Mathematics I chapter one

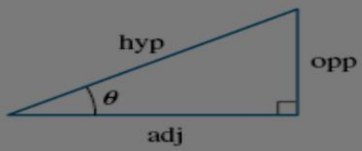
1.1 Trigonometric Function

RIGHT ANGLE TRIGONOMETRY

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

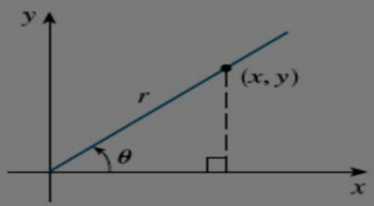


TRIGONOMETRIC FUNCTIONS

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$



FUNDAMENTAL IDENTITIES

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta \qquad \sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta \qquad \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$$

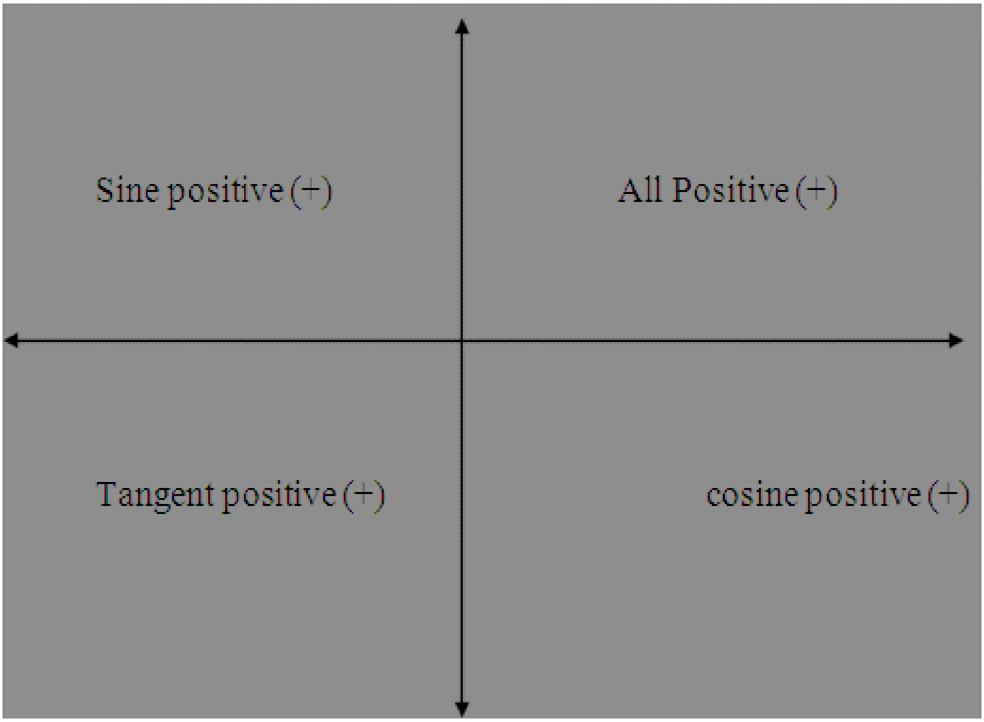
$$\cos(A \pm B) = \cos A \,.\, \cos B \mp \sin A \,.\, Sin\ B \ \ldots\ \ldots$$

$$\sin(A \pm B) = \sin A .\cos B \pm \cos A \,.\, \sin B \ \ldots\ldots$$

Double angle formula:

$$\cos 2\theta = cos^2\theta - sin^2\theta \ \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\sin 2\theta = 2 \sin\theta \,.\, \cos\theta \ \ldots\ldots\ldots\ldots\ldots\ldots$$



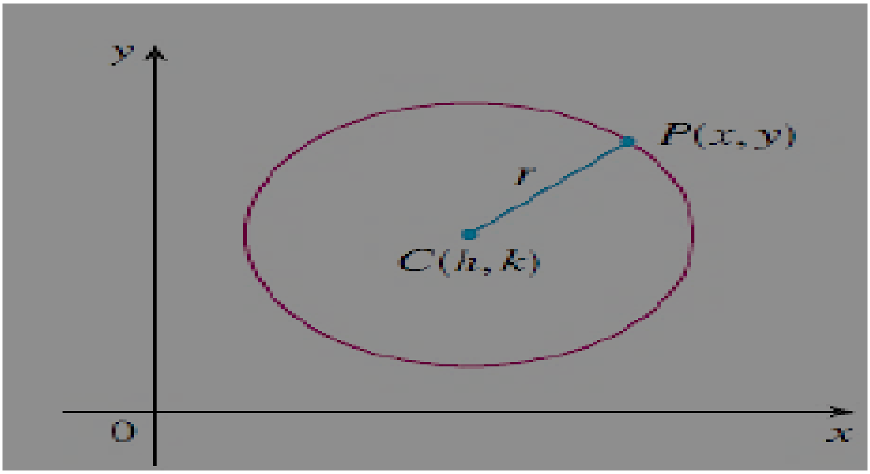
1.2 Equation of circle and straight line.

Circle equation.

To find an equation of the circle with radius(r) and center (h, k), by definition, the circle is the set of all points P (x, y) whose distance from the center C (h, k) is r.

$$(x-h)^2+(y-k)^2=r^2$$

$x^2+y^2=r^2$ **At the center origin point**



Example 1: Find the radius and the coordinate of the center of the equation.

$$x^2+y^2+2x+2y-4=0$$

Solution//

$$x^2+2x+1-1+y^2+2y+1-1-4=0$$

$$(x+1)^2+(y+1)^2=6 \leftrightarrow (x-h)^2+(y-k)^2=r^2$$

$$\rightarrow \mathbf{h=-1, k=-1, r=\sqrt[2]{6}}$$

Equation of straight line

The equation of the line through the point(x,y)with slope(m)is:-

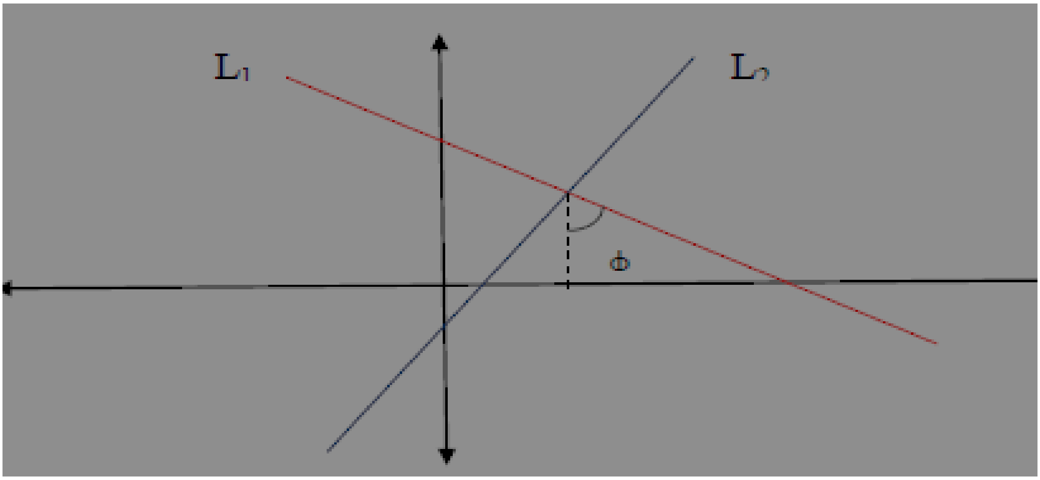
$$y - y_1 = m\,(x - x_1)$$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

PARALLEL AND PERPENDICULAR LINES

1. Two nonvertical lines are parallel if and only if they have the same slope.
2. Two lines with slopes m_1 and m_2 are perpendicular if and only if $m_1 m_2 = -1$; that is, their slopes are negative reciprocals:

$$m_2 = -\frac{1}{m_1}$$



Example 1: Find the equation of the line passes through the points $p_1(4,6)$ and $p_2(6,10)$.

Solution//

$$m = \frac{\Delta y}{\Delta x} = \frac{10-6}{6-4} = 2$$

$$y - y_1 = m\,(x - x_1) \Longrightarrow y-6=2(x-4) \Longrightarrow y=2x-2$$

Example 2: Find the equation of the line tangent to the curve $y=x^3-3x-3$ at the point (0,3).

Solution//

$$m=\frac{dy}{dx}=3x^2-3 \quad \text{at } (0,3) \Longrightarrow m=-3$$

$$y - y_1 = m\,(x - x_1) \Longrightarrow$$

$$\Longrightarrow y-3=-3(x-0) \Longrightarrow y-3=-3x$$

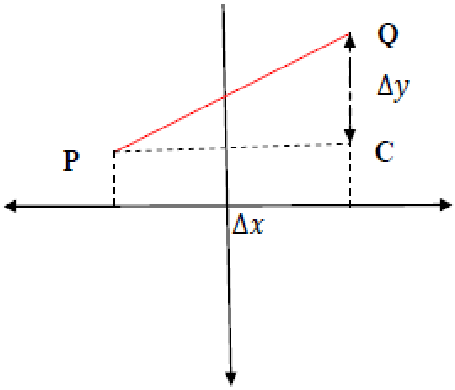
1.3 Distance from point to line

The distance between points in the plain is calculated with a formula that's comes from the Pythagorean Theorem.

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{\Delta x^2 + \Delta y^2}$$



Example 1: Find the distance between the line y=3x-2 and the point (1,3).

Solution//

$$m_1=3 \rightarrow m_2=(-1/m1) = -1/3$$

$$y - y_1 = m\,(x - x_1) \rightarrow y-3=-\frac{1}{3}(x-1)$$

$$\rightarrow y = -\frac{1}{3}x + \frac{1}{3} + 3$$

$$\rightarrow y = -\frac{1}{3}x + \frac{10}{3}$$

$$\text{At } y=3x-2 \rightarrow 3x - 2 = -\frac{1}{3}x + \frac{10}{3}$$

$$\rightarrow 9x-6=-x+10 \rightarrow x=\frac{8}{5}$$

$$\rightarrow y = -\frac{1}{3} * \frac{8}{5} + \frac{10}{3}$$

$$\rightarrow y=\frac{14}{5}$$

$$d = \sqrt{(x2 - x1)^2 + (y2 - y1)^2}$$

$$\boldsymbol{d = \sqrt{(\frac{8}{5} - 1)^2 + (\frac{14}{5} - 3)^2} \rightarrow d = \frac{\sqrt{10}}{5}}$$