<u>Heat Transfer by Convection</u>

Non-Dimensional Group Numbers Analysis

<u>The Objective:</u> Studying the non-dimensional group numbers which are used in the convection heat transfer.

<u>Introduction:</u>

In convection studies, it is common practice to nondimensionalize the governing equations and combines the variables, which group together into ***dimensionless numbers*** in order to ***reduce the number of total variables***.

<u>Nusselt Number</u>: (Nu)

It is common practice to nondimensionalize the heat transfer coefficient ***h*** with the Nusselt number, defined as:

Where ***k*** is the thermal conductivity of the fluid and L_c is the ***characteristic length***.

$$Nu = \frac{hL_c}{k}$$ ***for External flow*** (flat plate,…) (9)

$$Nu = \frac{hD}{k}$$ ***for Internal flow*** (pipe, duct,…)

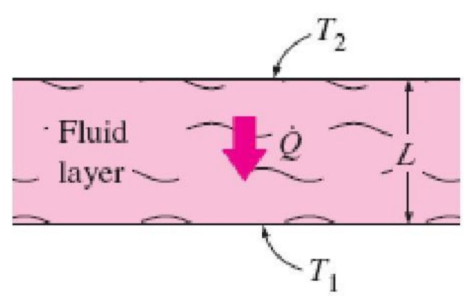
To understand the ***physical significance of the Nusselt number***, consider a ***fluid layer*** of thickness L_c and temperature difference $\Delta T = T_2 - T_1$, as shown in the figure. Heat transfer through the fluid layer will be by ***convection*** when the fluid involves some ***motion*** and by ***conduction*** when the fluid layer is ***motionless***.

Heat flux (the rate of heat transfer per unit time per unit surface area) in either case will be:

$$\dot{q}_{\text{conv}} = h\Delta T$$

and

$$\dot{q}_{\text{cond}} = k\frac{\Delta T}{L}$$



$\Delta T = T_2 - T_1$

Taking their ratio gives

$$\frac{\dot{q}_{\text{conv}}}{\dot{q}_{\text{cond}}} = \frac{h\Delta T}{k\Delta T/L} = \frac{hL}{k} = \text{Nu}$$

Which is the ***Nusselt number***. Therefore, the **Nusselt number** represents the enhancement of heat transfer through a fluid layer as a result of convection relative to conduction across the same fluid layer. The ***larger*** the Nusselt number, the ***more effective the convection***.

A Nusselt number of ***Nu = 1*** for a fluid layer represents heat transfer across the layer by ***pure conduction***.

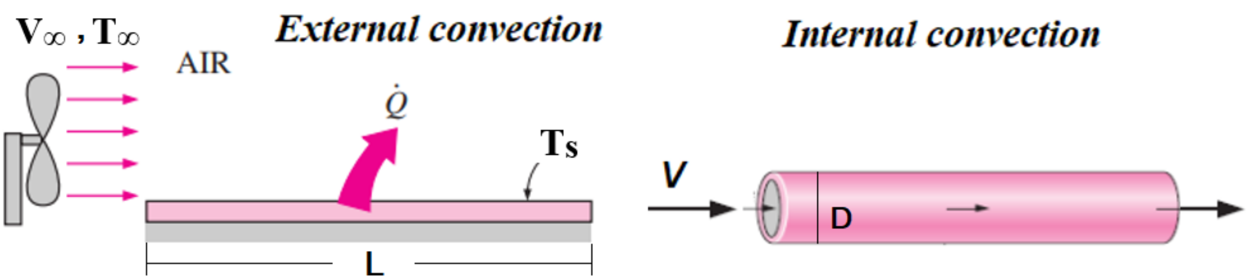
Reynolds Number: (Re)

The transition from ***laminar*** to ***turbulent*** flow depends on the ***surface geometry***, ***surface roughness, free-stream velocity, surface temperature***, and ***type of fluid***, among other things.

The flow regime depends mainly on the ratio of the ***inertia forces*** to ***viscous forces*** in the fluid. This ratio is called the ***Reynolds number,*** which is a ***dimensionless*** quantity, and is expressed for external flow as:

$$Re = \frac{\text{inertia forces}}{\text{viscous forces}} = \frac{VLc}{\nu} = \frac{\rho V Lc}{\mu} \qquad \textit{for External flow}$$

$$Re = \frac{\rho V D}{\mu} \qquad \textit{for Internal flow} \qquad (10) \qquad , \qquad \nu = \mu / \rho$$



Where V is the upstream velocity (equivalent to the free-stream velocity u_∞ for a flat plate), Lc is the ***characteristic length*** of the geometry, and $\nu = \mu/\rho$ is the ***kinematic viscosity*** of the fluid.

For a ***flat plate***, the characteristic length Lc is the distance x from the leading edge. The Reynolds number at which the flow becomes turbulent is called the ***critical Reynolds number*** which is equal to 5×10^5 for ***flat plate***. For flow on flat plate $Re < 5 \times 10^5$ the flow is ***laminar***, $Re > 5 \times 10^5$ the flow is ***turbulent***.

Prandtl Number: (Pr)

The relative ***thickness*** of the ***velocity and the thermal boundary layers*** is best described by the ***dimensionless*** parameter **Prandtl number,** defined as:

$$Pr = \frac{\textbf{Molecular diffusivity of momentum}}{\textbf{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu\, Cp}{k} \qquad (11)$$

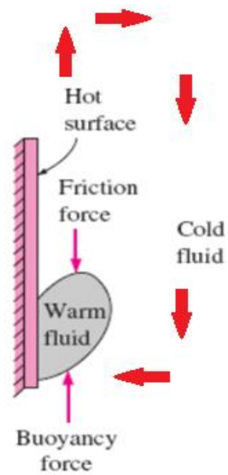
Where: ***kinematic viscosity*** $\upsilon = \mu/\rho$, and the ***thermal diffusivity*** (m^2/s) $\alpha = k/\rho\, C_p$

The ***Prandtl numbers*** of fluids range from less than **0.01** for ***liquid metals*** to more than **100,000 *for heavy oils***. Note that the ***Prandtl number*** is in the order of **10** for ***water***, and in order of **0.7** for ***air***.

Note: Prandtl number is given in ***tables*** depending on ***fluid temperature***.

The Grashof Number: (Gr)

The flow regime in forced convection is governed by the dimensionless ***Reynolds number***, which represents the ratio of ***inertial forces*** to ***viscous forces*** acting on the fluid. The ***flow regime*** in ***natural convection*** is governed by the ***dimensionless Grashof number***, which represents the ratio of the ***buoyancy force*** to **the *viscous force*** acting on the fluid.



$$Gr = \frac{Buoyancy\ forces}{Viscous\ forces} = \frac{g\,\Delta\rho V}{\rho v^2} = \frac{g\,\beta\,\Delta T\,V}{v^2} \qquad (12)$$

Since: $\Delta\rho \approx \rho\beta\Delta T$

The ***Grashof number*** characteristic length L_c is:

$$Gr_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2} \qquad (13)$$

$(\beta = 1/T)$

Where:

g = gravitational acceleration, m/s^2

β = coefficient of volume expansion, **1**/K, (β = **1**/T for ***ideal gases***)

T_s = temperature of the surface, $^\circ$**C**

T_∞ = temperature of the fluid sufficiently far from the surface, $^\circ$**C**

V = Volume (m^3)

L_c = characteristic length of the geometry, m

ν = kinematic viscosity of the fluid, m^2/**s**

Rayleigh number: (Ra)

It represents the product of the ***Grashof*** and ***Prandtl numbers***:

$$\mathrm{Ra}_L = \mathrm{Gr}_L \,\mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\mathrm{Pr} \tag{14}$$

The ***Reynolds analogy*** relates the convection coefficient h to the friction coefficient C_f for fluids with Prandtl Number, $P_r \approx 1$, C_p is specific heat at constant pressure, V is fluid velocity, ρ is density, and is expressed as:

$$h = \frac{C_f}{2}\frac{\rho \mathcal{V} C_p}{\mathrm{Pr}^{2/3}} \tag{15}$$

Analytical Solution of Convection Heat Transfer problems

Equations of motion (Governing equations)

The Objective: To derive the governing equations of fluid flow in the boundary layers.

Introduction:

In convection studies, the analytical solution of the problems is ***very complicated***. It can be done by solving the ***Conservation of Mass Equation, Conservation of Momentum Equations, and the Conservation of Energy Equation*** in ***one, two, or three dimension,*** for steady or unsteady flow.

To keep the analysis at a manageable level, ***we assume the flow to be steady and two-dimensional, and the fluid to be Newtonian with constant properties*** (density, viscosity, thermal conductivity, etc.).

1- Mass conservation (continuity Equation):

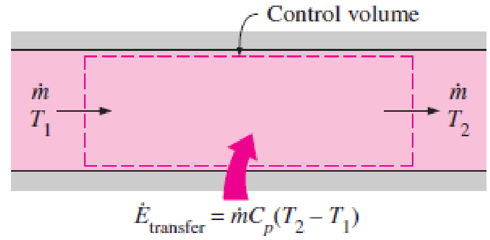
Conservation of mass principle is simply a statement that ***mass cannot be created or destroyed***, and all the mass must be accounted for during an analysis.

In steady flow within the control volume:

$$\begin{pmatrix}\text{Rate of mass flow}\\ \text{into the control volume}\end{pmatrix} = \begin{pmatrix}\text{Rate of mass flow}\\ \text{out of the control volume}\end{pmatrix}$$

Continuity Equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$



Where: **u** = fluid velocity in ***x-direction*** and **v** in ***y-direction***

2- Momentum conservation (Momentum Equation):

Newton's second law is an expression for the conservation of momentum, and can be stated as ***the net force acting on the control volume is equal to the mass times the acceleration of the fluid element within the control volume, which is also equal to the net rate of momentum outflow from the control volume.*** We express Newton's second law of motion for the control volume as,

$$(\text{Mass})\begin{pmatrix}\text{Acceleration}\\ \text{in a specified direction}\end{pmatrix} = \begin{pmatrix}\text{Net force (body and surface)}\\ \text{acting in that direction}\end{pmatrix}$$

X **- Momentum Equation:**

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x}$$

Where: **ρ** = fluid density, **P** = fluid flow pressure

3- Energy conservation (Energy Equation):

The energy balance for any system undergoing any process is expressed as

$$E_{in} - E_{out} = \Delta E_{system}$$

Which states that the ***change in the energy content of a system during a process*** is equal to the difference between the energy input and the energy output. During a ***steady-flow process***, the total energy content of a control volume remains constant (and thus **$\Delta E_{system} = 0$**), or:

$\dot{E}_{in} - \dot{E}_{out} = 0$

Energy Equation:

Where Φ is viscous dissipation function

$$\rho C_p\left[u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right] = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \mu\Phi$$

The ***continuity, momentum***, and ***energy equations*** for ***steady two-dimensional*** incompressible flow with constant properties are determined from mass, momentum, and energy balances to be:

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

x-momentum:

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x}$$

Energy:

$$\rho C_p\left[u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right] = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \mu\Phi$$

where the *viscous dissipation function* Φ is

$$\Phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2$$

With the ***boundary conditions for each case under investigation***, the problem can be solved ***analytically.*** It is more convenient to solve the problems by ***numerical consideration*** to the governing equation within a specified domain in the flow field.

Solution of one dimensional steady state forced convection on flat plate

Using the boundary layer approximations and a ***similarity variable*** (***Blasius solution***), these equations can be solved for parallel steady incompressible flow over a flat plate, with the following results:

Velocity boundary layer thickness: $\delta = \dfrac{5.0}{\sqrt{\mathcal{V}/\nu x}} = \dfrac{5.0x}{\sqrt{Re_x}}$

Local friction coefficient: $C_{f,x} = \dfrac{\tau_w}{\rho \mathcal{V}^2/2} = 0.664\, Re_x^{-1/2}$

Local Nusselt number: $Nu_x = \dfrac{h_x x}{k} = 0.332\, Pr^{1/3}\, Re_x^{1/2}$

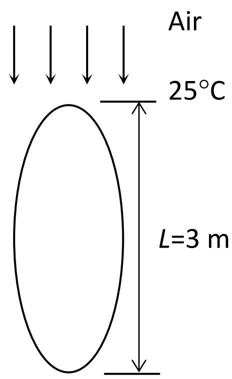
Thermal boundary layer thickness: $\delta_t = \dfrac{\delta}{Pr^{1/3}} = \dfrac{5.0x}{Pr^{1/3}\sqrt{Re_x}}$

The average friction coefficient and Nusselt number are expressed in functional form as,

$C_f = f_4(Re_L)$ and $Nu = g_3(Re_L, Pr)$ **($Re = \rho U_\infty x / \mu$)**

Example-1:

A metallic airfoil of elliptical cross section has a mass of 50 kg, surface area of 12 m^2, and a specific heat of 0.50 kJ/kg.˚C). The airfoil is subjected to air flow at 1atm, 25˚C, and 8 m/s along its 3m long side. The average temperature of the airfoil is observed to drop from 160 ˚C to 150 ˚C within 2 min of cooling. Assuming the surface temperature of the airfoil to be equal to its average temperature and using momentum-heat transfer analogy, ***determine the average friction coefficient of the airfoil surface***.



Solution:

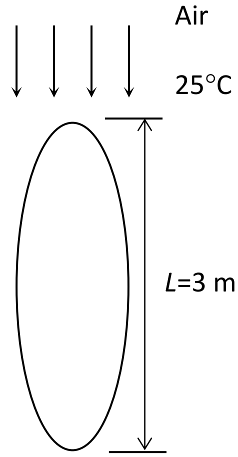
Assumptions: **1-** Steady operating conditions exist. **2-** The edge effects are negligible.

Properties The properties of air at 25°C and 1 atm are (from **Tables**)

$\rho = 1.184$ kg/m^3, C_p =1.007 kJ/kg.K, Pr = 0.7296

The rate of heat transfer from:

$$\dot{Q} = \frac{mC_{\text{p,airfoil}}(T_2 - T_1)}{\Delta t} = \frac{(50\ \text{kg})(500\ \text{J/kg}\cdot°\text{C})(160-150)°\text{C}}{(2\times 60\ \text{s})} = 2083\ \text{W}$$



The surface temperature of airfoil is taken as its average temperature, which is: $T_S = (150+160)/2 = 155\ °\text{C}$

Then the average heat transfer coefficient ***h*** is determined by:

$$\dot{Q} = hA_s(T_s - T_\infty) \longrightarrow h = \frac{\dot{Q}}{A_s(T_s - T_\infty)} = \frac{2083\ \text{W}}{(12\ \text{m}^2)(155-25)°\text{C}} = 1.335\ \text{W/m}^2\cdot°\text{C}$$

The average friction coefficient of the airfoil is determined from the modified ***Reynolds analogy*** to be:

From equation: $$h = \frac{C_f}{2}\frac{\rho \mathcal{V} C_p}{\text{Pr}^{2/3}}$$

$$C_f = \frac{2h\text{Pr}^{2/3}}{\rho \text{V} C_p} = \frac{2(1.335\ \text{W/m}^2\cdot°\text{C})(0.7296)^{2/3}}{(1.184\ \text{kg/m}^3)(8\ \text{m/s})(1007\ \text{J/kg}\cdot°\text{C})} = \mathbf{0.000227}$$

Test:

Q1: What is the physical significance of the Nusselt number? How is it defined?

Q2: What are the advantages of nondimensionalizing the convection equations?