THE BIG M METHOD

If an LP has any ≥ or = constraints, a starting **BFS** may not be readily apparent. When a **BFS** is not readily apparent, the Big M method or the two phase simplex method may be used to solve the problem.

The Big M method is a version of the Simplex Algorithm that first finds a **BFS** by adding "artificial" variables to the problem. The objective function of the original LP must, of course, be modified to ensure that the artificial variables are all equal to 0 at the conclusion of the simplex algorithm.

Steps

1) Modify the constraints so that the RHS of each constraint is nonnegative (This requires that each constraint with a negative RHS be multiplied by -1. Remember that if you multiply an inequality by any negative number, the direction of the inequality is reversed!). After modification, identify each constraint as ≥ , ≤ or = constraint.
2) Convert each inequality constraint to standard form .If constraint i is ≤ constraint, we add a slack variable s_i; and if constraint i is ≥ constraint, we subtract an excess (surplus) variable e_i
3) Add an artificial variable a_i to the constraints identified as ≥ or = constraints at the end of Step 1. Also add the sign restriction $a_i \geq 0$.
4) Let M denote a very large positive number. If the LP is a min problem, add (for each artificial variable) Ma_i to the objective function (before equal to 0). If the LP is a max problem, add (for each artificial variable) $-Ma_i$ to the objective function(before equal to 0).
5) Since each artificial variable will be in the starting basis, all artificial variables must be eliminated from objective row before beginning the simplex. Now solve the transformed problem by the simplex (In choosing the entering variable, remember that M is a very large positive number!).

If all artificial variables are equal to zero in the optimal solution, we have found the optimal solution to the original problem. If any artificial variables are positive in the optimal solution, the original problem is infeasible.

Example:

Maximize $Z = 3x + 2y$

Subject to

$2x + y \leq 9$

$x + 2y \geq 9$ (Note in this case we always subtract surplus and add artificial to avoid that when the initial solu. is $x = y = 0$ this leads to surplus variable will be negative number $= 9$. So we add nonnegative artificial variable)

$x, y \geq 0$

Convert the constrains to equations

$$2x + y + s = 9$$

$$x + 2y - e + a = 9$$

Since we have Maximization, add – Ma to the objective function

$$Z = 3x + 2y - Ma$$

Then the standard form

$$2x + y + s + 0e + 0a + 0Z = 9$$

$$x + 2y + 0s - e + a + 0Z = 9$$

$$-3x - 2y + 0s + 0e + Ma + Z = 0$$

$$x, y, s, e, a \geq 0$$

	x	y	s	e	a	Z	
s	2	1	1	0	0	0	9
	1	2	0	-1	1	0	9
Z	-3	-2	0	0	M	1	0

Apply step 5 by eliminating any artificial from the objective row (hint: row operation)

$R_2 = R_2/2$

	x	y	s	e	a	Z	
s	2	1	1	0	0	0	9
a	1	2	0	-1	1	0	9
Z	-3-M	-2- 2 M	0	M	0	1	-9M

The initial is s = 9 , a = 9 and other variables are zeros

Use Simplex Method

$R_1 = R_1 - R_2$

$R_3 = R_3 - (-2- 2M)R_2$

	x	y	s	e	a	Z	
s	$\frac{3}{2}$	0	1	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{9}{2}$
a	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{9}{2}$
Z	-2	0	0	-1	1+M	1	9

	x	y	s	e	a	Z	
x	1	0	$\frac{2}{3}$	$\frac{1}{3}$	$-\frac{1}{3}$	0	3
y	0	1	$-\frac{1}{3}$	$-\frac{2}{3}$	$\frac{2}{3}$	0	3
Z	0	0	$\frac{4}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$+M	1	15

The ratios of the 1st , 2nd rows respectively are (9 , -9/2) . Since we are looking for the smallest positive ratio then we will consider the 1st ratio is the one

	x	y	s	e	a	Z	
e	3	0	2	1	-1	0	9
y	2	1	1	0	0	0	9
Z	1	0	2	0	M	1	18

Since, the last row doesn't include negative number, we will stop

The optimal solution is the active variables y = 9 , e = 9 , Z = 18 . While the non-active variables x = 0 , s = 0 , a = 0 .

The artificial variable a is 0 , which means that it is true optimal solution

Example:

Maximize $Z = 3x + 2y$

Subject to

$2x + y \leq 4$

$x + 2y \geq 9$

$x, y \geq 0$

Then the standard form

$$2x + y + s + 0e + 0a + 0Z = 4$$

$$x + 2y + 0s - e + a + 0Z = 9$$

$$-3x - 2y + 0s + 0e + Ma + Z = 0$$

$$x, y, s, e, a \geq 0$$

The Table will be

	x	Y	s	e	a	Z	
s	2	1	1	0	0	0	4
	1	2	0	-1	1	0	9
Z	-3	-2	0	0	M	1	0

Then

	x	y	s	e	a	Z	
s	2	1	1	0	0	0	4
a	1	2	0	-1	1	0	9
Z	-3-M	-2- 2 M	0	M	0	1	-9M

Using Row operation until we get to the final table

	x	Y	s	e	a	Z	
y	2	1	1	0	0	0	4
a	-3	0	-2	-1	1	0	1
Z	1+3M	0	2+2M	M	0	1	8-M

The active variables are y = 4 , a = 1 , Z = 8-M and non-active variables x = 0 , s = 0 , e = 0

Since the artificial variable a is positive, then the problem is infeasible.

Also the objective function Z include the large –M mean that there will be no maximize