be left in the clearance volume. As a result, suction does not begin as soon as the piston starts moving away from the IDC, since the pressure inside the cylinder is higher than the suction pressure ($P_c > P_e$). As shown in Fig. 4, suction starts only when the pressure inside the cylinder falls to the suction pressure in an ideal compressor with clearance. This implies that even though the compressor swept volume, $V_{sw} = V_A - V_C$ the actual volume of the refrigerant that entered the cylinder during suction stroke is $V_A - V_D$. As a result, the volumetric efficiency of the compressor with clearance, $\eta_{V,cl}$ is less than 100 percent,

$$\eta_{v.cl} = \frac{actual\ volume\ of\ refrigerant\ comprassed}{Swept\ volume\ of\ the\ compressor} = \left[\frac{V_A - V_D}{V_A - V_C}\right] \text{i. e.,}$$
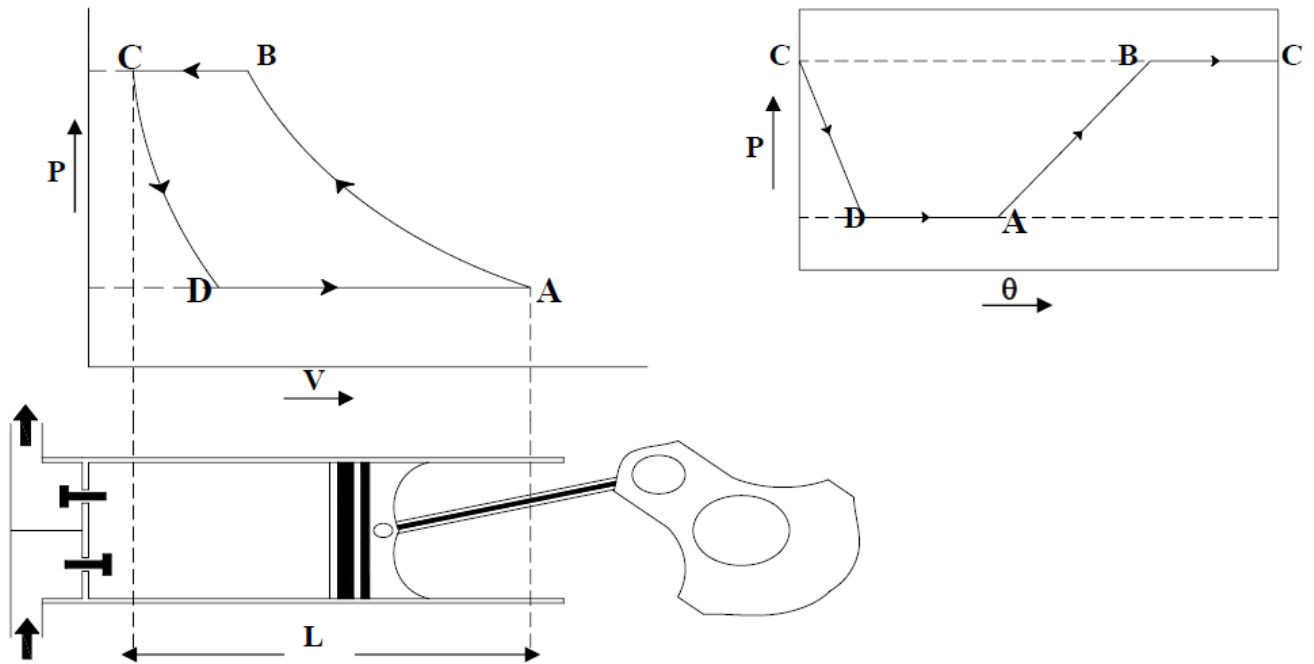
Fig.21 Ideal reciprocating compressor with clearance

From Fig. 4, the clearance volumetric efficiency can be written as:

$$\eta_{v.cl} = \left[\frac{V_A - V_D}{V_A - V_C}\right] = \frac{(V_A - V_C) + (V_C - V_D)}{(V_A - V_C)} = 1 + \left[\frac{(V_C - V_D)}{(V_A - V_C)}\right]$$

Since the clearance ratio, $\varepsilon = \frac{V_C}{V_{sw}} = \frac{V_C}{V_A - V_C} \Rightarrow (V_A - V_C) = \frac{V_C}{\varepsilon}$

Substituting the above equation in the expression for clearance volumetric efficiency, we can show that:

$$\eta_{Vcl} = 1 + \left(\frac{V_C - V_D}{V_A - V_C}\right) = 1 + \frac{\varepsilon(V_C - V_D)}{V_C} = 1 + \varepsilon - \varepsilon\left(\frac{V_D}{V_C}\right)$$

Since the mass of refrigerant in the cylinder at points C and D are same, we can express the ratio of cylinder volumes at points D and C in terms of ratio of specific volumes of refrigerant at D and C, i.e.,

$$\left(\frac{V_D}{V_C}\right) = \left(\frac{v_D}{v_C}\right)$$

Hence, the clearance volumetric efficiency is given by:

$$\eta_{V.cl} = 1 + \varepsilon - \varepsilon \left(\frac{V_D}{V_C}\right) = 1 + \varepsilon - \varepsilon \left(\frac{V_D}{V_C}\right)$$

If we assume the re-expansion process also to follow the equation $PV^\gamma =$ constant, then:

$$\left(\frac{V_D}{V_C}\right) = \left(\frac{Pc}{P_D}\right)^{\frac{1}{\gamma}} = \left(\frac{Pc}{Pe}\right)^{\frac{1}{\gamma}}$$

Hence, the clearance volumetric efficiency is given by:

$$\eta_{V.cl} = 1 + \varepsilon - \varepsilon \left(\frac{Pc}{P_e}\right)^{\frac{1}{\gamma}} = 1 - \varepsilon \left[r_p^{1/\gamma} - 1\right]$$

Where r_p is the pressure ratio, P_c/P_e .

The above expression holds good for any reversible compression process with clearance. If the process is not reversible, adiabatic (i.e., non-isentropic) but a reversible polytropic process with an index of compression and expansion equal to n, then k in the above equation has to be replaced by n, i.e., in general for any reversible compression process;

$$\eta_{V.cl} = 1 + \varepsilon - \varepsilon \left(\frac{Pc}{P_e}\right)^{1/n} = 1 - \varepsilon \left[r_p^{1/n} - 1\right]$$

The above expression shows that $\eta_{V,cl} \downarrow$ as $r_p \uparrow$ and $\varepsilon \uparrow$ as shown in Fig. 5. It can also be seen that for a given compressor with fixed clearance ratio ε, there is a limiting pressure ratio at which the clearance volumetric efficiency becomes zero. This limiting pressure ratio is obtained from the equation:

$$\eta_{V.cl} = 1 - \varepsilon \left[r_p^{\frac{1}{n}} - 1\right] = 0 \Rightarrow r_{p.max} = \left[\frac{1+\varepsilon}{\varepsilon}\right]^n$$

The mass flow rate of refrigerant compressed with clearance $\dot{m}_{cl}$ is given by:

$$\dot{m}_{cl} = \eta_{V.cl} \frac{\dot{V}_{sw}}{v_e}$$

Thus the mass flow rate and hence the refrigeration capacity of the system decreases as the volumetric efficiency reduces, in other words, the required size of the compressor increases as the volumetric efficiency decreases.
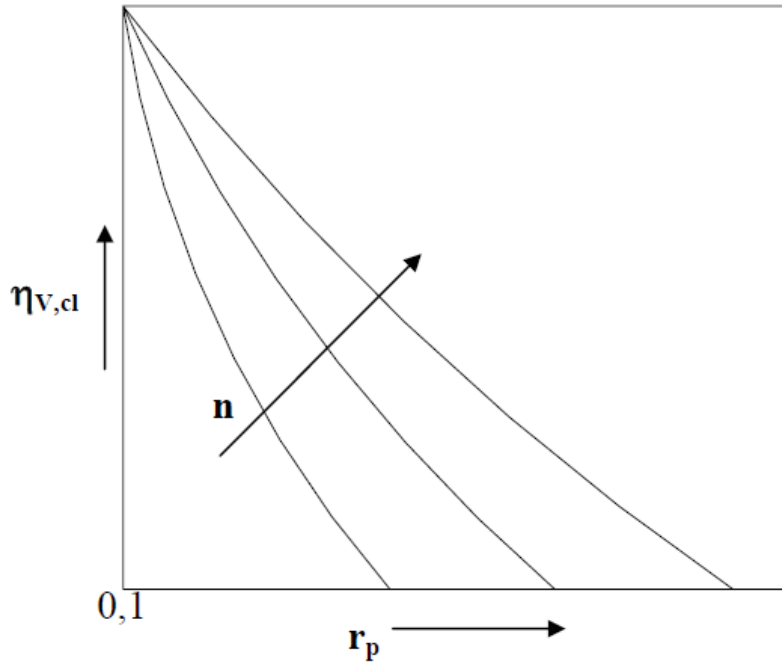
Fig 22 Effect of pressure ratio (r_p) and index of compression (n) clearance volumetric efficiency ($\eta_{V.cl}$)

Work input to the compressor with clearance:

If we assume that both compression and expansion follow the same equation Pv^n = constant (i.e., the index of compression is equal to the index of expansion), then the extra work required to compress the vapour that is left in the clearance volume will be exactly equal to the work output obtained during the re-expansion process. Hence, the clearance for this special case does not impose any penalty on work input to the compressor. The total work input to the compressor during one cycle will then be equal to the area A-B-C-D-A on P-V diagram.

The specific work with and without clearance will be given by the same expression:

$$W_{id} = \int_{P_e}^{P_c} V.dP = P_e V_e \left(\frac{n}{n-1}\right)\left[\left(\frac{P_c}{P_e}\right)^{\frac{n-1}{n}} - 1\right]$$

However, since the mass of refrigerant compressed during one cycle is different with and without clearance, the power input to the compressor will be different with and without

clearance. The power input to the compressor and mean effective pressure (mep) with clearance are given by:

$$W_c = \dot{m}\, w_{id} = \left(\eta_{V.cl}\, \frac{\dot{V}_{sw}}{V_e}\right) w_{id}$$

$$mep = \eta_{V.cl}\, \frac{w_{id}}{v_e}$$

Thus, the power input to the compressor and mep decrease with clearance due to decrease in mass flow rate with clearance.

If the process is reversible and adiabatic (i.e., n = k), then the power input to the compressor with clearance is given by:

$$W_c = \left(\eta_{V.cl}\, \frac{\dot{V}_{sw}}{v_e}\right)(h_B - h_A) = \left(\eta_{V.cl}\, \frac{\dot{V}_{sw}}{v_e}\right) \Delta h_{c,s}$$

Where Δh is the isentropic work of compression (kJ/kg).

VOLUMETRIC EFFICIENCY

In reality, the piston cannot expel all the gas and a clearance volume is needed between the piston and the cylinder head. This means that a small volume of compressed gas is trapped in the cylinder at point 3. When the piston moves away from the cylinder head, the compressed gas expands by the law $pV^n = C$ until the pressure falls to the level of the inlet pressure. At point 4 the inlet valve opens and gas is drawn in. The volume drawn in from 4 to 1 is smaller than the swept volume because of this expansion.
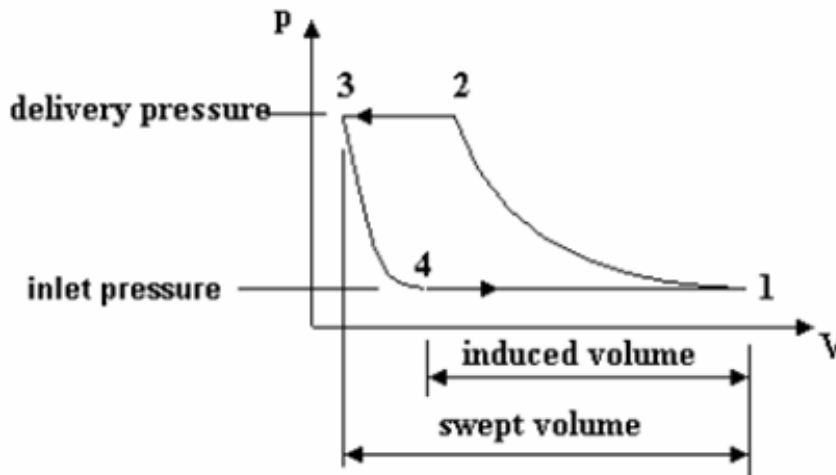
The volumetric efficiency is defined as

$$\eta_{vol} = \frac{\text{Induced Volume}}{\text{Swept Volume}}$$

This efficiency is made worse if leaks occur past the valves or piston.

The clearance ratio is defined as c = Clearance volume/Swept volume.

Ideally the process 2 to 3 and 4 to 1 are isothermal. That is to say, there is no temperature change during induction and expulsion.

V = 0.013/0.828 = 0.0157 m^3/s

Crank speed = 6 rev/s so the swept volume = 0.0157/6 = 2.62 dm^3.

V_1 = V + 0.05V = 1.05V

$$\eta_{vol} = \frac{\text{Induced Volume}}{\text{Swept Volume}}$$

$$\eta_{vol} = \frac{0.828V}{V} = 82.8 \%$$

WORKED EXAMPLE No.1

Gas is compressed in a reciprocating compressor from 1 bar to 6 bar. The FAD is 13 dm^3/s. The clearance ratio is 0.05. The expansion part of the cycle follows the law $pV^{1.2}$ = C. The crank speed is 360 rev/min. Calculate the swept volume and the volumetric efficiency.

Swept Volume = V   Clearance volume = 0.05 V

Consider the expansion from 3 to 4 on the p-V diagram.

p_4 = 1 bar p_3 = 6 bar. $p_3V_3^{1.2} = p_4V_4^{1.2}$

$6(0.05V)^{1.2} = 1(V_4^{1.2})$

V_4 = 0.222V or 22.2%% of V

F.A.D. = 0.013 m^3/s.

Induced volume = $V_1 - V_4$ = 1.05V - 0.222V = 0.828V

Induced volume = 0.013

THERMODYNAMI

Second Stage (2)

Dr.Abbas F mahmood

GAS-TURBINE WITH REHEATER
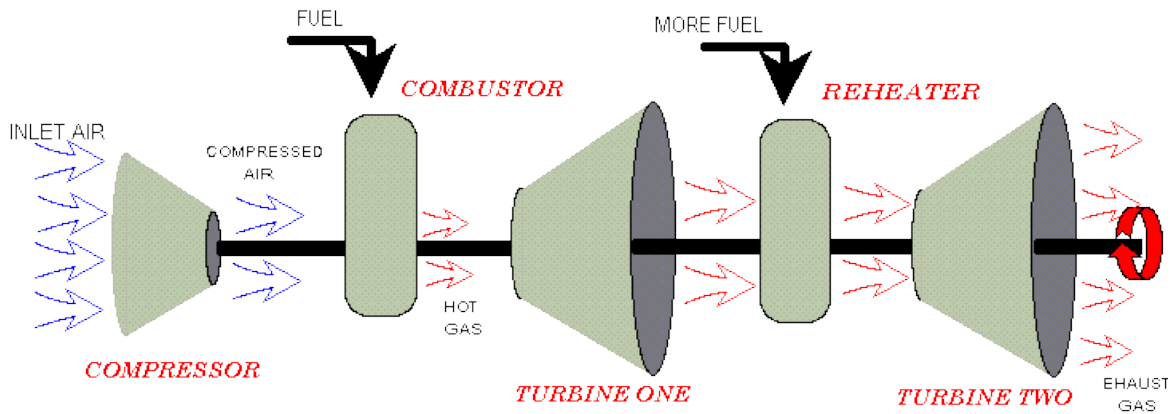
1. GAS TURBINE CYCLE—BRAYTON CYCLE
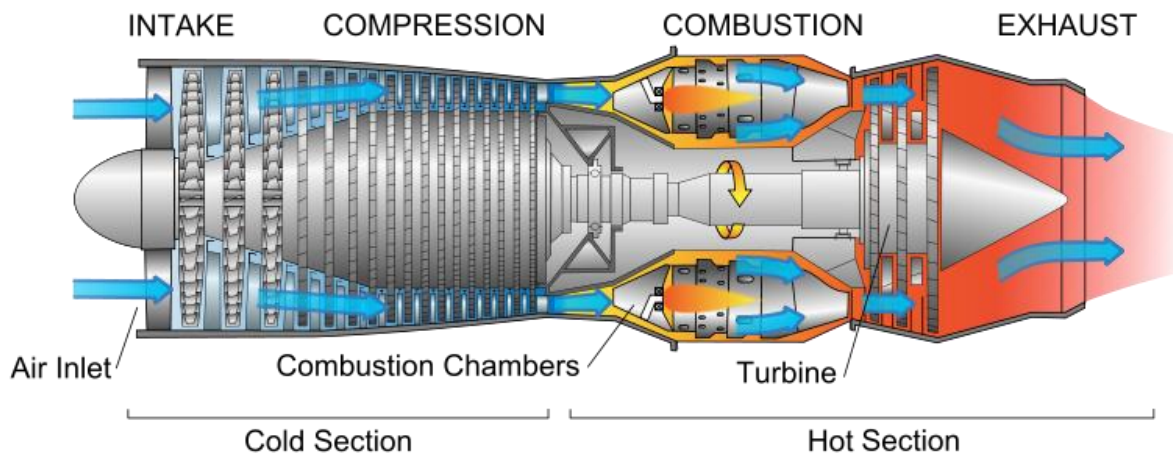
Ideal Brayton Cycle

Brayton cycle is *a constant pressure cycle for a perfect gas*. It is also called **Joule cycle.**

The heat transfers are achieved in reversible constant pressure heat exchangers. An ideal gasturbine plant would perform the processes that make up a Brayton cycle. The cycle isshown in theFig. 1 (*a*) and it is represented on *p-v* and *T-s* diagrams as shown in Figs. 1 (*b*)and (*c*).

The various operations are as follows:

Operation 1-2. The air is compressed isentropically from the lower pressure p_1 to the upper pressure p_2, the temperature rising from T_1 to T_2. No heat flow occurs.

Operation 2-3. Heat flows into the system increasing the volume from V_2 to V_3 and temperature from T_2 to T_3 whilst the pressure remains constant at p_2. Heat received = $mc_p(T_3 - T_2)$.

Operation 3-4. The air is expanded isentropically from p_2 to p_1, the temperature falling from T_3 to T_4. No heat flow occurs.

Operation 4-1. Heat is rejected from the system as the volume decreases from V_4 to V_1 and the temperature from T_4 to T_1 whilst the pressure remains constant at p_1. Heat rejected = $mc_p(T_4 - T_1)$.

2
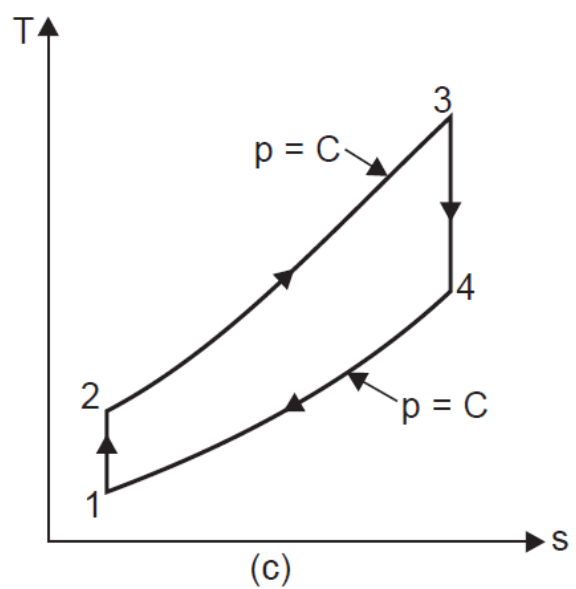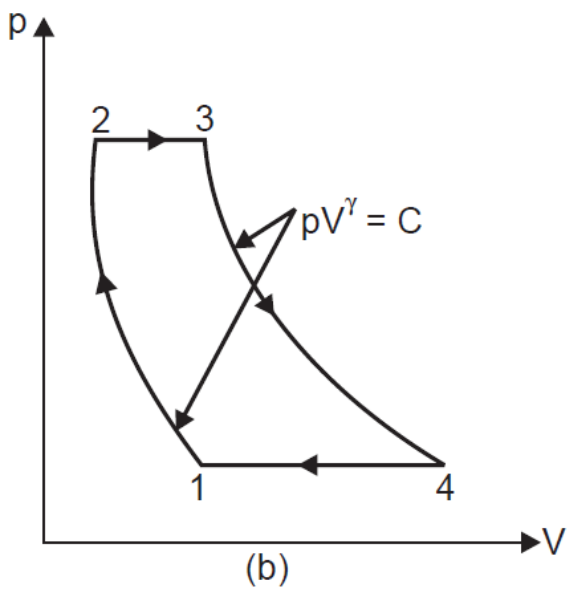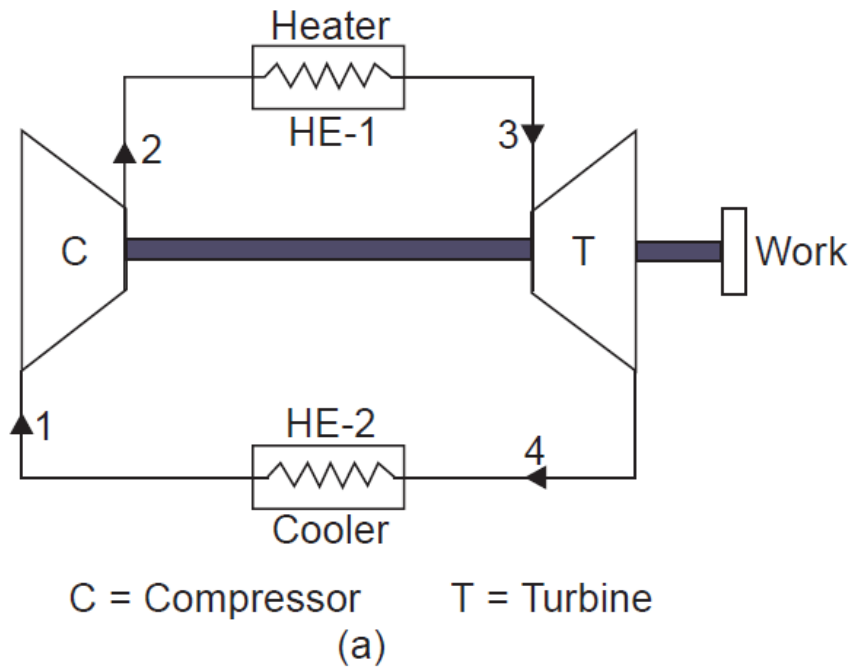
Fig. 1.1. Brayton cycle: (*a*) Basic components of a gas turbine power plant (*b*) *p-V* diagram (*c*) *T-s* diagram.

$$\eta_{\text{air-standard}} = \frac{\text{Work done}}{\text{Heat received}}$$

$$= \frac{\text{Heat received/cycle} - \text{Heat rejected/cycle}}{\text{Heat received/cycle}}$$

$$= \frac{mc_p\,(T_3 - T_2) - mc_p\,(T_4 - T_1)}{mc_p\,(T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

Now, from isentropic expansion,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$T_2 = T_1\left(r_p\right)^{\frac{\gamma-1}{\gamma}}$, where r_p= pressure ratio.

Similarly

$\frac{T_3}{T_4} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$ or $T_3 = T_4\left(r_p\right)^{\frac{\gamma-1}{\gamma}}$

$$\therefore \quad \eta_{air-standard} = \frac{T_4 - T_1}{T_4\left(r_p\right)^{\frac{\gamma-1}{\gamma}} - T_1\left(r_p\right)^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{1}{\left(r_p\right)^{\frac{\gamma-1}{\gamma}}} \dots\dots\dots\dots\dots\dots\dots\dots\dots (1.1)$$
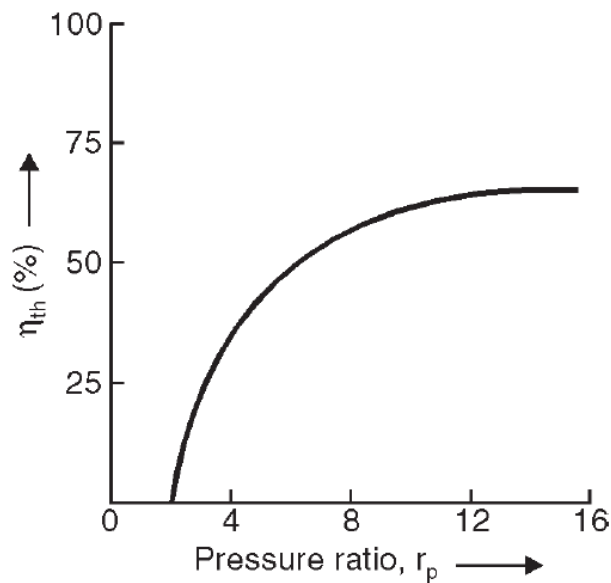
Fig. 1.2. Effect of pressure ratio on the efficiency of Brayton cycle.

The eqn. (1.1) shows that the *efficiency of the ideal joule cycle increases with the pressure ratio. The absolute limit of upper pressure is determined by the limiting temperature of the material of the turbine at the point at which this temperature is reached by the compression process alone, no further heating of the gas in the combustion chamber would be permissible and the work of expansion would ideally just balance the work of compression so that no excess work would be available for external use.*

Example 1.

Air enters the compressor of a gas turbine plant operating on Brayton cycle at 101.325 kPa, 27°C. The pressure ratio in the cycle is 6. Calculate the maximum temperature in the cycle and the cycle efficiency. Assume $W_T = 2.5\ W_C$, where W_T and W_C are the turbine and the compressor work respectively. Take $\gamma = 1.4$.

Example 2.

A gas turbine is supplied with gas at 5 bar and 1000 K and expands it adiabatically to 1 bar. The mean specific heat at constant pressure and constant volume are 1.0425 kJ/kg K and 0.7662 kJ/kg K respectively.

(i) Draw the temperature-entropy diagram to represent the processes of the simple gas turbine system.

(ii) Calculate the power developed in kW per kg of gas per second and the exhaust gas temperature.

Example 3.
An isentropic air turbine is used to supply 0.1 kg/s of air at 1bar and at 285 K to a cabin. The pressure at inlet to the turbine is 4bar. Determine the temperature at turbine inlet and the power developed by the turbine. Assume c_p= 1.0 kJ/kg K.

Open Cycle Gas Turbine—Actual Brayton Cycle

Refer Fig. 1.3. The fundamental gas turbine unit is one operating on the open cycle in which a rotary compressor and a turbine are mounted on a common shaft. Air is drawn into the compressor and after compression passes to a combustion chamber. Energy is supplied in the combustion chamber by spraying fuel into the air stream, and the resulting hot gases expand through the turbine to the atmosphere. In order to achieve net work output from the unit, the turbine must develop more gross work output than is required to drive the compressor and to overcome mechanical losses in the drive. The products of combustion coming out from the turbineare exhausted to the atmosphere as they cannot be used any more. The working fluids (air andfuel) must be replaced continuously as they are exhausted into the atmosphere.
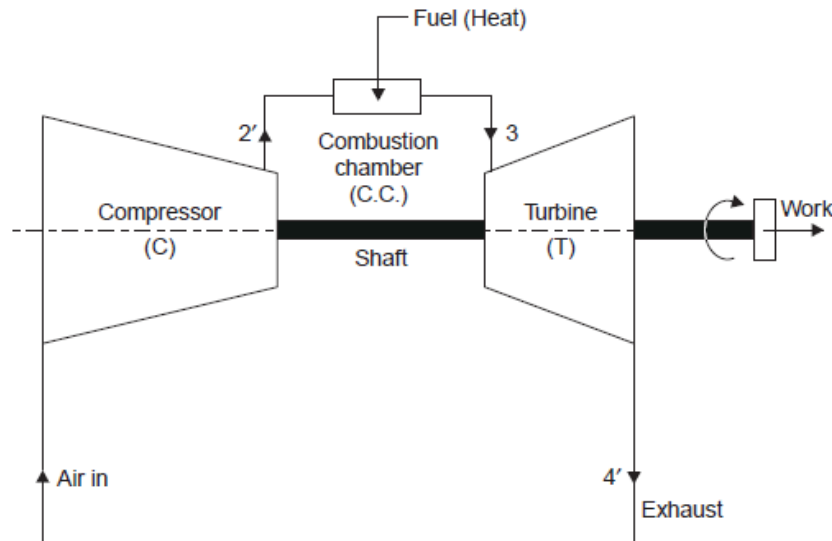
Fig. 1.3. Open cycle gas turbine.

If pressure loss in the combustion chamber is neglected, this cycle may be drawn on a *T-s* diagram as shown in Fig. 1.4.

_ 1-2′ represents : *irreversible adiabatic compression.*
_ 2′-3 represents : *constant pressure heat supply in the combustion chamber.*
_ 3-4′ represents : *irreversible adiabatic expansion.*
_ 1-2 represents : *ideal isentropic compression.*
_ 3-4 represents : *ideal isentropic expansion.*

Assuming change in kinetic energy between the various points in the cycle to be negligibly small compared with enthalpy changes and then applying the flow equation to each part of cycle, for unit mass, we have

Work input (compressor) $= cp\,(T2' - T1)$
Heat supplied (combustion chamber) $= cp\,(T3 - T2')$

Work output (turbine) = cp $(T3 - T4')$

∴ Net work output = Work output – Work input
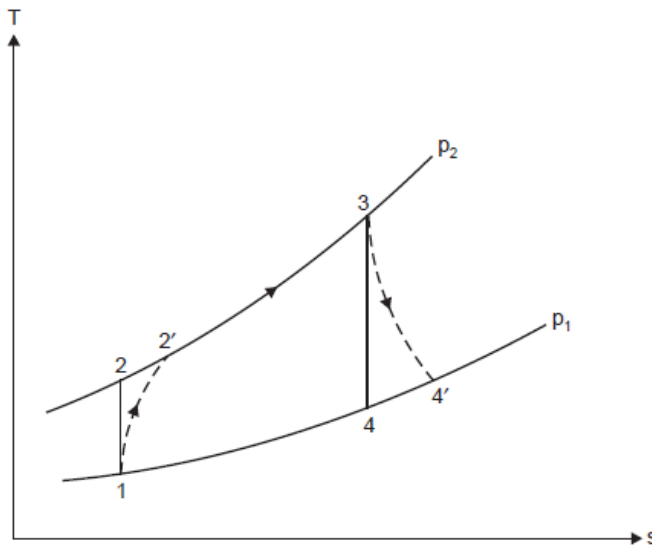
= cp $(T3 - T4') - cp(T2' - T1)$

Fig. 1.4. T-s diagram

$$\eta_{thermal} = \frac{\text{Net work output}}{\text{Heat supplied}} = \frac{c_p(T_3 - T_{4/}) - c_p(T_{2/} - T_1)}{c_p(T_3 - T_{2/})}$$

Compressor isentropic efficiency, η_{comp}

$$= \frac{\text{Work input required in isentropic compression}}{\text{Actual work required}} = \frac{c_p(T_2 - T_1)}{c_p(T_{2/} - T_1)} = \frac{(T_2 - T_1)}{(T_{2/} - T_1)}$$

Turbine isentropic efficiency, $\eta_{turbine}$

$$= \frac{\text{Actual work output}}{\text{Isentropic work output}} = \frac{c_p(T_3 - T_{4/})}{c_p(T_3 - T_4)} = \frac{(T_3 - T_{4/})}{(T_3 - T_4)}$$

Note. With the variation in temperature, the value of the specific heat of a real gas varies, and also in the open cycle, the specific heat of the gases in the combustion chamber and in turbine is different from that in the compressor because fuel has been added and a chemical change has taken place. Curves showing the variation of *cp* with temperature and air/fuel ratio can be used, and a suitable mean value of *cp* and hence γ can be found out. It is usual in gas turbine practice to assume fixed mean value of *cp* and γ for the expansion process, and fixed mean values of *cp* and γ for the compression process. In an open cycle gas turbine unit the mass flow of gases in turbine is greater than that in compressor due to mass of fuel burned, but it is possible to neglect mass of fuel, since the air/ fuel ratios used are large. Also, in many cases, air is bled from the

compressor for cooling purposes, or in the case of air-craft at high altitudes, bled air is used for de-icing and cabin air-conditioning. This amount of air bled is approximately the same as the mass of fuel injected therein.

Methods for Improvement of Thermal Efficiency of Open Cycle Gas Turbine Plant

The following methods are employed *to increase the specific output and thermal efficiency of the plant :*
1. *Intercooling 2. Reheating 3. Regeneration.*

1. Intercooling.

A compressor in a gas turbine cycle utilises the major percentage of power developed by the gas turbine. The work required by the compressor can be reduced by compressing the air in two stages and *incorporating an intercooler* between the two as shown in Fig. 1.5. The corresponding *T-s* diagram for the unit is shown in Fig. 1.6. The actual processes take place as follows :

1-2′ ... L.P. (Low pressure) compression
2′-3 ... Intercooling
3-4′ ... H.P. (High pressure) compression
4′-5 ... C.C. (Combustion chamber)-heating
5-6′ ... T (Turbine)-expansion
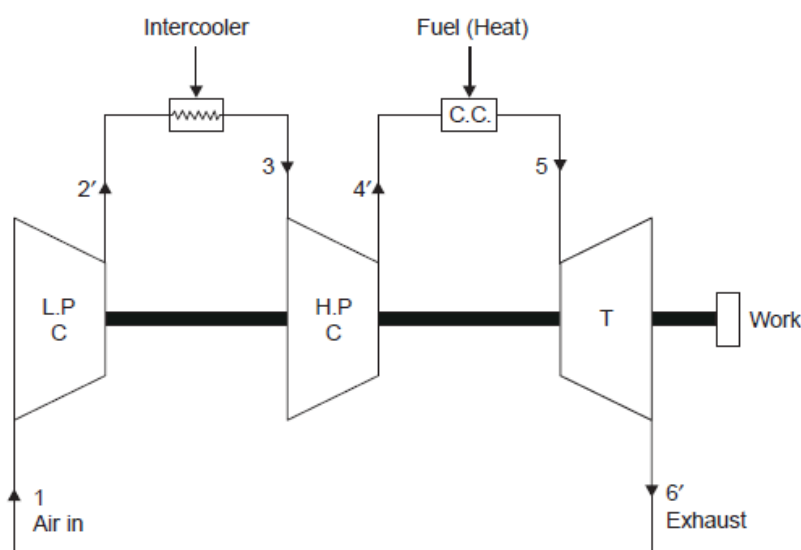
Fig. 1.5 Turbine plant with intercooler.

The ideal cycle for this arrangement is 1-2-3-4-5-6; the compression process without intercooling is shown as 1-*L'* in the actual case, and 1-*L* in the ideal isentropic case. Now,

Work input (*with intercooling*)

$$= c_p(T2' - T1) + c_p(T4' - T3) \ldots\ldots\ldots\ldots\ldots(1.2)$$

Work input (*without intercooling*)

$$= c_p(TL' - T1) = c_p(T2' - T1) + c_p(TL' - T2') \ldots\ldots\ldots\ldots\ldots(1.3)$$

By comparing equation (1.3) with equation (1.2) it can be observed that the *work input with intercooling is less than the work input with no intercooling*, when *cp* ($T4'$– $T3$) is *less* than $c_p(TL' - T2')$. This is so if it is assumed that isentropic efficiencies of the two compressors, operating separately, are each equal to the isentropic efficiency of the single compressor which would be required if no intercooling were used. Then ($T4'$ – $T3$) < (TL' – $T2'$) since the pressure lines diverge on the *T-s* diagram from left to the right.
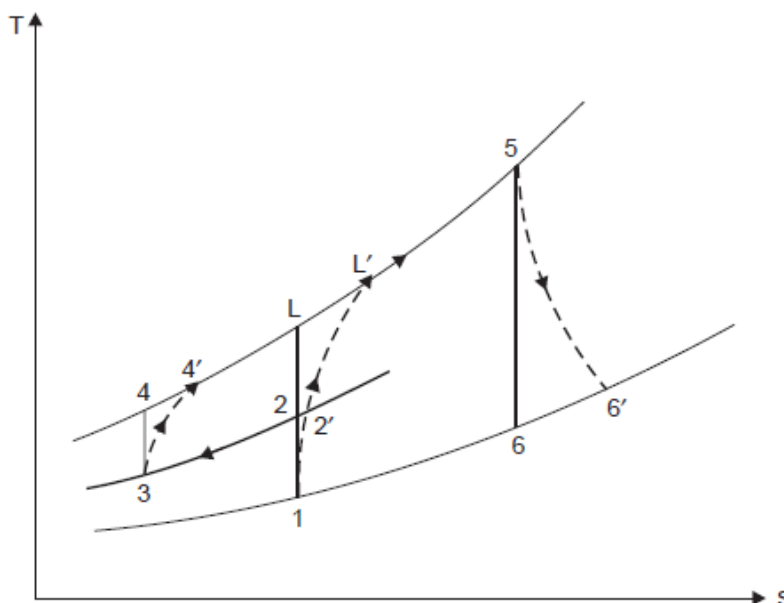
Fig.1.6. *T-s* diagram for the unit.

$$\text{Again, work ratio} = \frac{\text{Net work output}}{\text{Gross work output}}$$

$$= \frac{\text{Work of expansion Work of compression}}{\text{Work of expansion}}$$

From this we may conclude that *when the compressor work input is reduced then the work ratio is increased.*

However the heat supplied in the combustion chamber when intercooling is used in the cycle, is given by, Heat supplied *with intercooling* $= cp(T_5 - T_4')$

Also the heat supplied when intercooling is not used, with the same maximum cycle temperature T_5, is given by

Heat supplied *without intercooling* $= cp\ (T_5 - TL')$

Thus, the *heat supplied when intercooling is used is greater than with no intercooling. Although the net work output is increased by intercooling it is found in general that the increase in heat to be supplied causes the thermal efficiency to decrease.* When intercooling is used a supply of cooling water must be readily available. The additional bulk of the unit may offset the advantage to be gained by increasing the work ratio.

2. Reheating.

The output of a gas turbine can be amply improved by expanding the gases in two stages with a *reheater* between the two as shown in Fig. 13.39. The H.P. turbine drives the compressor and the L.P. turbine provides the useful power output. The corresponding *T-s* diagram is shown in Fig. 13.40. The line 4′-L′ represents the expansion in the L.P. turbine if reheating is *not* employed.
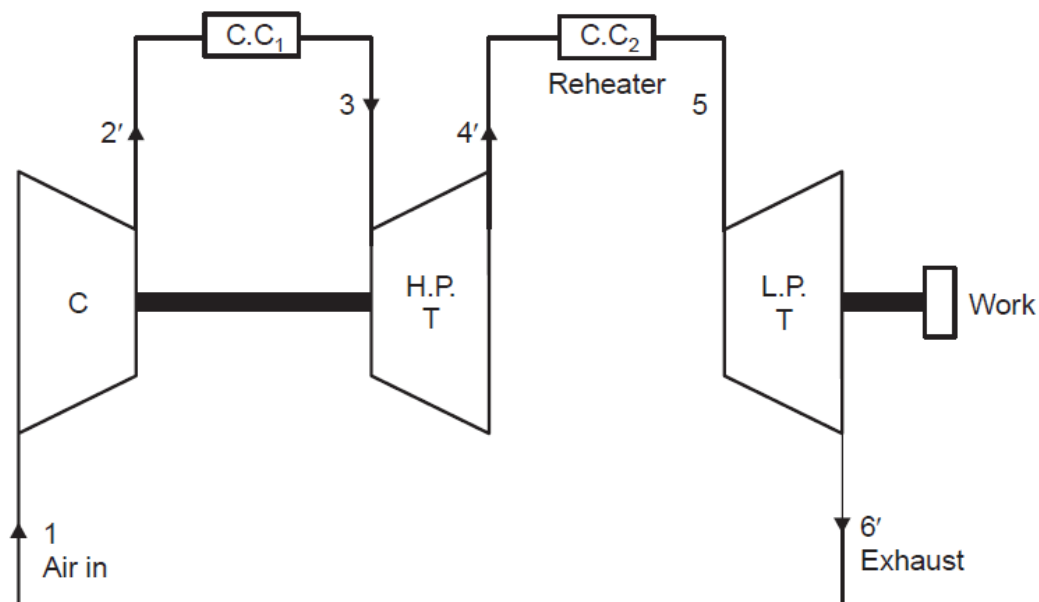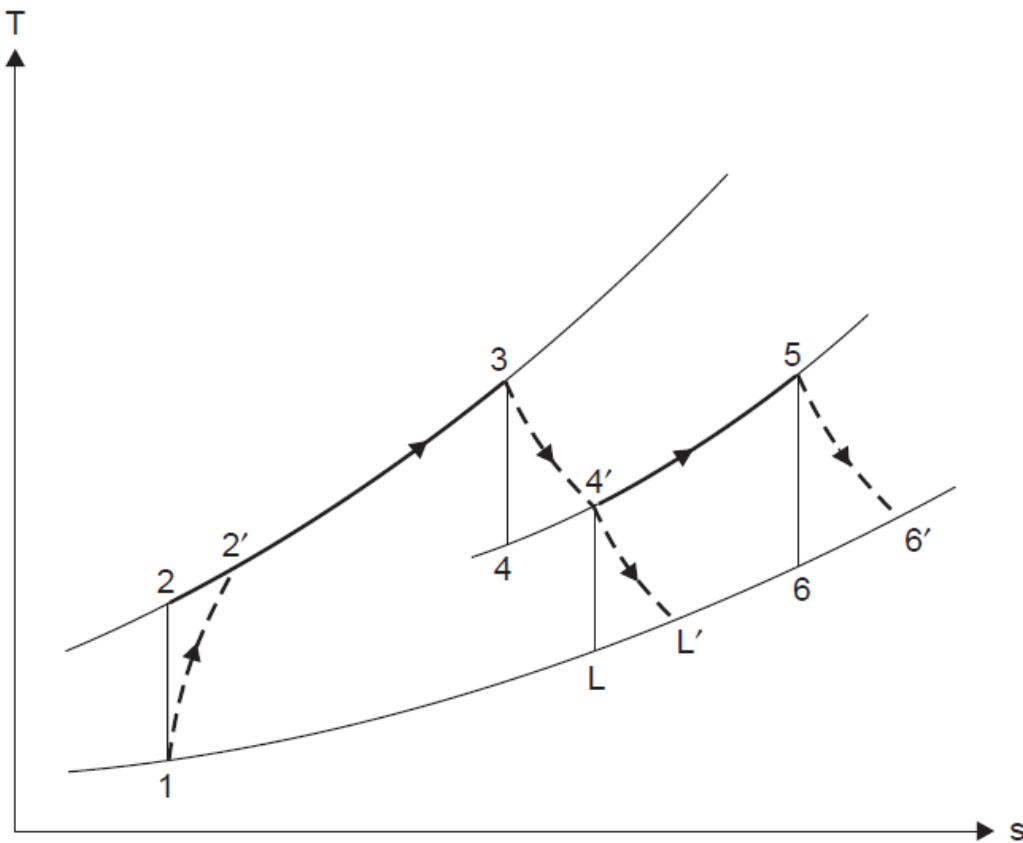

Fig. 1.7. Gas turbine with reheater.

Fig. 1.8. *T-s* diagram for the unit.

Neglecting mechanical losses the work output of the H.P. turbine must be exactly equal to the work input required for the compressor i.e., $c_{pa}(T_2' - T_1) = c_{pg}(T_3 - T_4')$
The work output (net output) of L.P. turbine is given by,

$$\textit{Net work output (with reheating)} = c_{pg}\ (T_5 - T_6')$$
$$\textit{and Net work output (without reheating)} = cpg\ (T_4' - T_L')$$

Since the pressure lines diverge to the right on *T-s* diagram it can be seen that the temperature difference $(T_5 - T_6')$ is always *greater* than $(T_4' - T_L')$, so that *reheating increases the net work output.*

Although net work is increased by reheating the heat to be supplied is also increased, and the net effect *can be to reduce the thermal efficiency*

$$\textit{Heat supplied} = c_{pg}\ (T_3 - T_2') + c_{pg}\ (T_5 - T_4').$$

Note. c_{pa} and c_{pg} stand for specific heats of air and gas respectively at constant pressure.

3. Regeneration.

The exhaust gases from a gas turbine carry a large quantity of heat with them since their temperature is far above the ambient temperature. They can be used to heat the air coming from the compressor thereby reducing the mass of fuel supplied in the combustion chamber. Fig. 1.9 shows a gas turbine plant with a regenerator. The corresponding *T-s* diagram is shown in Fig. 1.10. 2'-3 represents the heat flow into the compressed air during its passage through the heat exchanger and 3-4 represents the

heat taken in from the combustion of fuel. Point 6 represents the temperature of exhaust gases at discharge from the heat exchanger. The maximum temperature to which the air could be heated in the heat exchanger is ideally that of exhaust gases, but less than this is obtained in practice because a temperature gradient must exist for an unassisted transfer of energy. The *effectiveness* of the heat exchanger is given by:
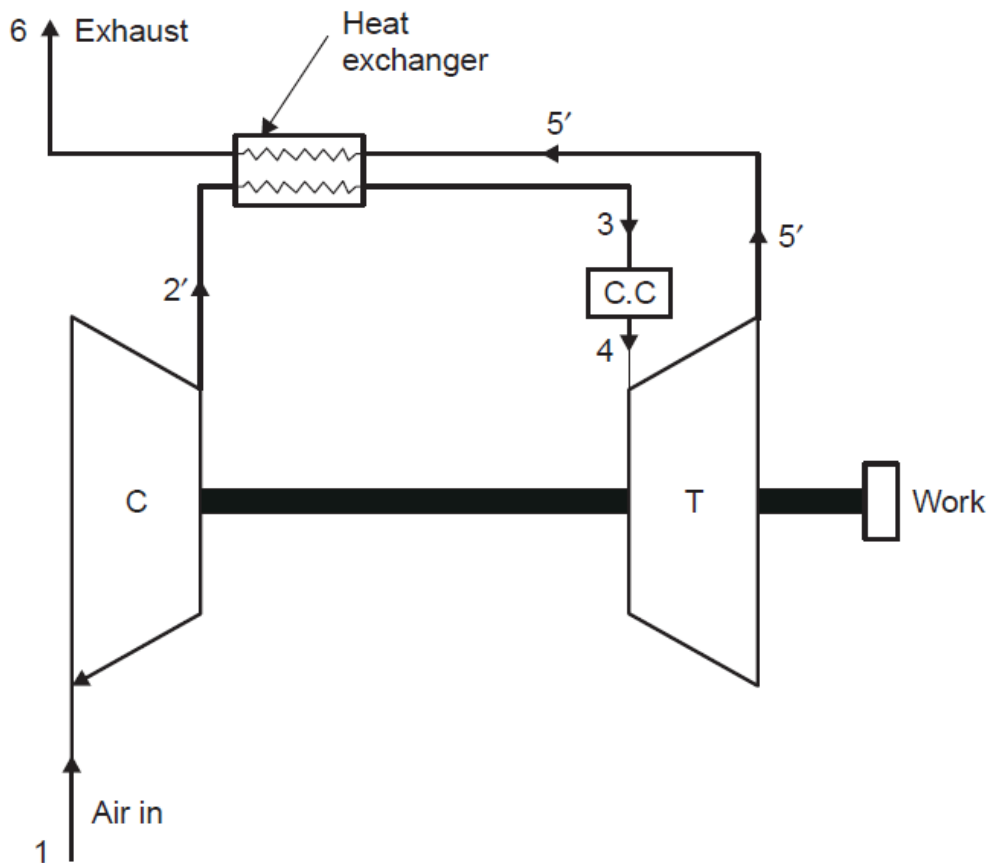

Fig. 1.9 Gas turbine with regenerator.

$$Effectiveness \quad \varepsilon = \frac{Increase\ in\ enthalpy\ per\ kg\ of\ air}{Available\ increase\ in\ enthalpy\ per\ kg\ of\ air}$$

$$= \frac{(T3 - T2')}{(T5' - T2')}$$

(assuming *cpa* and *cpg* to be equal)

A heat exchanger is usually used in large gas turbine units for marine propulsion or industrial power.
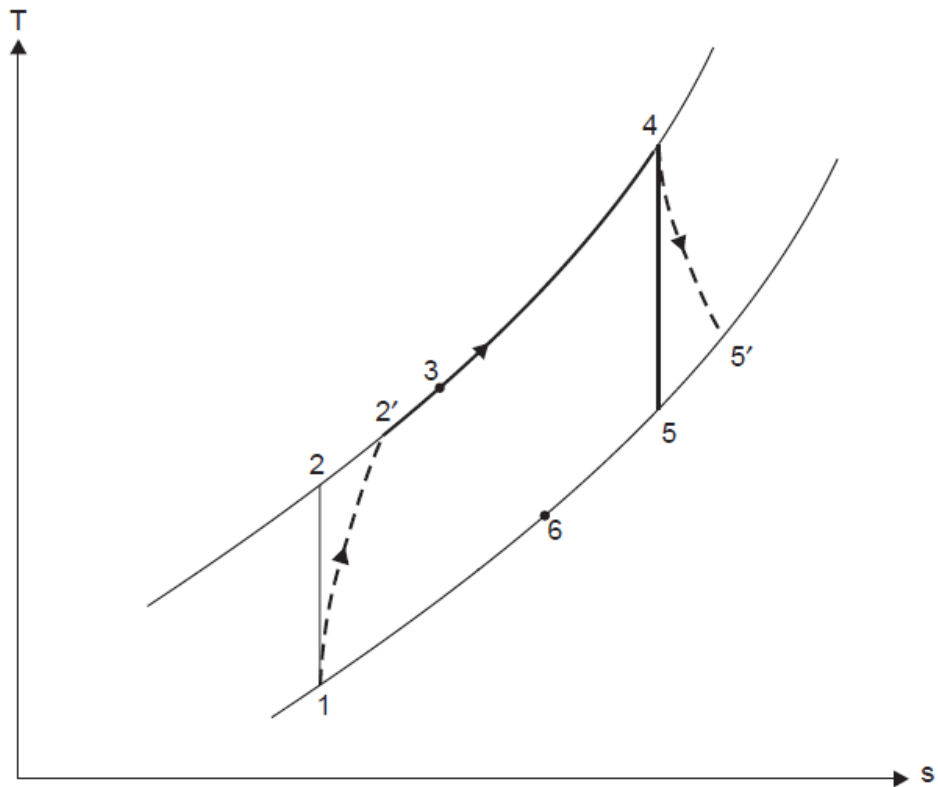
Fig. 1.10. *T-s* diagram for the unit.

Example.
Air enters the compressor of a gas turbine plant operating on Brayton cycle at 101.325 kPa, 27°C. The pressure ratio in the cycle is 6. Calculate the maximum temperature in the cycle and the cycle efficiency. Assume $W_T = 2.5 \, W_C$, where W_T and W_C are the turbine and the compressor work respectively. Take γ = 1.4.

Types of gas turbines

1. **Impulse**: There is no change in the pressure of the steam as it passes through the moving blades. There is change only in the velocity of the gas flow.

2. **Reaction**: There is change in both pressure and velocity as the gas flows through the moving blades.

velocity compounded Impulse turbine

The velocity compounded Impulse turbine was first proposed by C G Curtis to solve the problem of single stage Impulse turbine for use of high gas pressure and temperature.

The rings of moving blades are separated by rings of fixed blades. The moving blades are keyed to the turbine shaft and the fixed blades are fixed to the casing. The high pressure gas coming from the boiler is expanded in the nozzle first. The Nozzle converts the pressure energy of the gas into kinetic energy. It is interesting to note that the total enthalpy drop and hence the pressure drop occurs in the nozzle. Hence, the pressure thereafter remains constant.
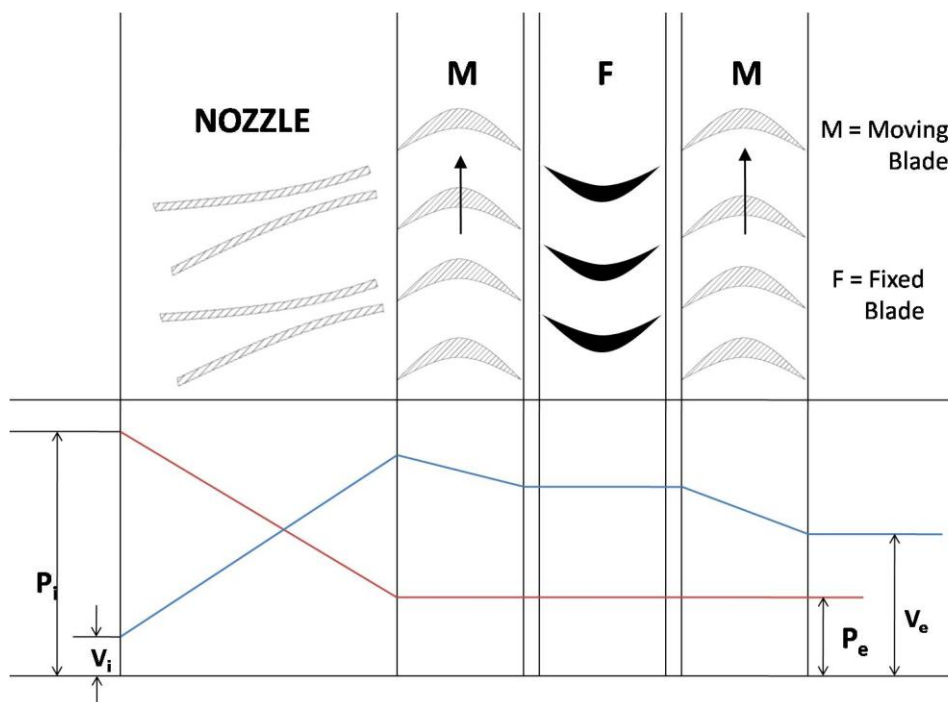
Fig-1.11: Schematic Diagram of Curtis Stage Impulse Turbine

This high velocity steam is directed on to the first set (ring) of moving blades. As the gas flows over the blades, due the shape of the blades, it imparts some of its momentum to the blades and loses some velocity. Only a part of the high kinetic energy is absorbed by these blades. The remainder is exhausted on to the next ring of fixed blade. The function of the fixed blades is to redirect the gas leaving from the first

ring of moving blades to the second ring of moving blades. There is no change in the velocity of the steam as it passes through the fixed blades. The gas then enters the next ring of moving blades; this process is repeated until practically all the energy of the gas has been absorbed.

A schematic diagram of the Curtis stage impulse turbine, with two rings of moving blades one ring of fixed blades is shown in *figure 1.3*. The figure also shows the changes in the pressure and the absolute steam velocity as it passes through the stages.

Where,

P_i = *pressure of gas at inlet* V_i = *velocity of gas at inlet* P_o = *pressure of gas at outlet* V_o = *velocity of gas at outlet*

In the above figure there are two rings of moving blades separated by a single of ring of fixed blades. As discussed earlier the entire pressure drop occurs in the nozzle, and there are no subsequent pressure losses in any of the following stages. Velocity drop occurs in the moving blades and not in fixed blades.

The Single-Stage Impulse Turbine

The *single-stage impulse turbine* is also called the *de Laval turbine* after its inventor. The turbine consists of a single rotor to which impulse blades are attached. The steam is fed through one or several convergent-divergent nozzles which do not extend completely around the circumference of the rotor, so that only part of the blades is impinged upon by the steam at any one time. The nozzles also allow governing of the turbine by shutting off one or more them.The velocity diagram for a single-stage impulse has been shown in *Fig. 1.12*shows the velocity diagram indicating the flow through the turbine blades.
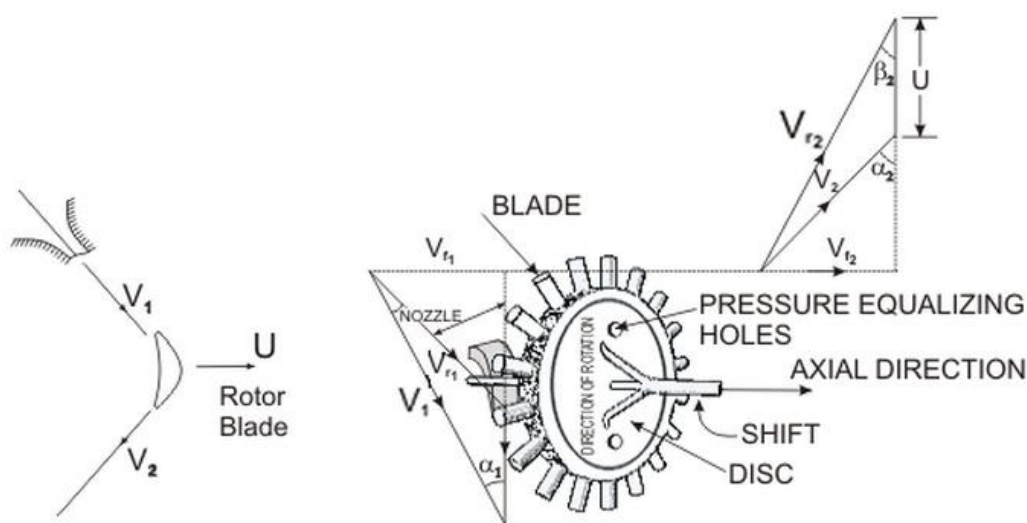
Figure

1.12 Schematic diagram of an Impulse turbine

V_1 and V_2 = Inlet and outlet absolute velocity

V_{r_1} and V_{r_2} = Inlet and outlet relative velocity (Velocity relative to the rotor blades.)

U = mean blade speed

α_1 = nozzle angle, α_2 = absolute fluid angle at outlet

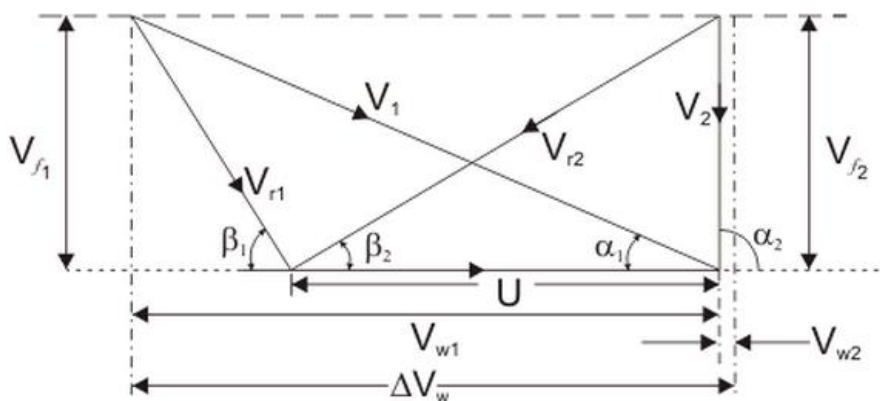It is to be mentioned that all angles are with respect to the tangential velocity (in the direction of U)

Figure 1.13   Velocity diagram of an Impulse Turbine

β_1 and β_2 = Inlet and outlet **blade angles**

V_{w1} and V_{w2} = Tangential or whirl component of absolute velocity at inlet and outlet

V_{f1} and V_{f2} = Axial component of velocity at inlet and outlet

Tangential force on a blade,

$$F_u = \dot{m}(V_{w1} - V_{w2})$$

(mass flow rate X change in velocity in tangential direction)

or,

$$F_u = \dot{m}\,\Delta V_w$$

$$\dot{m}\,U\,\Delta V_w$$

Power developed

18

Blade efficiency or Diagram efficiency or Utilization factor is given by

$$\eta_b = \frac{\dot{m} \cdot U \cdot \Delta V_w}{m(V_1^2/2)} = \frac{Workdone}{K.E \quad supplied}$$

$$\text{stage efficiency} = \eta_s = \frac{Work\ done\ by\ the\ rotor}{Isentropic\ enthalpy\ drop}$$

$$\eta_b = \frac{2U\Delta V_w}{V_1^2}$$

$$\eta_s = \frac{\dot{m}U\Delta V_w}{\dot{m}(\Delta H)_{isen}} = \frac{\dot{m}U\Delta V_w}{\dot{m}\left(\dfrac{V_1^2}{2}\right)} \cdot \frac{\dot{m}(V_1^2/2)}{\dot{m}(\Delta H)_{isen}}$$

or,

or, $\eta_s = \eta_b \times \eta_n$ [$\eta_n = Nozzle\ efficiency$]

Optimum blade speed of a single stage turbine

$$\Delta V_w = V_{r1}\cos\beta_1 + V_{r2}\cos\beta_2$$

$$= V_{r1}\cos\beta_1 + \left(1 + \frac{V_{r2}}{V_{r1}} \cdot \frac{\cos\beta_2}{\cos\beta_1}\right)$$

$$= (V_1\cos\alpha_1 - U) + (1 + kc)$$

α_1 is of the order of 18^0 to 22^0 or,

Now, $(\rho)_{opt} = \left(\dfrac{U}{V_1}\right)_{opt} = \dfrac{\cos\alpha_1}{2}$ (For single stage impulse turbine)

∴ The maximum value of blade efficiency

$$(\eta_b)_{max} = 2(\rho\cos\alpha_1 - \rho^2)(1 + kc)$$

$$= \frac{\cos^2\alpha_1}{2}(1 + kc)$$

For equiangular blades,

$$(\eta_b)_{max} = \frac{\cos^2\alpha_1}{2}(1 + k)$$

If the friction over blade surface is neglected

$$(\eta_b)_{max} = \cos^2\alpha_1$$

Form the above figure it can be seen that the steam, after exiting from the moving blades, enters into the fixed blades. The fixed blades redirect the steam into the next set of moving blades. Hence, steam losses its velocity in multiple stages rather than in a single stage.

Optimum Velocity

It is the velocity of the blades at which maximum power output can be achieved. Hence, the optimum blade velocity for this case is,

$$V_{b,optimum} = \frac{V_{a1}cos\theta_1}{2n}$$

Where n is the number of stages.

It is interesting to note that this value of optimum velocity is 1/n times that of the single stage turbine. This means that maximum power can be produced at much lower blade velocities.

However, the work produced in each stage is not the same. The ratio of work produced in a 2 stage turbine is 3:1 as one move from higher to lower pressure. This ratio is 5:3:1 in three stage turbine and changes to 7:5:3:1 in a four stage turbine.

Disadvantages of Velocity Compounding

❖ Due to the high steam velocity there are high friction losses
❖ Work produced in the low-pressure stages is much less.
❖ The designing and fabrication of blades which can withstand such high velocities is difficult.

Pressure compounding of reaction turbine

As explained earlier a reaction turbine is one which there is pressure and velocity loss in the moving blades. The moving blades have a converging gas nozzle. Hence when the steam passes over the fixed blades, it expands with decrease in steam pressure and increase in kinetic energy.

This type of turbine has a number of rings of moving blades attached to the rotor and an equal number of fixed blades attached to the casing. In this type of turbine the pressure drops take place in a number of stages.

The steam passes over a series of alternate fixed and moving blades. The fixed blades act as nozzles i.e. they change the direction of the steam and also expand it. Then gas is passed on the moving blades, which further expand the steam and also absorb its velocity.

This is explained in **figure 1.14**. Where, symbols have the same meaning as above.
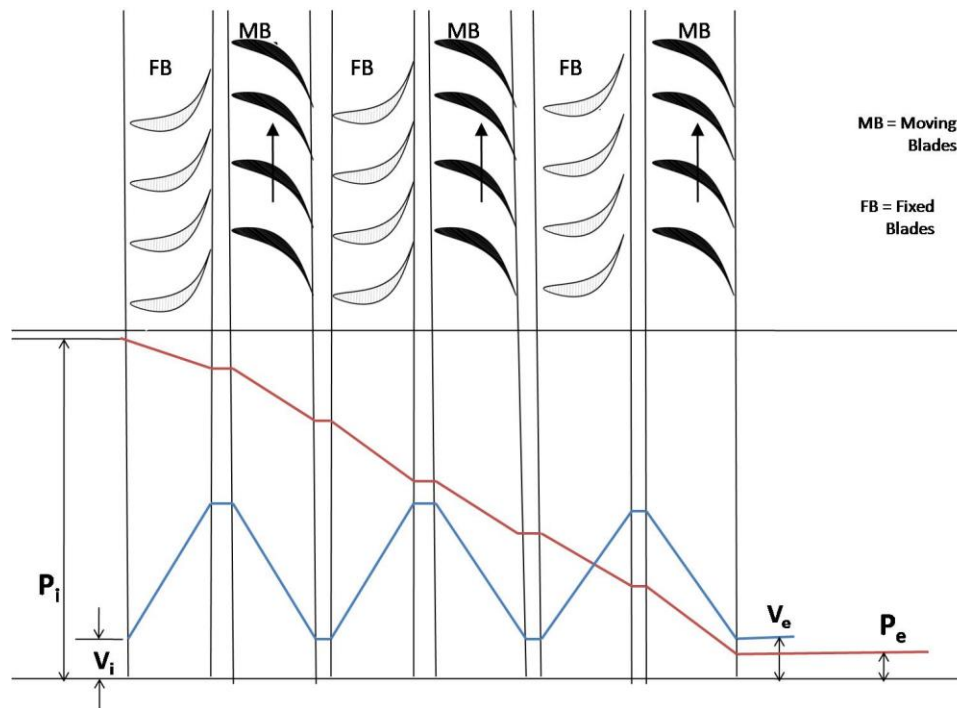
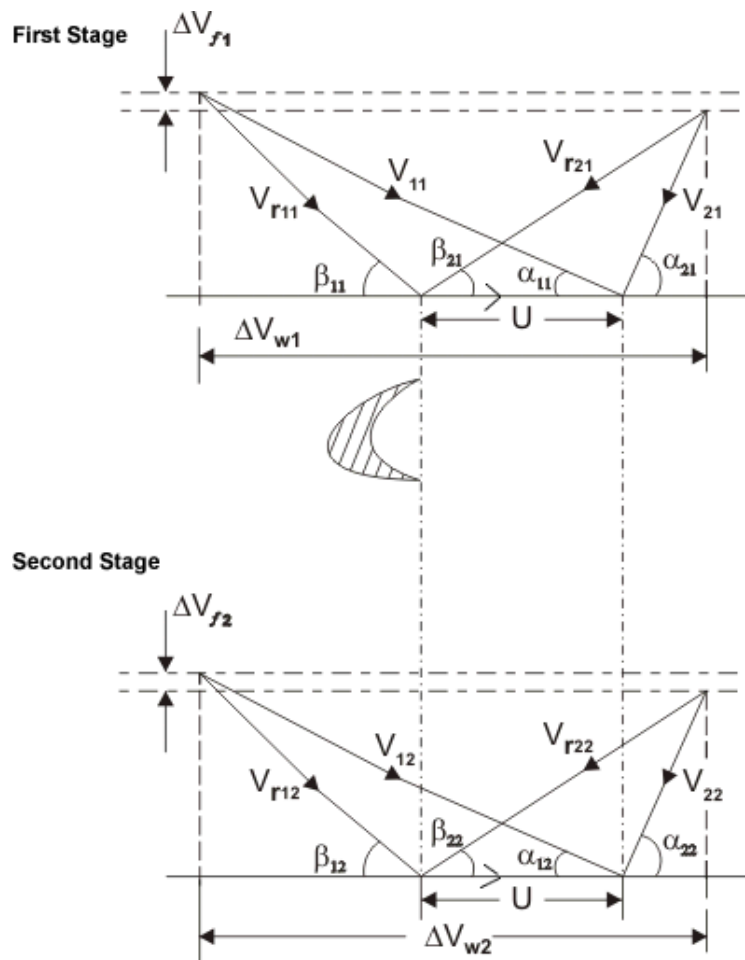*Fig1.14: Schematic Diagram of Pressure compounded Reaction Turbine*

Figure 1.15.Velocity diagrams for the Velocity-Compounded Impulse turbine

The fixed blades are used to guide the outlet steam/gas from the previous stage in such a manner so as to smooth entry at the next stage is ensured. K, the blade velocity coefficient may be different in each row of blades

Work done $= \dot{m} \, U (\Delta V_{w1} + \Delta V_{w2})$

End thrust $= \dot{m} (\Delta V_{f1} + \Delta V_{f2})$

The optimum velocity ratio will depend on number of stages and is given by $P_{opt} = \dfrac{\cos \alpha_{11}}{2n}$

2. *THERMODYNAMIC RELATIONS*

Types of Thermodynamic Properties

The thermodynamic state of a system can be characterized by its properties that can be classified as measured, fundamental, or derived properties. We want to develop relationships to relate the changes in the fundamental and derived properties in terms of the measured properties that are directly accessible from laboratory measurements. Some of the measured properties are P, v, T, composition, cp, and cv. The small letters are used to denote specific quantities for example v is specific volume.

The fundamental properties are internal energy u and entropy s. These properties arrive from the first and second law of thermodynamics. The first law states that energy is conserved, and the second law states that entropy of the universe always increases.

The derived properties are defined to facilitate the energy balance of systems in which the combination of internal energy and other properties often occurs. In open systems, the mass that crosses the boundary between the surroundings and the system always contributes to two terms in the energy balance: internal energy and flow work (Pv). For convenient we can define an enthalpy (h) as

$$h = u + Pv \ldots\ldots\ldots (1.2)$$

In terms of the total enthalpy H, we have

$$H = U + PV \ldots\ldots\ldots(2.2)$$

We can then make an enthalpy balance for an open system in which the flow work is included in the enthalpy term. *Figure 2.1* shows a raindrop created from the surrounding super saturated vapor in the atmosphere. Not only the energy U of the raindrop is needed but also some additional energy, equal to PV, is required to push the atmosphere out of the way to make room for the drop.
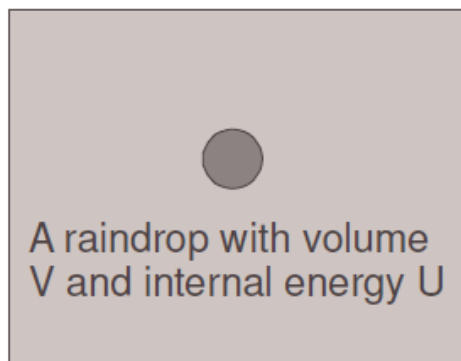

A raindrop with volume
V and internal energy U

Figure 2.1 An energy of U + PV is required to create a raindrop.

Enthalpy is the total energy we would need, to create a system out of nothing and put it in an environment with constant pressure P. Or, if we could completely annihilate a system, H isthe energy we could recover: the system's energy plus the work done by the collapsing atmosphere. However, we usually are not interested in the total energy needed or the total energy that can be recovered from a system. We will be more interested in the work involved in a system. For isothermal surroundings, the system can extract heat from the surroundings for free, so the work required to create the system from nothing is equal to the internal energy minus the heat received. And if we annihilate the system, we generally cannot recover all its energy as work since we have to dispose of its entropy by dumping some heat into the surroundings.

Therefore it is more convenient to define the *Helmholtz free energy*, *A*, for an environment at constant temperature *T*

$$A = U - TS …………….. (2.3)$$

A is the energy that must be provided as work if we create the system out of nothing. The heat extracted from the surroundings is TDS = $T(S_f - S_i)$ = TS_f where S_f is the system final entropy and S_i the system zero initial entropy. If we annihilate a system with initial entropy S_i, A is the amount of recovered work, since we have to dump some heat, equal to TSi, into the environment to get rid of the system's entropy.

Equation (3) includes all work, even the work done by the system's surroundings. If the system is in an isothermal and isobaric environment, it is more convenient to use the Gibbs free energy

$$G = U - TS + PV ………….... (2.4)$$

Gibbs free energy is the work required to create a system from nothing in an environment with constant P and constant temperature T. We usually are more interested in the change in states of a system rather than its creation or annihilation. We then want to look at the changes in A and G. The change in A at constant temperature is given by

$$dA = dU - TdS = Q + W - TdS ………. (2.5)$$

In this expression *Q* is the heat added and *W* is the work done on the system. If the process is reversible then *Q* = *T*D*S* and the change in *A* is precisely equal to the work

done on the system. If the process is irreversible then $Q < TDS$ and $DA < W$, the change in A is less than the work done on the system. For an environment with constant P and constant temperature T, the change in G is given by

$$dG = dU - TdS + PdV = Q + W - TdS + PdV \ldots \ldots \ldots (2.6)$$

For any process we have

$$Q - TdS \leq 0 (equal\ sign\ for\ reversible\ processes) \ldots \ldots (2.7)$$

The work term W consists of the work done by the environment, - PDV, and any "other" work done on the system.

$$W = - PdV + W_{other} \ldots \ldots \ldots (2.8)$$

Substituting equations (2.7) and (2.8) into equation (2.6) we obtain

$$dG \leq W_{other}\ at\ constant\ T,\ P \ldots \ldots (2.9)$$

THE MAXWELL RELATIONS

The equations that relate the partial derivatives of properties P, v, T, and s of a simple compressible system to each other are called the Maxwell relations. They are obtained from the four Gibbs equations by exploiting the exactness of the differentials of thermodynamic properties.

Two of the Gibbs relations were derived and expressed as

$$du = Tds - Pdv \ldots \ldots \ldots (2.10)$$
$$dh = Tds + vdP \ldots \ldots \ldots (2.11)$$

The other two Gibbs relations are based on two new combination properties—the **Helmholtz function** *a* and the **Gibbs function** *g*, defined as

$$a = u - Ts \ldots \ldots (2.12)$$
$$g = h - Ts \ldots \ldots (2.13)$$

Differentiating, we get

$$da = - sdT - Pdv \ldots \ldots \ldots (2.14)$$
$$dg = -sdT + vdP \ldots \ldots \ldots (2.15)$$

A careful examination of the four Gibbs relations reveals that they are of the form

$$dz = Mdx + Ndy \ldots\ldots\ldots\ldots(2.16)$$

with

$$\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial P}{\partial s}\right)_v \ldots\ldots\ldots\ldots\ldots (2.17)$$

$$\left(\frac{\partial T}{\partial P}\right)_s = \left(\frac{\partial v}{\partial s}\right)_p \ldots\ldots\ldots\ldots\ldots (2.18)$$

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial P}{\partial T}\right)_v \ldots\ldots\ldots\ldots\ldots (2.19)$$

$$\left(\frac{\partial s}{\partial P}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_P \ldots\ldots\ldots\ldots\ldots (2.20)$$

These are called the **_Maxwell relations_**. They are extremely valuable in thermodynamics because they provide a means of determining the change in entropy, which cannot be measured directly, by simply measuring the changes in properties P, v, and T. Note that the Maxwell relations given above are limited to simple compressible systems. However, other similar relations can be written just as easily for nonsimple systems such as those involving electrical, magnetic, and other effects.

Example

Verify the validity of the last Maxwell relation $\left(\frac{\partial s}{\partial P}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_P$ for steam at 250°C and 300 kPa.

THE CLAPEYRON EQUATION

The Maxwell relations have far-reaching implications in thermodynamics and are frequently used to derive useful thermodynamic relations. The Clapeyron equation is one such relation, and it enables us to determine the enthalpy change associated with a phase change (such as the enthalpy of vaporization h_{fg}) from a knowledge of P, v, and T data alone. Consider the third Maxwell relation, Eq. 9:

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial P}{\partial T}\right)_v$$

During a phase-change process, the pressure is the saturation pressure, which depends on the temperature only and is independent of the specific volume. That is, P_{sat} _ f (T_{sat}). Therefore, the partial derivative $\left(\frac{\partial P}{\partial T}\right)_v$ can be expressed as a total derivative $(dP/dT)_{sat}$, which is the slope of the saturation curve on a *P-T* diagram at a specified saturation state (Fig. 2).

This slope is independent of the specific volume, and thus it can be treated as a constant during the integration of Eq. (9) between two saturation states at the same temperature. For an isothermal liquid–vapor phase-change process, for example, the integration yields
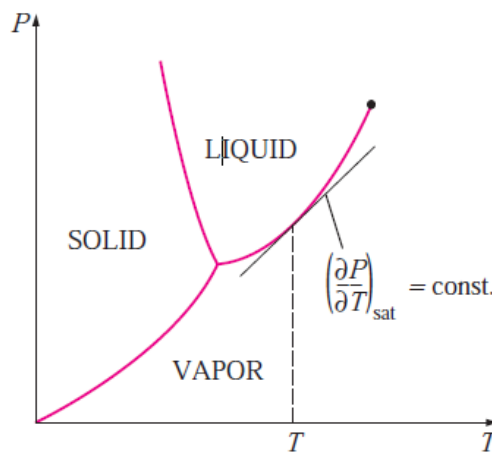

FIGURE 2.2. The slope of the saturation curve on a P-T diagram is constant at a constant T or P.

$$s_g - s_f = \left(\frac{dP}{dT}\right)_{sat}(v_g - v_f)\ldots\ldots\ldots\ldots(2.21)$$

Or

$$\left(\frac{dP}{dT}\right)_{sat} = \frac{s_{fg}}{v_{fg}} \ldots\ldots\ldots\ldots(2.22)$$

During this process the pressure also remains constant. Therefore, from

Eq. 2.2,

$$dh = Tds + vdP^{\nearrow 0} \rightarrow \int_f^g dh = \int_f^g Tds \rightarrow h_{fg} = Ts_{fg}$$

Substituting this result into Eq. 2.22, we obtain

$$\left(\frac{dP}{dT}\right)_{sat} = \frac{h_{fg}}{Tv_{fg}} \ldots\ldots\ldots\ldots(2.23)$$

which is called the Clapeyron equation after the French engineer and physicist E. Clapeyron (1799–1864). This is an important thermodynamic relation since it enables us to determine the enthalpy of vaporization h_{fg} at a given temperature by simply measuring the slope of the saturation curve on a P-T diagram and the specific volume of saturated liquid and saturated vapor at the given temperature. The Clapeyron equation is applicable to any phase-change process that occurs at constant temperature and pressure. It can be expressed in a general form as

$$\left(\frac{dP}{dT}\right)_{sat} = \frac{h_{12}}{Tv_{12}}$$

where the subscripts 1 and 2 indicate the two phases.

GENERAL RELATIONS FOR du, dh, ds, cv, AND cp

The state postulate established that the state of a simple compressible system is completely specified by two independent, intensive properties. Therefore, at least theoretically, we should be able to calculate all the properties of a system at any state once two independent, intensive properties are available.

This is certainly good news for properties that cannot be measured directly such as internal energy, enthalpy, and entropy. However, the calculation of these properties from measurable ones depends on the availability of simple and accurate relations between the two groups. In this section we develop general relations for changes in internal energy, enthalpy, and entropy in terms of pressure, specific volume,

temperature, and specific heats alone. We also develop some general relations involving specific heats. The relations developed will enable us to determine the changes in these properties. The property values at specified states can be determined only after the selection of a reference state, the choice of which is quite arbitrary.

✓ **The state of a simple compressible system is completely specified by two independent, intensive properties.**

✓ **Able to calculate all the properties of a system (u, h, and s) at any state once two independent, intensive properties are available.**

✓ **The calculation of these properties from measurable ones depends on the availability of simple and accurate relations between the two groups.**

✓ **Development of general relations for Δu, Δh, Δs in terms of P, v, T, and cv, cp.**

✓ **The property values at specified states can be determined only after the selection of a reference state, the choice of which is quite arbitrary**

INTERNAL ENERGY CHANGES

Consider u = u (T, v) and take its total differential (Eq.)

$$du = \left(\frac{\partial u}{\partial T}\right)_v dT + \left(\frac{\partial u}{\partial v}\right)_T dv$$

Using the definition of cv, we have

$$du = c_v\, dT + \left(\frac{\partial u}{\partial v}\right)_T dv \qquad \ldots\ldots(2.24)$$

Consider entrophy to be a function of T and v → s = s(T,v) and take its total differential

$$ds = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv \qquad \ldots\ldots(2.25)$$

Substituting this into the T ds relation du = T ds – P dv yields

$$du = T\left(\frac{\partial s}{\partial T}\right)_v dT + \left[T\left(\frac{\partial s}{\partial v}\right)_T - P\right] dv$$

......(2.26)

Equating the coefficients of dT and dv in Eqn. 12-25 & 12-27 gives

$$\left(\frac{\partial s}{\partial T}\right)_v = \frac{c_v}{T}$$

$$\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial s}{\partial v}\right)_T - P$$

......(2.27)

Using the 3ʳᵈ Maxwell relation (Eq. 12-28), we get

$$\left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_v - P$$

Substituting this into Eq. 12-25, we obtain the desired relation for du:

Δu general relation

$$du = c_v dT + \left[T\left(\frac{\partial P}{\partial T}\right)_v - P\right] dv$$

......(2.28)

The change in internal energy of a simple compressible system associated with a change of state from (T1, v1) to (T2,v2) is determined by integration of Eqn.12-29:

$$u_2 - u_1 = \int_{T_1}^{T_2} c_v \, dT + \int_{v_1}^{v_2} \left[T\left(\frac{\partial P}{\partial T}\right)_v - P\right] dv$$

......(2.29)

ENTHALPHY CHANGES

Consider the enthalpy to be a function of T and P → h = h(T,P), and take its total differential,

$$dh = \left(\frac{\partial h}{\partial T}\right)_P dT + \left(\frac{\partial h}{\partial P}\right)_T dP$$

Using the definition of cp, we have

$$dh = c_p\, dT + \left(\frac{\partial h}{\partial P}\right)_T dP$$

……**(2.29)**

Consider again the entrophy to be a function of T and P → s = s(T,P) and take its total differential,

$$ds = \left(\frac{\partial s}{\partial T}\right)_P dT + \left(\frac{\partial s}{\partial P}\right)_T dP$$

……(2.30)

Substituting this into the T ds relation dh = T ds + v dP gives

$$dh = T\left(\frac{\partial s}{\partial T}\right)_P dT + \left[v + T\left(\frac{\partial s}{\partial P}\right)_T\right] dP$$

……(2.31)

Equating the coefficients of dT and dP in Eq. 12-31 & 12-33, we obtain

$$\left(\frac{\partial s}{\partial T}\right)_P = \frac{c_p}{T}$$

$$\left(\frac{\partial h}{\partial P}\right)_T = v + T\left(\frac{\partial s}{\partial P}\right)_T$$

……(2.32)

Using the *4th Maxwell relation*, we have

$$\left(\frac{\partial h}{\partial P}\right)_T = v - T\left(\frac{\partial v}{\partial T}\right)_P$$

Substituting this into Eq. (29), we obtain the desired relation for dh:

Δh general relation

$$dh = c_p\, dT + \left[v - T\left(\frac{\partial v}{\partial T}\right)_P\right] dP$$

……(2.33)

The change in enthalpy of a simple compressible system associated with a change of state from (T1,P1) to (T2, P2) is determined by integration:

31

$$h_2 - h_1 = \int_{T_1}^{T_2} c_p \, dT + \int_{P_1}^{P_2} \left[v - T \left(\frac{\partial v}{\partial T} \right)_P \right] dP$$
......(2.34)

The other case can easily be determined by using the definition of enthalpy h = u + Pv:

$$h_2 - h_1 = u_2 - u_1 + (P_2 v_2 - P_1 v_1)$$
......(2.35)

ENTROPY CHANGES

The first relation is obtained by replacing the 1st partial derivative in the total differential ds (Eq. 12-26) by Eq. 12-28 and the second partial derivative by 3rd Maxwell relation (Eq. 12-18), yielding

$$ds = \frac{c_v}{T} \, dT + \left(\frac{\partial P}{\partial T} \right)_v dv$$
......(2.36)

And

$$s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_v}{T} \, dT + \int_{v_1}^{v_2} \left(\frac{\partial P}{\partial T} \right)_v dv$$
......(2.37)

The 2nd relation is obtained by replacing the 1st partial derivative in the total differential of ds (Eq. 30) by Eq. 32and the 2nd partial derivative by the 4th Maxwell relation

And

$$ds = \frac{c_P}{T} \, dT - \left(\frac{\partial v}{\partial T} \right)_P dP$$

Δs general relation

$$s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_p}{T} \, dT - \int_{P_1}^{P_2} \left(\frac{\partial v}{\partial T} \right)_P dP$$

Either relation can be used to determine the entropy change. The proper choice depends on the available data

3. REAL GAS RELATIONSHIPS

For a real gas,

$$PV \neq nRT$$

The best method for obtaining the PVT relationship for a real gas is thru experimentation. In the absence of an experimental data, the following methods may be used:
1. Compressibility Chart
2. Equations of State
3. Estimated Properties

THE COMPRESSIBILITY FACTOR (Z)

The volume of a real gas is usually less than what the volume of an ideal gas would be at the same temperature and pressure; hence, a real gas is said to be super compressible. The ratio of the real volume to the ideal volume, which is a measure of the amount that the gas deviates from perfect behavior, is called the supercompressibility factor, sometimes shortened to the compressibility factor. It is also called the gas deviation factor and given the symbol z. The gas deviation factor is by definition the ratio of the volume actually occupied by a gas at a given pressure and temperature to the volume it would occupy if it behaved ideally, or:

$$z = \frac{v_{real}}{v_{ideal}} = \frac{v_r}{v_i}$$

V_r – volume of container.

V_i – volume that is available for gas molecule to move.

The General equation for ideal gas

$$PV_i = nRT$$

$$V_i v_i = \frac{nRT}{P}$$

$$\therefore z = \frac{v_r}{\frac{nRT}{P}} = \frac{Pv_r}{nRT}$$

The **compressibility factor** (**Z**) is a useful thermodynamic property for modifying the ideal gas law to account for behavior of real gases. It is a measure of how much the thermodynamic properties of a real gas deviate from those expected of an ideal gas. It may be thought of as the ratio of the actual volume of a real gas to the volume predicted by the ideal gas at the same temperature and pressure as the actual volume.

For an ideal gas, Z always has a value of 1. For real gases, the value may deviate positively or negatively, depending on the effect of the intermolecular forces of the gas. The closer a real gas is to its critical point or to its saturation point, the larger are the deviations of the gas from ideal behavior.

What causes non-ideal behaviour?

In the compression factor expression, pV / nRT, everything on the bottom of the expression is either known or can be measured accurately. But that's not true of pressure and volume.

In the assumptions we make about ideal gases, there are two statements which say things which can't be true of a real gas, and these have an effect on both pressure and volume.

The volume problem

The kinetic theory assumes that, for an ideal gas, the volume taken up by the molecules themselves is entirely negligible compared with the volume of the container.

For a real gas, that assumption isn't true. The molecules themselves do take up a proportion of the space in the container. The space in the container available for things to move around in is less than the measured volume of the container.
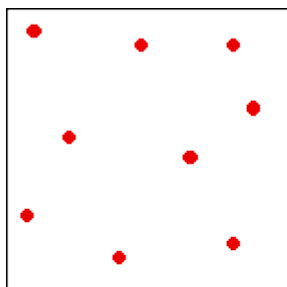
Kinetic Theory assumes
that all of this space is
available for the molecules
to move around in . . .
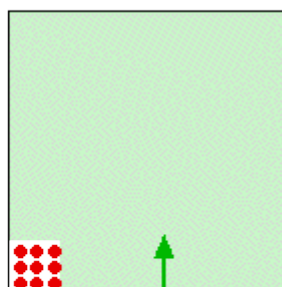
. . . but, in fact, only this
amount is available, because
the rest is taken up by the
molecules themselves.

Effect of Pressure

The extent of deviations which real gases show from Ideal behave our at different pressures can be studied from the graph plotted for different gases between compressibility factor and pressure keeping temperature constant.

Two types of curves are generally obtained:

(1). For gases like hydrogen and helium, the compressibility factor is continuously increasing with increase in pressure. That is, they always show positive deviation.

(2). For gases like CO_2, O_2, N_2, CH_4 the compressibility factor first decreases with increase in pressure reaches a minimum value and then begins to increase. Therefore, for these gas, $Z < 1$ (negative deviation) at low pressure and $Z > 1$ (positive deviation) at high pressure
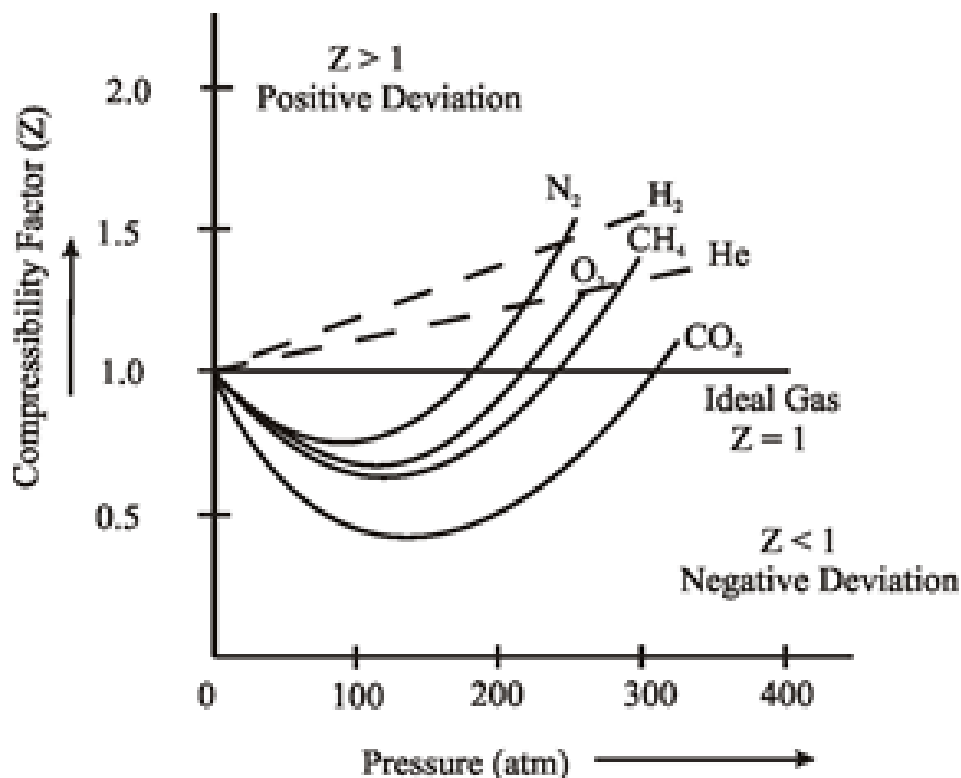
4. GAS MIXTURES

Up to this point in our development of thermodynamics, we have considered primarily pure substances. A large number of thermodynamic problems involve mixtures of different pure substances. Sometimes these mixtures are referred to as solutions, particularly in the liquid and solid phases.

In this chapter we shall turn our attention to various thermodynamic considerations of gas mixtures. We begin by discussing a rather simple problem: mixtures of ideal gases.

This leads to a description of a simplified but very useful model of certain mixtures, such as air and water vapor, which may involve a condensed (solid or liquid) phase of one of the components.

General considerations and mixtures of ideal gases

Let us consider a general mixture of N components, each a pure substance, so the total mass and the total number of moles are

$$m_{tot} = m_1 + m_2 + \cdots + m_N = \sum m_i$$

$$n_{tot} = n_1 + n_2 + \cdots + n_N = \sum n_i$$

The mixture is usually described by a mass fraction (concentration)

$$c_i = \frac{m_i}{m_{tot}}$$

or a mole fraction for each component as

$$y_i = \frac{n_i}{n_{tot}}$$

Which are related through the molecular mass, M_i (kg/kmol) of each species i., as $m_i = n_i\, M_i$. We may then convert from a mole basis to a mass basis as

$$c_i = \frac{m_i}{m_{tot}} = \frac{n_I M_I}{\sum n_j M_j} = \frac{n_I M_I / n_I}{\sum n_j M_j / n_j} = \frac{Y_I M_I}{\sum Y_j M_j}$$

and from a mass basis to a mole basis as

$$y_i = \frac{n_i}{n_{tot}} = \frac{m_i/M_i}{\sum m_j M_j} = \frac{m_i/(M_i m_{tot})}{\sum m_j/(M_j m_{tot})} = \frac{c_i/M_i}{\sum c_j/M_j}$$

The molecular mass for the mixture becomes

$$M_{mix} = \frac{m_{tot}}{n_{tot}} = \frac{\sum n_i M_i}{n_{tot}} = \sum y_i M_i$$

Partial pressures

The fractional composition is the percent composition divided by 100.

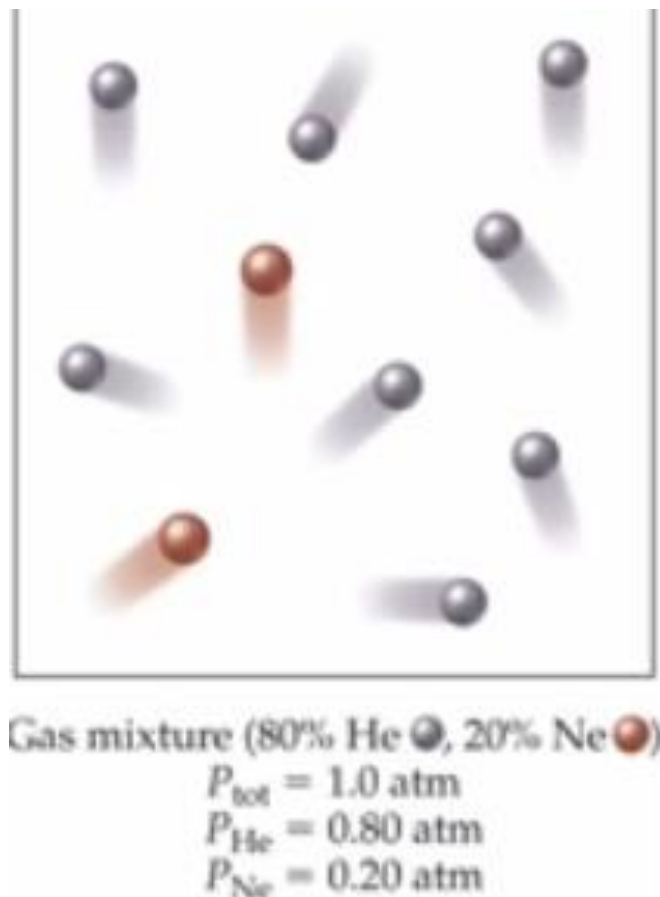Partial pressures of a component = Fractional composition ×Total pressure.

For example the total pressure of air is 1.0 *atm*

Helium occupies 80% of the total pressure,

So P_{He} = 0,80 (1,0 atm) = 0.8atm

Nitrogen occupies 20% of the total pressure,

So P_{Ne} = 0,20 (1,0 atm) = 0.2atm

Gas mixture (80% He ⬤, 20% Ne ⬤)
P_{tot} = 1.0 atm
P_{He} = 0.80 atm
P_{Ne} = 0.20 atm

Example

A gas mixture contains 64% methane and 36% ethane. If the total pressure is 1.44 atm, what are the partial pressures of each component?

Dalton's law of Partial pressures:

"The total pressure of gases that do not reactwith each otherand are available ina certain volume equals the sum of the partial pressures of these gasesthat caused if every gas fills this size alone"

$$P_{tot} = P_a + P_b + P_c + \ldots\ldots$$

Where P_{tot} is the total pressure

And P_a , P_b , P_c are the *partial pressures of the components*

For 1 atm air:

$$P_{tot} = P_{Ne} + P_{O2} + P_{Ar}$$

=0.78 atm + 0.21 atm + 0,01 atm

= 1.00 atm

Example

A sample of hydrogen gas is mixed with water vapor. The mixture has a total pressure of 745 toor , and the water vapour has a partial pressure of 34 torr. What is the a partial pressure of the hydrogen gas?