Electricity and Magnetism

Lecture Four

Flux of the Electric Field and Gauss's Law

Dr. Mohammed Hashim Abbas   **Dr. Ameen Alwan**

first stage

Department of medical physics

Al-Mustaqbal University-College

2023- 2024

Outline

1. **Flux of the Electric Field**

2. **Gauss' Law**

3. **References**

1. Flux of the Electric Field

Electric fluxالفيض **is the rate of flow of the electric field through a given area**

(Fig. 1). Electric flux is proportional to the number of electric field lines going through a virtual surface.
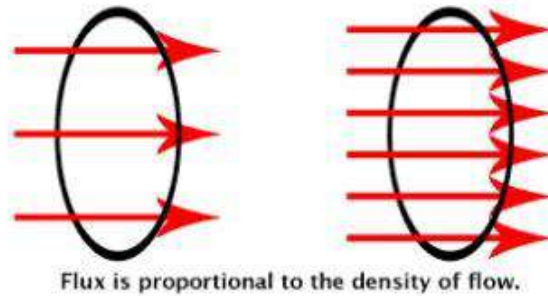

Flux is proportional to the density of flow.

Figure 1: Electric Flux: Electric flux visualized. The ring shows the surface boundaries. The red arrows for the electric field lines.

Flat Surface, Uniform Field: We begin with a flat surface (Fig. 2) with area A in a uniform electric field $\vec{E}$. The total flux Ø is then:

When the electric field is uniform and the surface is flat:

$$\Phi = (E\cos\theta)A \quad \text{(uniform field, flat surface)}$$

$$\Phi = \int \vec{E} \cdot d\vec{A} \quad (total\ flux)$$

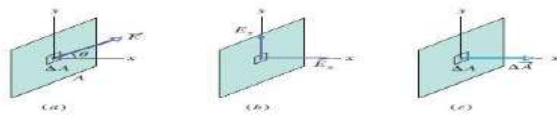$$\Phi = \int (E\cos\theta)\, d\vec{A} \quad (total\ flux$$

Figure 2: An electric field vector pierces a small square patch on a flat surface.

Example: A nonuniform electric field given by $\vec{E} = 3x\hat{i} + 4\hat{j}$ pierces the Gaussian Square area dA with length 2 m. what is the flux throw the surface when x= 3m point in the positive direction of the x axis.?

Solution:

$$\Phi_r = \int \vec{E} \cdot d\vec{A} = \int (3.0x\hat{i} + 4.0\hat{j}) \cdot (dA\hat{i})$$

$$= \int [(3.0x)(dA)\hat{i} \cdot \hat{i} + (4.0)(dA)\hat{j} \cdot \hat{i}]$$

$$= \int (3.0x \, dA + 0) = 3.0 \int x \, dA.$$

The integral $\int dA$ gives us the area A = 4 m^2 of the surface:

$$\Phi_r = 3.0 \int (3.0) \, dA = 9.0 \int dA.$$

$$\Phi_r = (9.0 \text{ N/C})(4.0 \text{ m}^2) = 36 \text{ N} \cdot \text{m}^2/\text{C}.$$

Closed Surface. Let's use the closed surface in (Fig.3) that sits in a nonuniform electric field. To use Gauss' law to relate flux and charge, we need a closed surface.

An inward field is negative flux. An outward field is positive flux

$$\Phi = \oint \vec{E} \cdot d\vec{A} \quad \text{(net flux).}$$

The loop on the integral sign indicates that we must integrate over the entire closed surface, to get the net flux through the surface (flux might enter on one side and leave on another side).
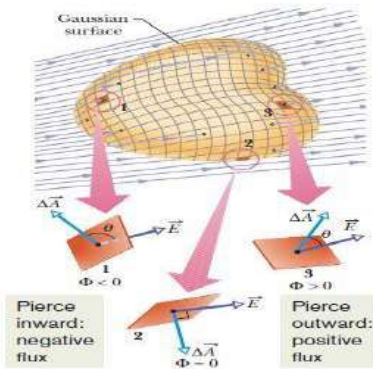
Figur 3: A Gaussian surface of arbitrary shape immersed in an electric field.

2. Gauss' Law

Gauss' law relates the net flux Ø of an electric field through a closed surface (a Gaussian surface) to the *net* charge q_{enc} that is *enclosed* by that surface. It tells us that:

$$\varepsilon_0 \Phi = q_{enc} \quad \text{(Gauss' law).}$$

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc} \quad \text{(Gauss' law).}$$

The net charge q_{enc} is the algebraic sum of all the *enclosed* positive and negative charges, and it can be positive, negative, or zero.

If q_{enc} is positive, the net flux is outward; if q_{enc} is negative, the net flux is inward.
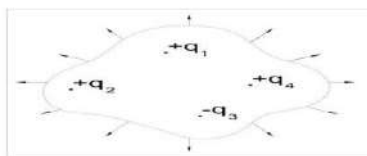
q_{enc} in figure above given below:

$$q_{enc} = q_1 + q_2 - q_3 + q_4$$

3. Refrences

Walker, Jearl, Robert Resnick, and David Halliday. Halliday and resnick fundamentals of physics. Wiley, 2014.

What is electric flux, and how is it related to the number of electric field lines passing through a virtual surface?

- A) Electric flux is unrelated to electric field lines.

- B) Electric flux is inversely proportional to the number of field lines.

- **C)** Electric flux is proportional to the number of electric field lines.

- D) Electric flux is constant regardless of field lines.

- E) Electric flux depends on the color of field lines.

What does Gauss' law relate?

- A) Magnetic field and charge

- **B)** Electric field and charge

- C) Electric field and mass

- D) Gravitational field and charge

- E) Magnetic field and mass

Gauss' law relates the net flux Ø of an electric field through a closed surface to:

- A) Velocity of charged particles

- B) Temperature of the surface

- **C)** Net charge enclosed by the surface

- D) Distance from the surface

- E) Color of the surface