17. Histogram modification and histogram equalization

An alternate perspective to gray-level modification that performs a similar function is referred to as histogram modification. The gray-level histogram of an image is the distribution of the gray levels in an image. In figure 11 we can see an image and its corresponding histogram. In general a histogram with a ***small-spread*** has a ***low-contrast***. And a histogram with a ***wide spread*** has a ***high contrast***, whereas an image with its histogram clustered at the ***low end*** of the range is ***dark***, and a histogram with the values clustered at the ***high end*** of the range corresponds to a ***bright*** image.

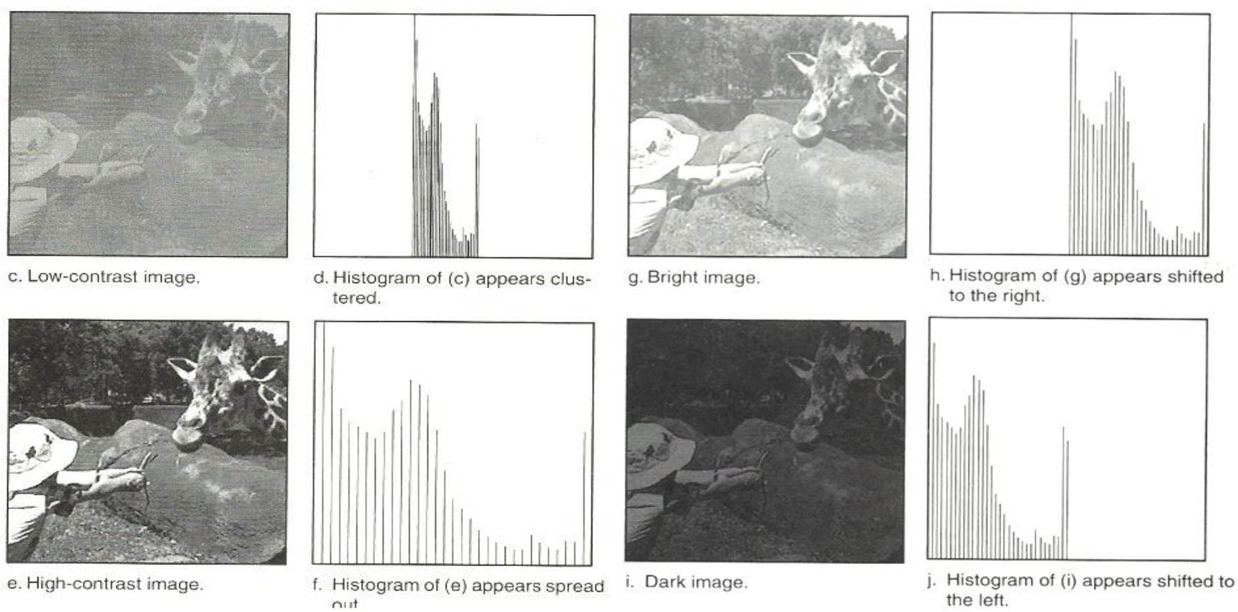


Figure 11: *a variety types of histograms*

The histogram can also be modified by a **mapping function**, which will either **stretch, shrink** (compress), or **slide** the histogram. Histogram stretching and histogram shrinking are forming a gray-level modification, sometimes referred to as histogram scaling. In figure **12** we see a graphical representation of histogram stretch, shrink, and slide.

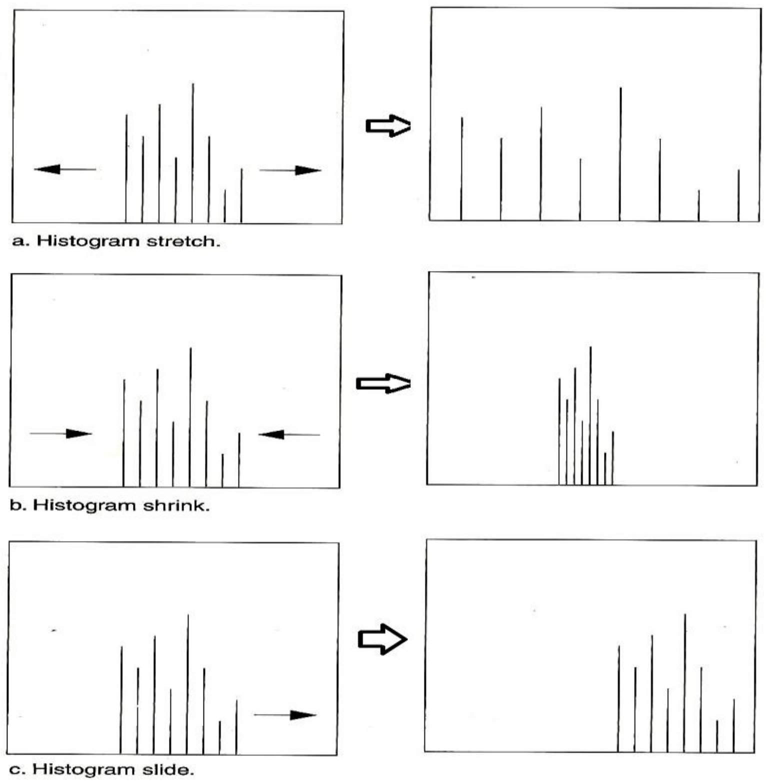


Figure 12: histogram modification

a. Histogram stretch

The mapping function for histogram stretch can be found by the equation :

$$\text{Stretch}(I(r,c)) = \left[\frac{I(r,c) - I(r,c)_{\text{MIN}}}{I(r,c)_{\text{MAX}} - I(r,c)_{\text{MIN}}}\right][\text{MAX} - \text{MIN}] + \text{MIN}$$

Where :

- ***I(r,c)***$_{MAX}$ *is the largest gray-level value in the image I(r,c)*
- ***I(r,c)***$_{MIN}$ *is the smallest gray-level value in the image I(r,c)*
- ***MAX*** *and* ***MIN*** *correspond to the maximum and minimum gray-level values possible (for 8-bitimages these are 0 and 255).*

This equation will take an image and stretch the histogram across the entire gray-level range, which has the effect of increasing the contrast of a low contrast image . ***If a stretch is desired over a smaller range, differentMAX and MIN values can be specified.***

In general, histogram stretch will ***increase image contrast***.

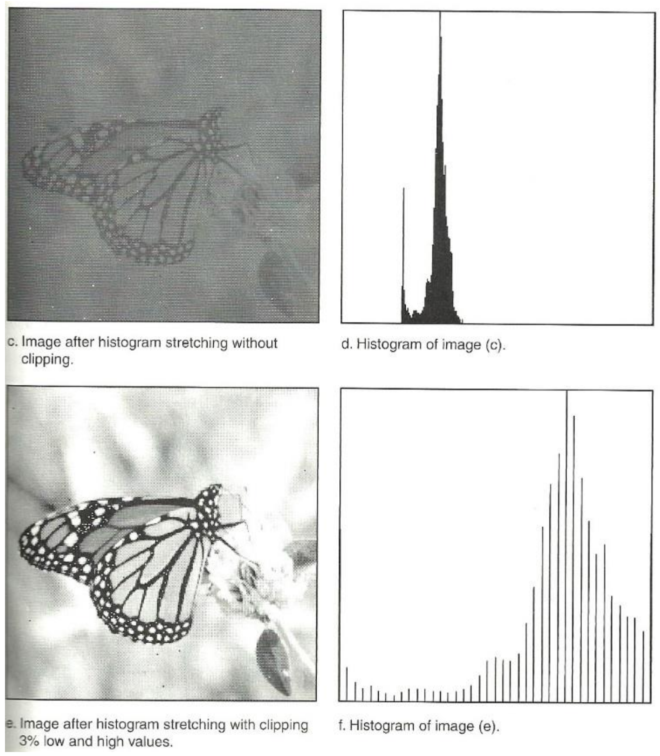


Figure 13: histogram stretch

b. Histogram shrink

The opposite of a histogram stretch is a histogram shrink, which will decrease image contrast by compressing the gray levels. The mapping function for a histogram shrink can be found by the following equation:

$$\text{Shrink}(I(r,c)) = \left[\frac{\text{Shrink}_{\text{MAX}} - \text{Shrink}_{\text{MIN}}}{I(r,c)_{\text{MAX}} - I(r,c)_{\text{MIN}}}\right]\left[I(r,c) - I(r,c)_{\text{MIN}}\right] + \text{Shrink}_{\text{MIN}}$$

Where :$I(r,c)_{MAX}$ is the largest gray-level value in the image I(r,c)

$I(r,c)_{MIN}$ is the smallest gray-level value in the image I(r,c)

$\mathbf{Shrink}_{MAX}$ and $\mathbf{Shrink}_{MIN}$ correspond to the maximum and minimum gray-level values ***desired*** in the compressed histogram.

In general, this process produces an image of ***reduced contrast***an may not seem to be useful as an image enhancement tool.

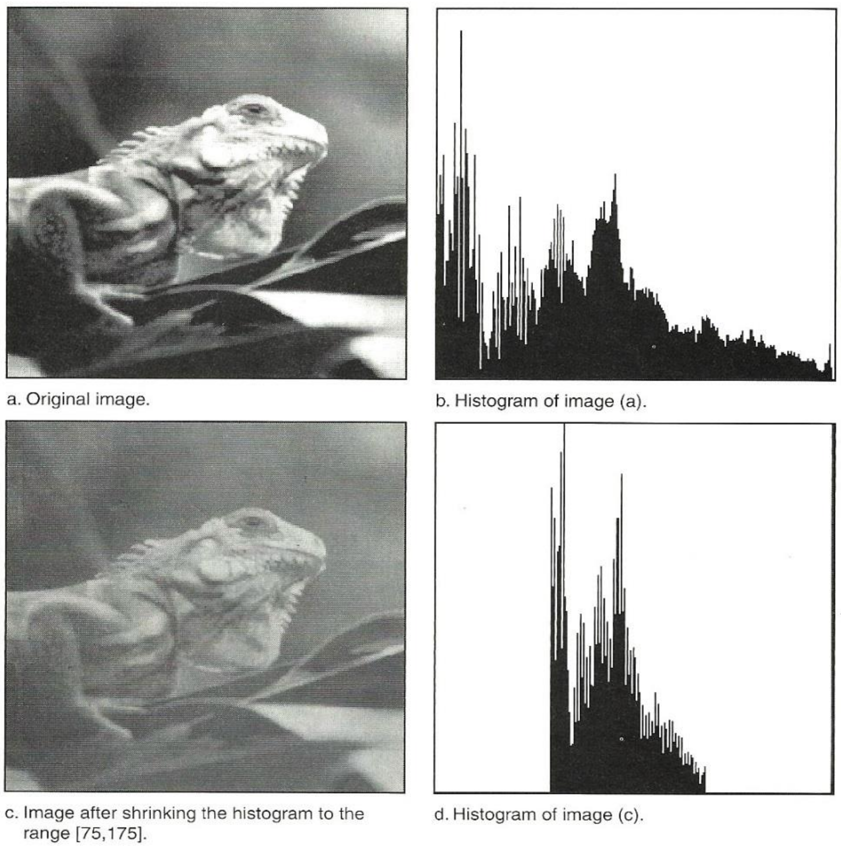


Figure 14: histogram shrink

c. Histogram slide

The histogram slide technique can be used to make an image either ***darker***or***lighter***but retain the relationship between gray-levels values. Thiscan be accomplished by simply **adding** or **subtracting** a fixed number from all the gray level values as follow:

$$\text{Slide}(I(r,c)) = I(r,c) + \text{OFFSET}$$

Where OFFSET value is the amount to slide the histogram**.**

In this equation, we assume that any values slide past the minimum and maximum value will be clipped to the respective minimum or maximum. Aposative OFFSET value will increase the overall brightness, whereas a negative OFFSET will create a darker image. Figure 15 shows a dark image that has been brightened by a histogram slide with a positive OFFSET value.

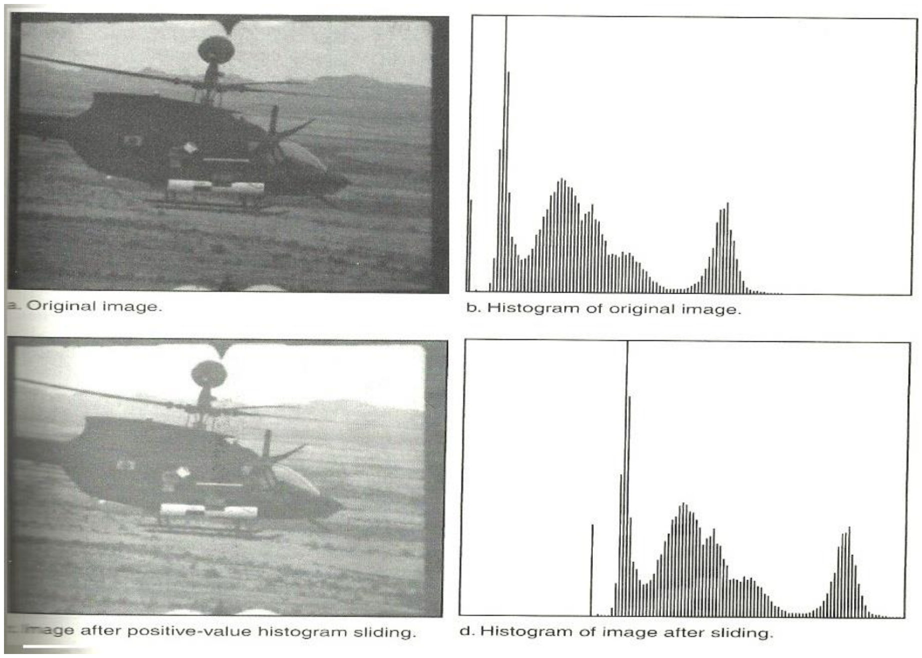


Figure 15: histogram slide

Example: Apply histogram stretching for the following sub image :

$$\begin{bmatrix} 7 & 12 & 8 \\ 20 & 9 & 6 \\ 10 & 15 & 1 \end{bmatrix}$$

Where: ***Max* =255 ;** ***Min* =0**

Solution:

$$St(r,c) = \left[\frac{I(r,c) - I(r,c)_{min}}{I(r,c)_{max} - I(r,c)_{min}}\right](Max - Min) + Min$$

***I(r,c)**$_{min}$* **= 1 ;** ***I(r,c)**$_{max}$* **= 20 ;** ***Max* =255 ;** ***Min* =0**

$I_{(0,0)}$ = [7-1 / 20-1] * [255 – 0] + 0 = 80.5

$I_{(0,1)}$ = [12-1 / 20-1] * [255 – 0] + 0 = 147.6

$I_{(0,2)}$ = [8-1 / 20-1] * [255 – 0] + 0 = 93.9

$I_{(1,0)}$ = [20-1 / 20-1] * [255 – 0] + 0 = 255

$I_{(1,1)}$ = [9-1 / 20-1] * [255 – 0] = 107.3

$I_{(1,2)}$ = [6-1 / 20-1] * [255 – 0] + 0 = 67.1

$I_{(2,0)}$ = [10-1 / 20-1] * [255 – 0] + 0 = 120.7

$I_{(2,1)}$ = [15-1 / 20-1] * [255 – 0] + 0 = 187.8

$I_{(2,2)}$ = [1-1 / 20-1] * [255 – 0] + 0 = 0

$$\begin{bmatrix} 80 & 147 & 93 \\ 255 & 107 & 67 \\ 120 & 187 & 0 \end{bmatrix}$$

Example: Apply histogram shrink for the following sub image :

$$\begin{bmatrix} 70 & 120 & 80 \\ 200 & 90 & 60 \\ 100 & 150 & 10 \end{bmatrix}$$

Where: **Shrink max =100 ;** **shrink min =20**

Solution:

$$\text{Shrink}(I(r,c)) = \left[\frac{\text{Shrink}_{\text{MAX}} - \text{Shrink}_{\text{MIN}}}{I(r,c)_{\text{MAX}} - I(r,c)_{\text{MIN}}}\right]\left[I(r,c) - I(r,c)_{\text{MIN}}\right] + \text{Shrink}_{\text{MIN}}$$

I(r,c) $_{min}$ **= 10 ;** ***I(r,c)*** $_{max}$ **= 200 ;** **Shrink max =100 ;** **shrink min =20**

$I_{(0,0)}$ = [100-20 / 200-10] * [**70** – 10] + 20= 45.2

$I_{(0,1)}$ = [100-20 / 200-10] * [**120** – 10] + 20= 66.3

$I_{(0,2)}$ = [100-20 / 200-10] * [**80** – 10] + 20= 49.4

$I_{(1,0)}$ = [100-20 / 200-10] * [**200**– 10] + 20= 100

$I_{(1,1)}$ = [100-20 / 200-10] * [**90** – 10] + 20=53.68

$I_{(1,2)}$ = [100-20 / 200-10] * [**60** – 10] + 20= 41.05

$I_{(2,0)}$ = [100-20 / 200-10] * [**100** – 10] + 20= 57.89

$I_{(2,1)}$ = [100-20 / 200-10] * [**150** – 10] + 20= 78.94

$I_{(2,2)}$ = [100-20 / 200-10] * [**10** – 10] + 20= 20

$$\begin{bmatrix} 45 & 66 & 49 \\ 100 & 53 & 41 \\ 57 & 78 & 20 \end{bmatrix}$$

Example: Apply histogram slide for the following sub image, where OFFSET= 10 :

$$\begin{bmatrix} 7 & 12 & 8 \\ 20 & 9 & 6 \\ 10 & 15 & 1 \end{bmatrix}$$

Solution:

Slide(*I(r,c)*) =*I(r,c)* + OFFSET

$$\begin{bmatrix} 17 & 22 & 18 \\ 30 & 19 & 16 \\ 20 & 25 & 11 \end{bmatrix}$$

Histogram equalization

Histogram equalization: is a technique for adjusting image intensities to enhance **contrast**.

Histogram equalization often produces unrealistic effects in photographs; however, it is very useful for scientific images like thermal, satellite or x-ray images. To find the histogram equalization must follow:

1- Count the total number of pixels associated with each pixel intensity.

2- Cumulative distribution function (CDF)

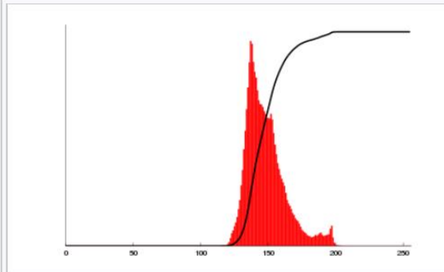
3- Calculate as transformation function

$$h(v) = \mathrm{round}\left(\frac{cdf(v) - cdf_{min}}{(M \times N) - 1} \times (L-1)\right)$$

- cdf_{min} is the minimum non-zero value of the cumulative distribution function.

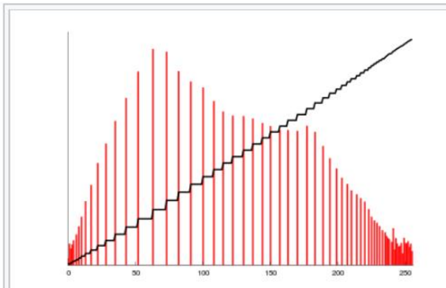
- ***M*N*** gives the image's number of pixels.
- ***L*** is the number of grey levels used (in most cases 256)


Before Histogram Equalization



Corresponding histogram (red) and cumulative histogram (black)


After Histogram Equalization



Corresponding histogram (red) and cumulative

Example:

Apply histogram equalization for the following sub image, where image is gray scale :

$$\begin{bmatrix} 50 & 55 & 150 & 150 \\ 51 & 50 & 55 & 55 \\ 70 & 80 & 90 & 100 \\ 50 & 55 & 70 & 80 \end{bmatrix}$$

Solution:

$$h(v) = \text{round}\left(\frac{cdf(v) - cdf_{min}}{(M \times N) - 1} \times (L-1)\right)$$

M*N = 4 * 4 = 16***L*** = 256 (because its gray scale)

cdf_{min} = **3**

$h(50) = round\left(\frac{3-3}{16-1}\right) * (255-1)$ = 0

$h(51) = round\left(\frac{4-3}{16-1}\right) * (255-1)$ = 17

$h(55) = round\left(\frac{8-3}{16-1}\right) * (255-1)$ = 85

$h(70) = round\left(\frac{10-3}{16-1}\right) * (255-1)$ = 119

$h(80) = round\left(\frac{12-3}{16-1}\right) * (255-1)$ = 153

$h(90) = round\left(\frac{13-3}{16-1}\right) * (255-1)$ = 170

$h(100) = round\left(\frac{14-3}{16-1}\right) * (255-1)$ = 187

$h(150) = round\left(\frac{16-3}{16-1}\right) * (255-1)$ = 221

Pixel Intensity	Count	Cdf_r	H(r)
50	3	3	**0**
51	1	4	**17**
55	4	8	**85**
70	2	10	**119**
80	2	12	**153**
90	1	13	**170**
100	1	14	**187**
150	2	16	**221**

$$\begin{bmatrix} 0 & 85 & 221 & 221 \\ 17 & 0 & 85 & 85 \\ 119 & 153 & 170 & 187 \\ 0 & 85 & 119 & 153 \end{bmatrix}$$