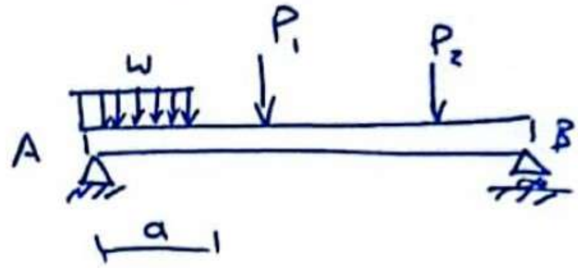
Beams

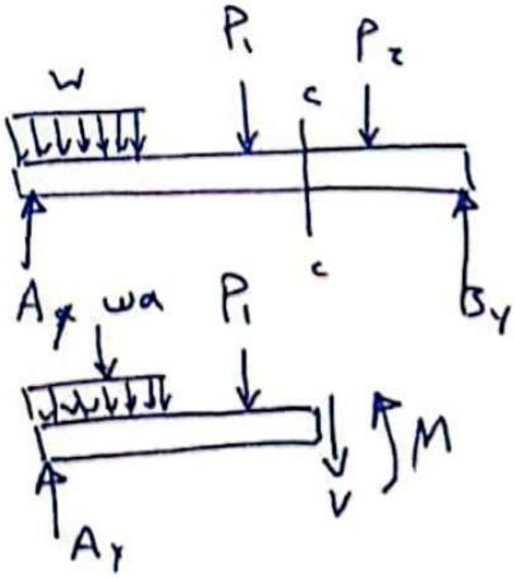
Introduction :

* Beams - structural members supporting loads at various points along the member.



* Transverse loading of beam are classified as concentrated loads or distributed loads.

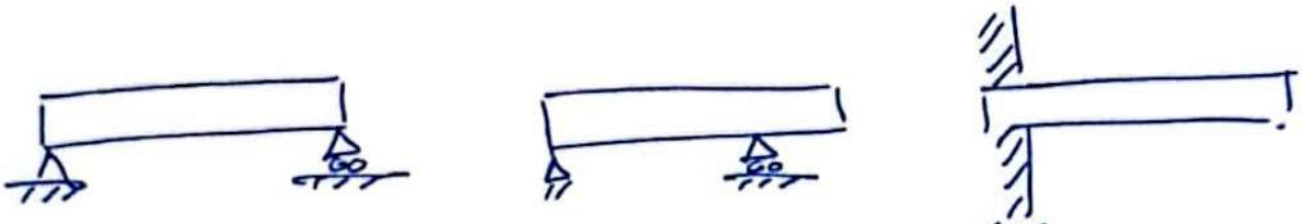
* Applied loads result in internal forces consisting of a shear force (from the shear stress distribution) and a bending couple (from the normal stress distribution).



Classification of Beams

1- Statically Determinate Beams

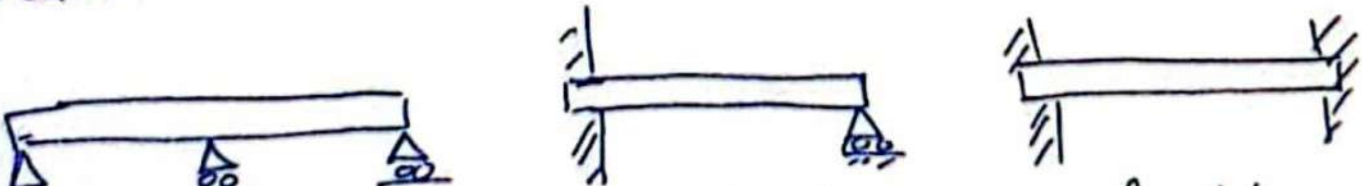
Statically determinate beams are those the beam supports may be determined by the use of the equation of static equilibrium. For examples



(a) Simply supported beam (b) overhangin beam (c) Cantilever beam

2. Statically Inderterminate Beams

If the number of reactions exerted upon a beam exceeds the number of equations in static equilibrium the beam is said to be statically inderterminate In order to solve the reaction of the beam, the static equations must be supplemented by equations based upon the elastic deformations of the beam.



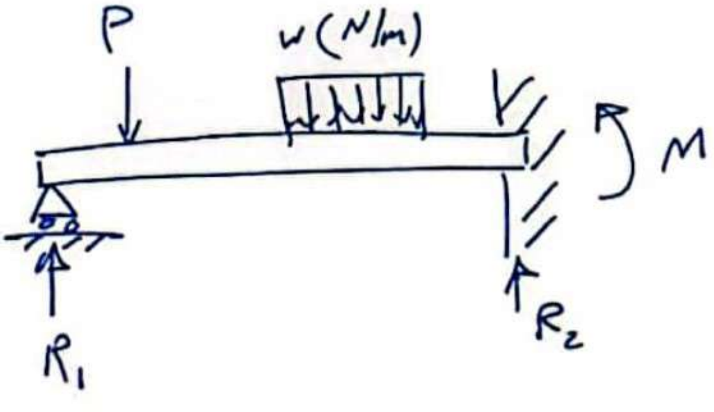
Continuous beam

beam fixed at one end and simply suproted at the other end

fixed beam

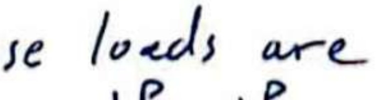
The degree of indeterminacy :-

is taken as the difference between the number of reactions to the number of equations in static equilibrium that can be applied. for example, below beam shown, there are 3 reactions, (R_1, R_2, M) while only two equations (ΣM, ΣF_v) can be applied, thus the beam is indeterminate to the first degree ($3 - 2 = 1$)



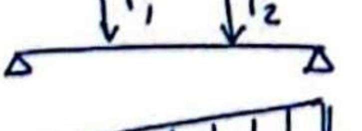
Type of loading

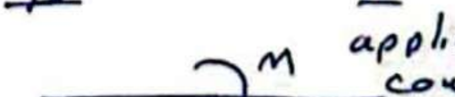
may be consist of concentrated load (load applied at a point ↓), uniform load, uniformly varying load, or an applied couple or moment. These loads are shown as:-



Concentrated load


uniform Load

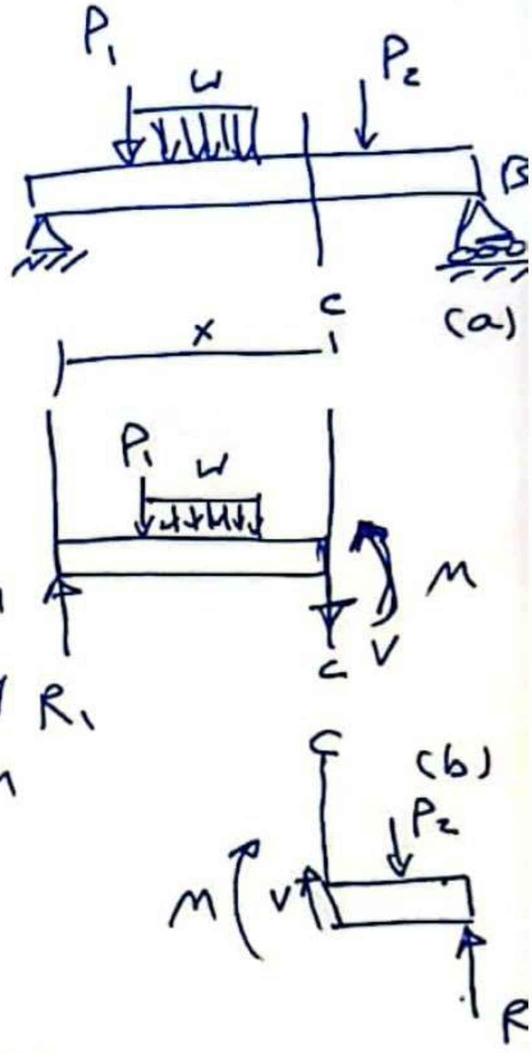

Uniformly varying load



applied coup

Shear force and Bending Moment diagrams :-

are plots of the shear forces and bending moment, respectively, along the length of a beam, The purpose of these plots is to clearly Show maximum of shear force and bending moment, which are important in the design of beams.



The most common sign convention for the shear force and bending moment in beams is shown in Fig.

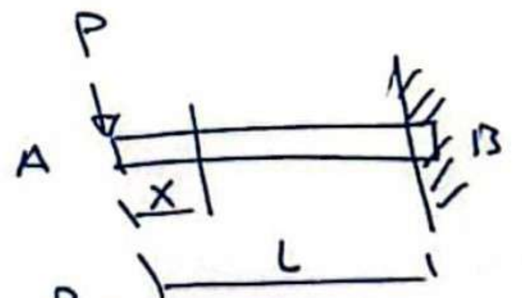
postive bending moment   postive shear

One method of determining the shear and moments diagram is by:-

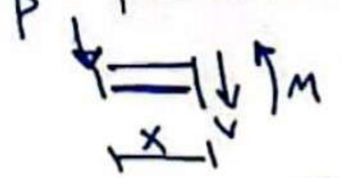
1- Determine the reactions from equilibrium of the entire beam.
2. Cut the beam at an arbitrary point
3. Show the unknown shear and moment on the cut using the positive sign convention.
4. Sum forces in the vertical direction to determine the unknown shear force.
5. Sum moments about the cut to determine the unknown moment.

Ex-1

For the beam shown, derive equations for shear force and bending moment at any point along the beam.

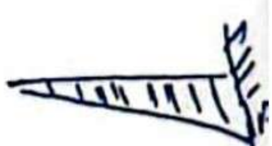


$\Sigma F_y = 0$	$\Sigma M_x = 0$
$P + V = 0$	$P \cdot x + M = 0$
$V = -P$ ↓	$M = -P \cdot x$

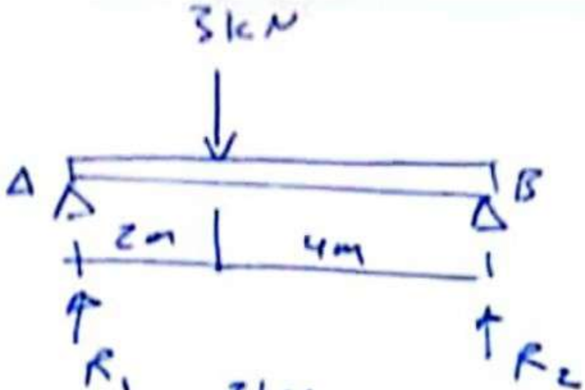


<u>Note</u> 1- Shear force is constant $= P$, along the beam

2- bending moment is a linear function of (x)



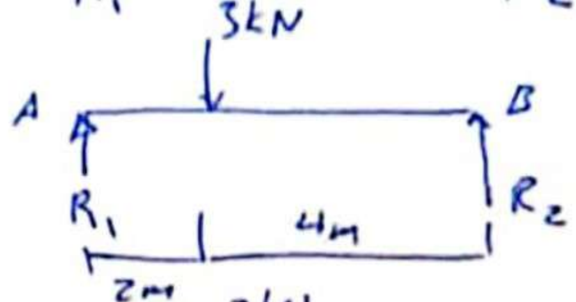
Ex-2 Plot the shear and moment diagram for the beam shown.



$\Sigma F_y = 0$

$R_1 + R_2 - 3kN = 0$

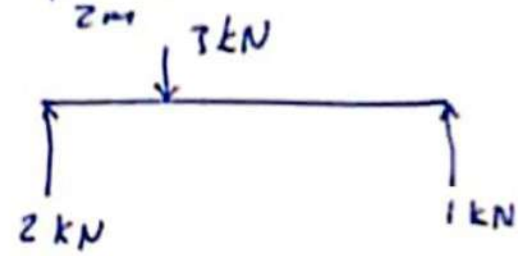
$R_1 + R_2 = 3$



$\Sigma M_A = 0$

$-3 \times 2 + 6R_2 = 0$

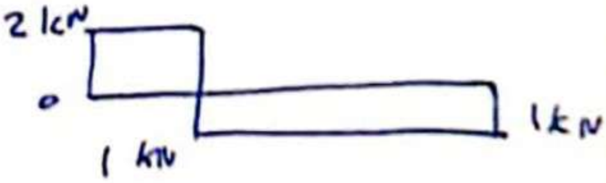
$6R_2 = 6 \quad \Rightarrow R_2 = 1\ kN$



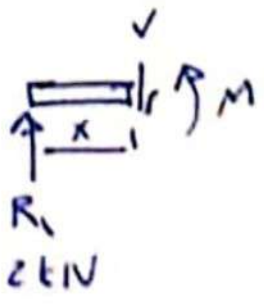
$\therefore R_1 + 1 - 3 = 0$

$R_1 = 2\ kN$

S.F.D



Cut beam between the left end and the load, show the unkown moment and shear on cut using positive sign convention. Sum the vertical forces to get :-

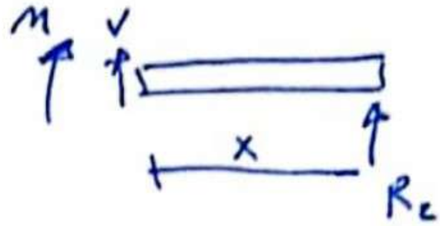


$V = 2kN$ (independent of x)

Sum moments about the cut to get

$M = R_1 x = 2x$

Repeat the procedure by making a cut between the right end of beam and 3kN load as shown



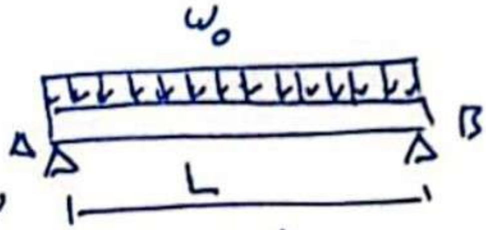
sum vertical forces and sum moments about the cut to get:

$V = 1\ kN$ (indepedent of x)

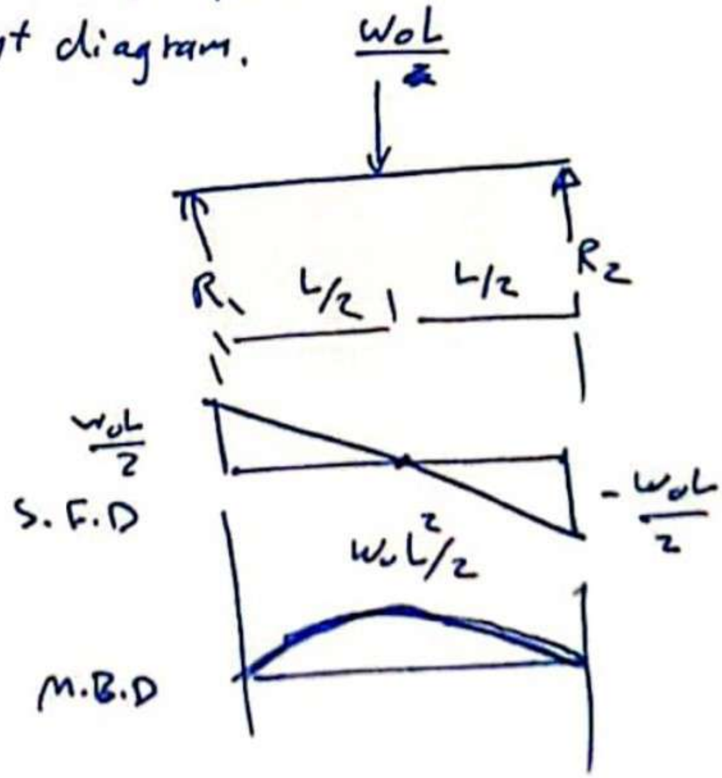
$M = 1X$

plots shear and moment diagrams.

Ex-2



The simply supported beam carries a uniform load of w_o, plot the shear and moment diagram.



$\Sigma F_y = 0$

$R_1 + R_2 - w_oL = 0$

$+\circlearrowleft \Sigma M_A = 0$

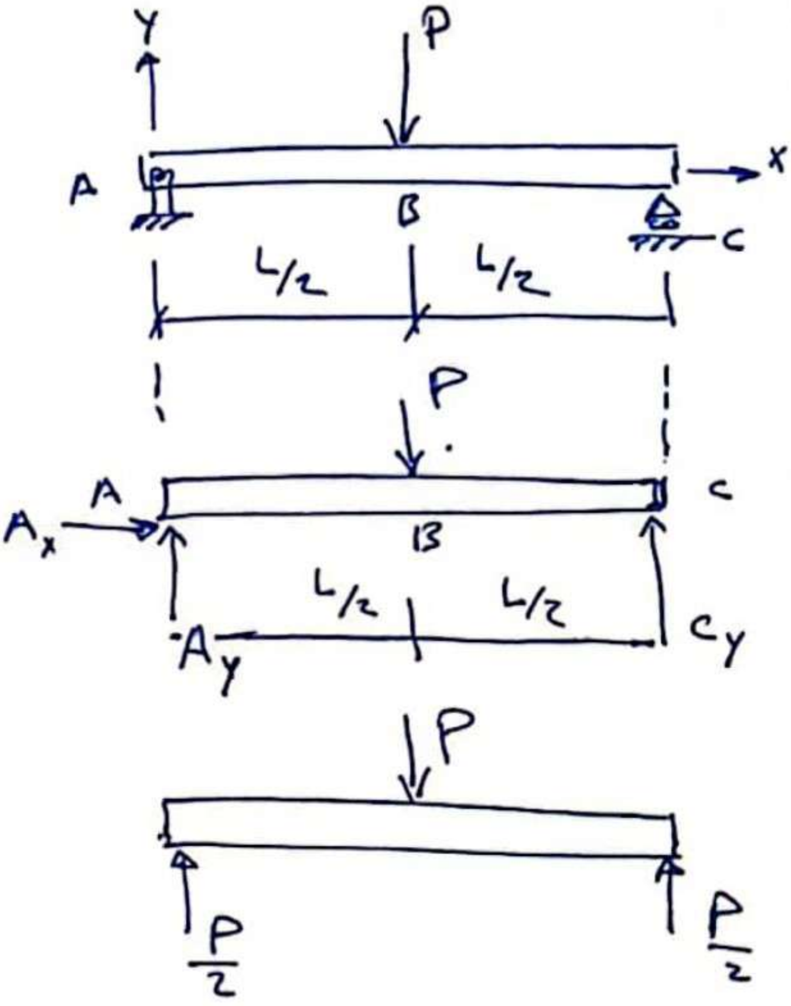
$R_2L - W_oL \cdot \frac{L}{2} = 0$

$R_2 = W_oL/2$

$\Rightarrow R_1 = w_oL/2$

It can be seen that the shear diagram is a straight line, and the moment varies parabolically with x. It can be seen the maximum bending moment occurs at center of the beam where the shear stress is zero.

Q1 - Draw the shear and bending moment diagrams for the simply supported beam shown



$\Sigma F_x = 0$ — (1)

$A_x = 0$

$\Sigma F_y = 0$

$A_y + C_y - P = 0$ — (2)

$+\circlearrowleft \Sigma M_A = 0$ — (3)

$-\frac{L}{2}P + L C_y = 0$

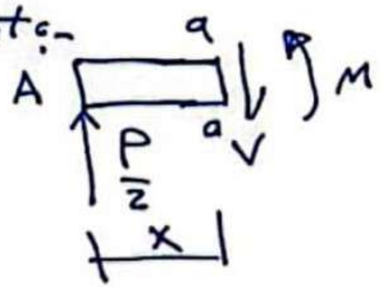
$\Rightarrow C_y = \frac{1}{2}P$

$\Rightarrow A_y + \frac{1}{2}P - P = 0$

$\Rightarrow A_y = \frac{1}{2}P$

For shear force ~~and~~ moment:-

$0 \leq x < \frac{L}{2}$



The beam cut on section a-a

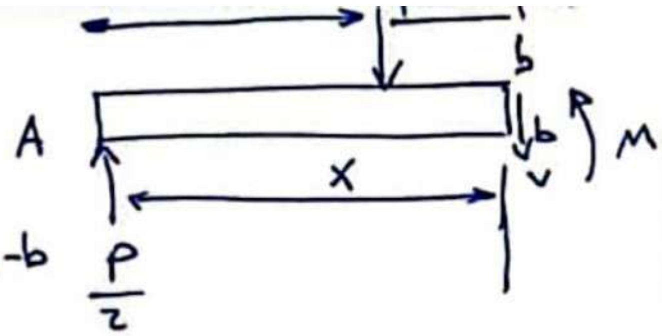
$\Sigma F_y = 0 = \frac{P}{2} - V$

$\Rightarrow \frac{P}{2} = V$

$\Sigma M_{a\text{-}a} = 0 = -\frac{P}{2}x + M$

$\Rightarrow M = \frac{P}{2}x$

$\Rightarrow$ shear force V is constant and bending moment M varies linearly in interval $0 \leq x < \frac{L}{2}$



$$\frac{L}{2} \leq x < L$$

the beam is cut on section b-b at distance x from A

$$\Sigma F_y = 0 = \frac{1}{2}P - P - V$$

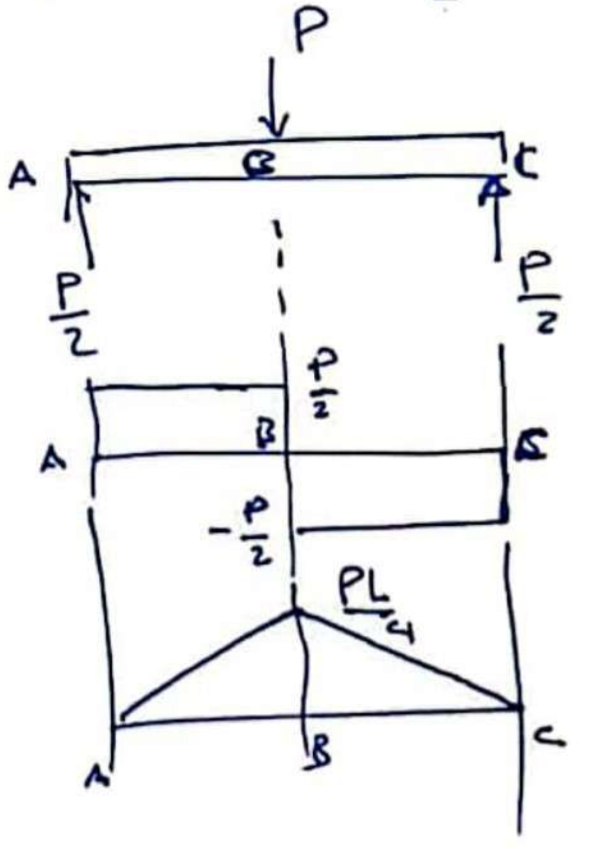
$$\Rightarrow \therefore V = -\frac{P}{2}$$

$$\Sigma M_{b-b} = 0 = -\frac{P}{2}x + P\left(x - \frac{L}{2}\right) + M$$

$$\Rightarrow M = -\frac{P}{2}x + \frac{PL}{2}$$

$\Rightarrow$ shear force V is constant and bending moment M varies linearly in interval $\frac{L}{2} < x < L$

Plot the shear and bending moment



$$0 \leq x < \frac{L}{2}$$

$$V = \frac{P}{2}$$

$$\frac{L}{2} < x < L$$

$$V = -\frac{P}{2}$$

$$M_A = 0$$

$$M_C = 0$$

$$M_B = \frac{P}{2} \times \frac{L}{2} = \frac{PL}{4}$$