Signals and Vectors

Vector components

To define any vector, two points or aspects are needed in this definition: the magnitude of the vector (its length) and its direction. Moreover, a vector can be represented as a sum of its components in various ways.

In the mathematical view, the representation of the vector is: $\vec{X}$ and sometimes is written by bold font. However, the length of this vector is $|\vec{X}|$. Suppose there are two vectors x and g as shown in Figure 1, the component of g along x is cx that is the projection of g on x.

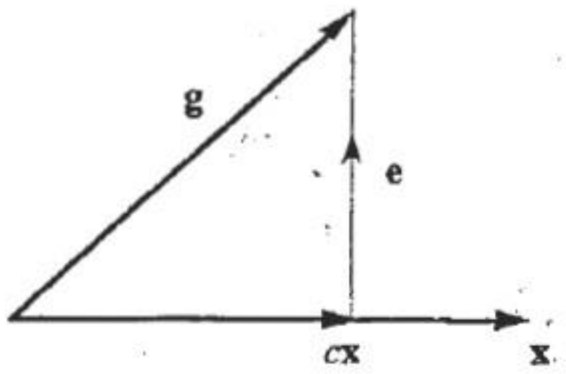

Figure 1: A vector representation

From Figure 1:

$$g = cx + e \tag{1}$$

where: e is the error vector.

In the same context, there are many methods to specify the vectors (g on x) as which displayed in Figure 2.

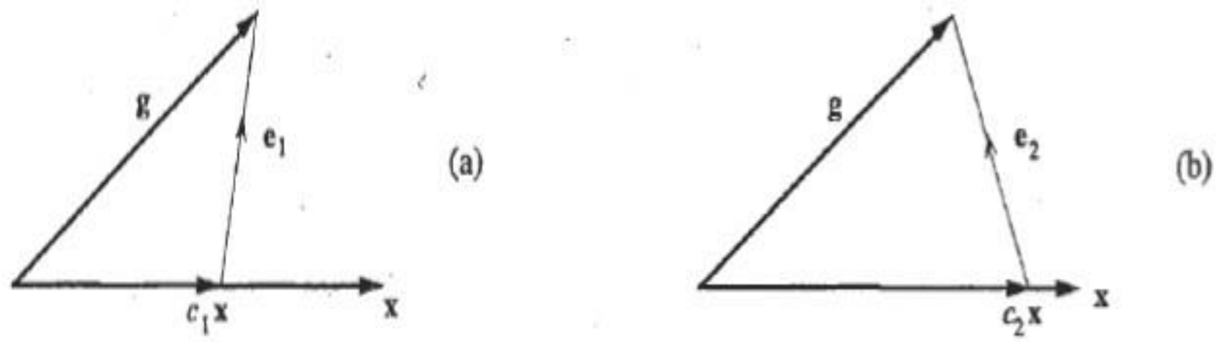

Figure 2: different methods to represent the vector (another method)

Based on equation (1) and Figure 2 (a), the vector g is:

$$g = c1x + e1 \qquad (2)$$

As well as from Figure 2 (b):

$$g = c2x + e2 \qquad (3)$$

If e $\rightarrow$ 0, then vector g is:

$g \simeq cx$

From Eq.1, the error vector e can be computed by Eq. 4 as follows:

$$e = g - cx \qquad (4)$$

Dot product of two vectors

Now, let us define the dot (inner or scalar) product of two vectors g and x as:

$$\vec{g} \cdot \vec{x} = |\vec{g}| \cdot |\vec{x}| . \cos\theta \qquad (5)$$

where, θ is the angle between two specified vectors, in our case these vectors are: $\vec{g}$ and $\vec{x}$

The length of the component g along x is: $|x| \cos \theta$, but remember that this also equals to c multiplied by the length of x (i.e. $c|x|$), so:

$$c\,|x| = |\vec{g}|\,.\cos\theta \qquad (6)$$

If we multiply both sides by $|x|$ then, we have:

$$c\,|x|^2 = |\vec{g}| * |\vec{x}| \cos\theta = \vec{g}\,.\,\vec{x} \qquad (7)$$

From this Eq. (7):

$$c = \frac{1}{|x|^2} * \vec{g}\,.\,\vec{x} \qquad (8)$$

Note: When $\vec{g}$ and $\vec{x}$ are perpendicular (orthogonal), then $\vec{g}$ has a zero component along x (i.e. c = 0). Thus, we say that $\vec{g}$ and $\vec{x}$ are orthogonal if the inner (scalar or dot) product of the two vectors is zero.

Indeed, that in this case:

$$\vec{g}\,.\,\vec{x} = 0 \qquad (9)$$

Signal components

The concepts of "vector component" and "orthogonally" can also be applied to signals. Consider the two signals g(t) and x(t) over the interval $[t_1, t_2]$. The signal g(t) can be expressed in terms of x(t) as:

$$g(t) = cx(t) + e(t) \qquad t_1 \leq t \leq t_2 \qquad (10)$$

Where cx(t) is the component of g(t) along x(t), and the signal e(t) is the error signal. The term cx(t) can also be considered as the approximation of g(t) by x(t).

$$g(t) \simeq cx(t) \qquad t_1 \leq t \leq t_2 \qquad (11)$$

The error e(t) in this approximation is:

$$e(t) = \begin{cases} g(t) - cx(t) & t_1 \le t \le t_2 \\ 0 & \text{otherwise} \end{cases} \qquad (12)$$

To minimize e(t), we minimize its energy as a suitable measure for the signal size. The energy of e(t) is known as E_e, over the interval $[t_1,t_2]$ is calculated by Eq. 13 as follows:

$$E_e = \int_{t_1}^{t_2} e^2(t)\, dt = \int_{t_1}^{t_2} [g(t) - cx(t)]^2\, dt$$

From that we conclude:

$$c = \frac{1}{E_x} \int_{t_1}^{t_2} g(t)\, x(t)\, dt \qquad (14)$$

Where E_x is the energy of the signal x(t).

Orthogonal

If the dot product of two signals or vectors equals zero, then these signals or vectors are orthogonal (perpendicular). In other words, two signals are orthogonal if they are mutually independent.

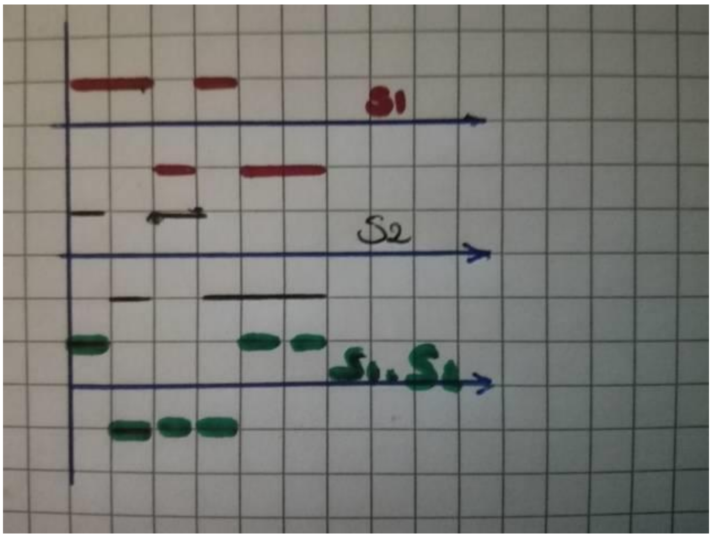

Figure 3: Orthogonal of signals

Correlation in signals

First, the signal is sent. If there is a reflection of it, then there is a target. Otherwise, if there is no reflection, it means there is no target. As a result, the concept of the appearance or non-appearance of the reflex that leads to the appearance or non-appearance of the target.

The correlation of two signals or waveforms is defined as the measure of similarity between those signals. There are two types of correlations:

- ⊗ Cross-correlation
- ⊗ Autocorrelation

Cross-correlation

The cross-correlation between two different signals or functions or waveforms is defined as the measure of similarity or coherence between one signal and the time-delayed version of another signal. The cross-correlation between two different signals indicates the degree of relatedness between one signal and the time-delayed version of another signal. The cross-correlation of energy (or aperiodic) signals and power (or periodic) signals is defined separately.

Autocorrelation

The autocorrelation function is defined as the measure of similarity or coherence between a signal and its time delayed version. Therefore, the autocorrelation is the correlation of a signal with itself.

Like cross-correlation, autocorrelation is also defined separately for energy (or aperiodic) signals and power (periodic) signals.