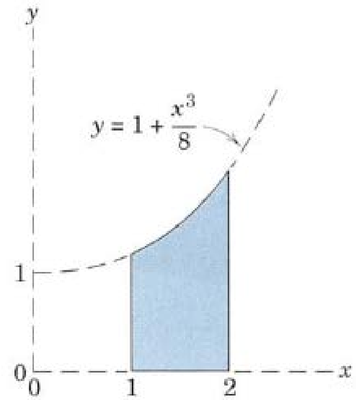
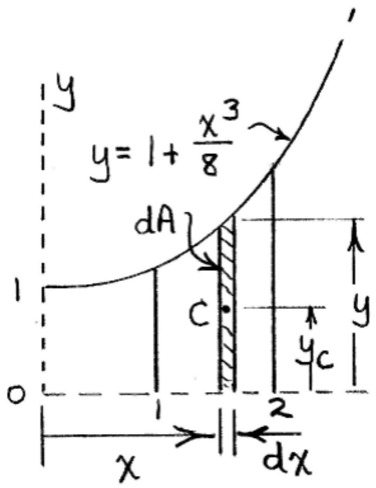
Problem 4

Determine the x and y – coordinates of the centroid of the shaded area



Solution



$$dA = y\,dx = \left(1 + \frac{x^3}{8}\right)dx$$

$$A = \int dA = \int_1^2 \left(1 + \frac{x^3}{8}\right)dx$$

$$= \left(x + \frac{x^4}{32}\right)\Big|_1^2 = \frac{47}{32}$$

$$\int x_c\,dA = \int_1^2 x\left(1 + \frac{x^3}{8}\right)dx$$

$$= \left(\frac{x^2}{2} + \frac{x^5}{40}\right)\Big|_1^2 = \frac{91}{40}$$

$$\int y_c\,dA = \int \frac{y}{2}\,y\,dx = \frac{1}{2}\int y^2\,dx$$

$$= \frac{1}{2}\int_1^2 \left(1 + \frac{x^3}{8}\right)^2 dx = \frac{1}{2}\int_1^2 \left(1 + \frac{x^3}{4} + \frac{x^6}{64}\right)dx$$

$$= \frac{1}{2}\left(x + \frac{x^4}{16} + \frac{x^7}{448}\right)\Big|_1^2 = \frac{995}{896}$$

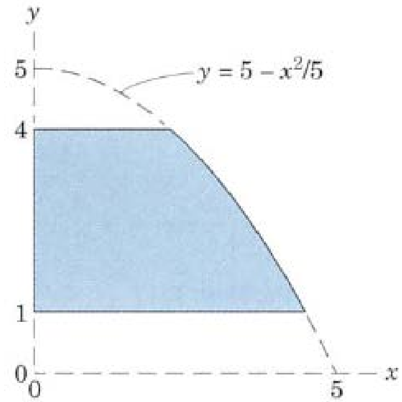
So

$$\bar{x} = \frac{\int x_c\,dA}{\int dA} = \frac{91/40}{47/32} = \underline{1.549}$$

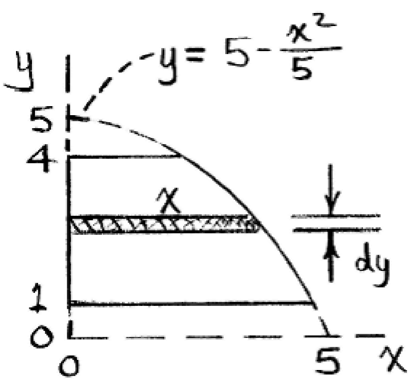
$$\bar{y} = \frac{\int y_c\,dA}{\int dA} = \frac{995/896}{47/32} = \underline{0.756}$$

Problem 5

Determine the x and y – coordinates of the centroid of the shaded area



Solution



$$A = \int x\,dy = \int_1^4 \sqrt{5(5-y)}\,dy$$

$$= \sqrt{5}\left(-\frac{2}{3}[5-y]^{3/2}\right)\Big|_1^4 = \frac{14\sqrt{5}}{3}$$

$$\int x_c\,dA = \int_1^4 \frac{x}{2}\,x\,dy = \int_1^4 \frac{5}{2}(5-y)\,dy$$

$$= \frac{5}{2}\left(5y - \frac{y^2}{2}\right)\Big|_1^4 = \frac{75}{4}$$

$$\bar{x} = \frac{1}{A}\int x_c\,dA = \frac{75/4}{14\sqrt{5}/3} = \underline{1.797}$$

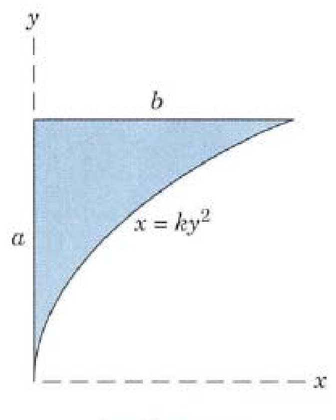
$$\int y_c\,dA = \int_1^4 y\,x\,dy = \int_1^4 y\sqrt{5(5-y)}\,dy$$

$$= \sqrt{5}\left(\frac{-2}{15}\right)(3y+10)(5-y)^{3/2}\Big|_1^4 = \frac{164\sqrt{5}}{15}$$

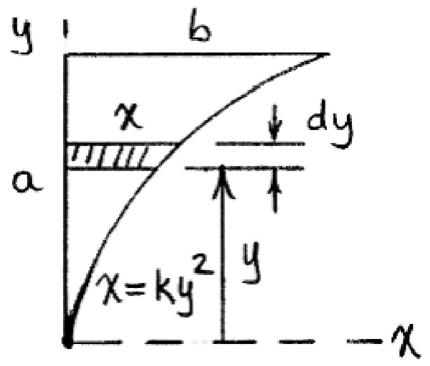
$$\bar{y} = \frac{1}{A}\int y_c\,dA = \frac{164\sqrt{5}/15}{14\sqrt{5}/3} = \underline{2.34}$$

Problem 6

Determine the x and y – coordinates of the centroid of the shaded area



Solution



$$b = ka^2, \quad k = \frac{b}{a^2}, \quad x = \frac{b}{a^2}\, y$$

$$A = \int x\, dy = \int_0^a \frac{b}{a^2} y^2\, dy = \frac{b}{a^2}\,\frac{a^3}{3} = \frac{1}{3}ab$$

$$\int x_c\, dA = \int \frac{x}{2}\, x\, dy = \int_0^a \frac{b^2 y^4}{2a^4}\, dy = \frac{ab^2}{10}$$

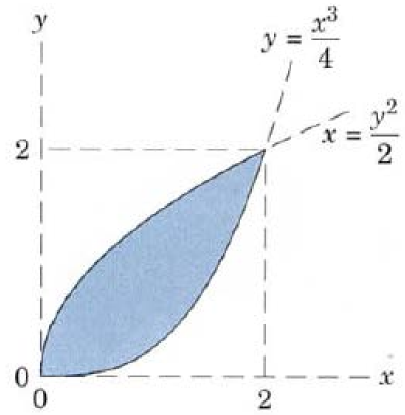
$$\bar{x} = \int x_c\, dA / A = \frac{ab^2/10}{ab/3} = \underline{\frac{3}{10}\, b}$$

$$\int y_c\, dA = \int y\, x\, dy = \int_0^a y\, \frac{b}{a^2} y^2\, dy = \frac{ba^2}{4}$$

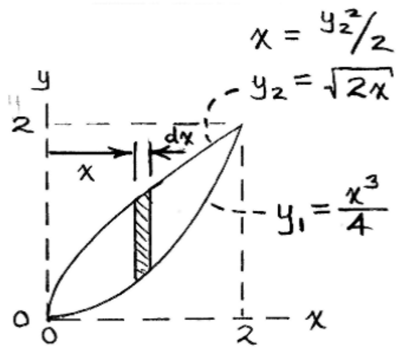
$$\bar{y} = \int y_c\, dA / A = \frac{ba^2/4}{ba/3} = \underline{\frac{3}{4}\, a}$$

Problem 7

Locate the centroid of the shaded area between the two curves



Solution



$$A = \int dA = \int_0^2 (y_2 - y_1)\,dx = \int_0^2 \left(\sqrt{2x} - \frac{x^3}{4}\right) dx$$
$$= \left(\frac{2\sqrt{2}}{3} x^{3/2} - \frac{x^4}{16}\right)_0^2 = 5/3$$

$$\int x_c\,dA = \int_0^2 x\left(\sqrt{2x} - \frac{x^3}{4}\right) dx$$
$$= \left(\frac{2\sqrt{2}}{5} x^{5/2} - \frac{x^5}{20}\right)\Big|_0^2 = 8/5$$

$$\int y_c\,dA = \int_0^2 \left(\frac{y_1+y_2}{2}\right)(y_2 - y_1)\,dx = \int_0^2 \frac{1}{2}(y_2^2 - y_1^2)\,dx$$
$$= \frac{1}{2}\int_0^2 \left(2x - \frac{x^6}{16}\right) dx = \frac{1}{2}\left[x^2 - \frac{x^7}{7(16)}\right]_0^2 = 10/7$$

$$\bar{x} = \int x_c\,dA / A = \frac{8/5}{5/3} = \underline{\frac{24}{25}}$$

$$\bar{y} = \int y_c\,dA / A = \frac{10/7}{5/3} = \underline{\frac{6}{7}}$$

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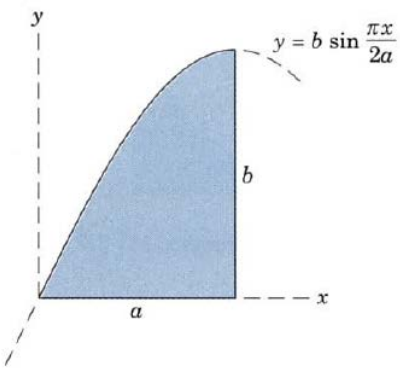
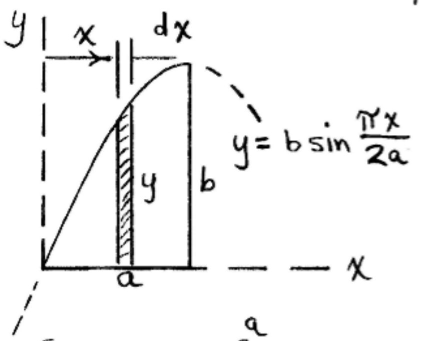
Problem 8

Determine the x and y – coordinates of the centroid of the shaded area



Solution



$$A = \int y\,dx = \int_0^a b \sin \frac{\pi x}{2a}\,dx$$

$$= -\frac{2ab}{\pi} \cos \frac{\pi x}{2a} \Big|_0^a = \frac{2ab}{\pi}$$

$$\int x_c\,dA = \int_0^a x y\,dx = \int_0^a b x \sin \frac{\pi x}{2a}\,dx$$

$$= b\left(\frac{2a}{\pi}\right)^2 \left[\sin \frac{\pi x}{2a} - \frac{\pi x}{2a} \cos \frac{\pi x}{2a}\right]_0^a$$

$$= 4a^2 b / \pi^2$$

$$\bar{x} = \frac{\int x_c\,dA}{A} = \frac{4a^2 b/\pi^2}{2ab/\pi} = \underline{\frac{2a}{\pi}}$$

$$\int y_c\,dA = \int_0^a \frac{y}{2}\, y\,dx = \frac{b^2}{2} \int_0^a \sin^2 \frac{\pi x}{2a}\,dx$$

$$= \frac{ab^2}{\pi} \left[\frac{\pi x}{4a} - \frac{1}{4} \sin \frac{\pi x}{a}\right]_0^a = \frac{ab^2}{4}$$

$$\bar{y} = \frac{\int y_c\,dA}{A} = \frac{ab^2/4}{2ab/\pi} = \underline{\frac{\pi b}{8}}$$

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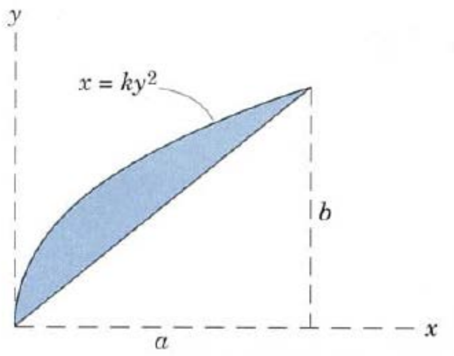
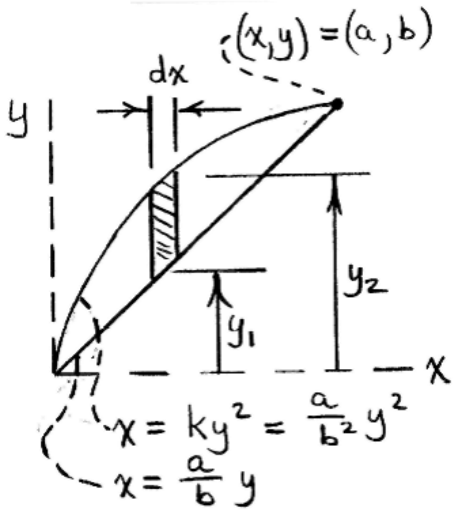
Problem 9

Determine the x and y – coordinates of the centroid of the shaded area



Solution



$$A = \int_0^a (y_2 - y_1)\, dx$$

$$= \int_0^a \left(b\sqrt{\frac{x}{a}} - x\,\frac{b}{a} \right) dx$$

$$= b \left[\frac{1}{\sqrt{a}} \frac{2x^{3/2}}{3} - \frac{1}{2a} x^2 \right]_0^a$$

$$= \frac{ab}{6}$$

$$\int x_c\, dA = \int_0^a x (y_2 - y_1)\, dx = \int_0^a \left[\frac{b}{\sqrt{a}} x^{3/2} - \frac{b}{a} x^2 \right] dx$$

$$= b \left[\frac{2x^{5/2}}{5\sqrt{a}} - \frac{x^3}{3a} \right]_0^a = \frac{a^2 b}{15}$$

$$\bar{x} = \frac{\int x_c\, dA}{A} = \frac{ab^2/15}{ab/6} = \underline{\frac{2}{5} a}$$

$$\int y_c\, dA = \int_0^a \left(\frac{y_1 + y_2}{2} \right) (y_2 - y_1)\, dx = \frac{1}{2} \int_0^a (y_2^2 - y_1^2)\, dx$$

$$= \frac{1}{2} \int_0^a \left(\frac{x b^2}{a} - \frac{x^2 b^2}{a^2} \right) dx = \frac{1}{12} a b^2$$

$$\bar{y} = \frac{\int y_c\, dA}{A} = \frac{ab^2/12}{ab/6} = \underline{\frac{b}{2}}$$