$F(s)$ ⟷ $f(t)$ Initial Value Theorem

$f(t=0) = f(0)$

$= \lim_{s \to \infty} sF(s)$

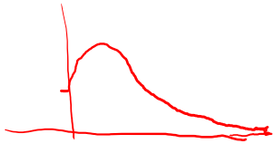
o/p f(t) at $t = \infty$ [steady state] ⟵ o/p at

$f(\infty) = \lim_{t=\infty} = \lim_{s \to 0} sF(s)$

$= \lim_{s \to 0} s \cdot \frac{(s+2)}{(s+1)^2+5} =$ Zero

$= 0$

Example



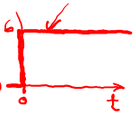
$\frac{d^3y}{dt^3} + 7\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y(t) = 6 \leftarrow x(t)$

Find $y(0)$ and $y(\infty)$ [o/p at steady state]

$\mathcal{L}\left[\frac{d^3y}{dt^3}\right] = s^3Y(s)$; $\mathcal{L}\left[7\frac{d^2y}{dt^2}\right] = 7s^2Y(s)$

$\mathcal{L}\left[3\frac{dy}{dt}\right] = 3sY(s)$; $\mathcal{L}[2y(t)] = 2Y(s)$

$x(t)$ = input = 6 ⟵ step Function



$\mathcal{L}[x(t)] = \mathcal{L}[6] = \frac{6}{s}$

$s^3Y(s) + 7s^2Y(s) + 3sY(s) + 2Y(s) = \frac{6}{s}$

$[s^3 + 7s^2 + 3s + 2]Y(s) = \frac{6}{s}$

$Y(s) = \frac{6}{s[s^3+7s^2+3s+2]}$

steady state o/p $y(\infty) = 3$

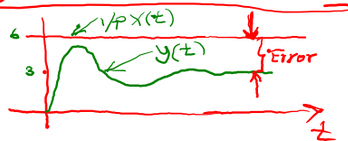
I/p = 6

1) $y(t=0) = \lim_{s \to \infty} s \cdot Y(s) = \lim_{s \to \infty} \frac{s \cdot (6)}{s[s^3+7s^2+3s+2]}$

steady state Error = I/p − o/p $(t=\infty)$

$= 0$

2) $y(t=\infty) = \lim_{s \to 0} \frac{s \cdot (6)}{s[s^3+7s^2+3s+2]} = \frac{6}{2} = 3$



$x(t) = I/P = \text{driving Function}$

Step Function

$A \xrightarrow{\mathcal{L}} \frac{A}{s}$

Ramp Function

$t \xrightarrow{\mathcal{L}} \frac{1}{s^2}$

Impulse Function

$\delta(t) \xrightarrow{\mathcal{L}} 1$

Cosine function

$\cos 3t \xrightarrow{\mathcal{L}} \frac{s}{s^2+3^2}$

exponential

$e^{-6t} \xrightarrow{\mathcal{L}} \frac{1}{s+6}$

Q)

$$y(t) = 4\cos 2t + 3e^{-7t} + 6$$

$$= \mathcal{L}\,4\cos 2t + \mathcal{L}\,3e^{-7t} + \mathcal{L}\,6$$

$$Y(s) = \frac{4s}{s^2+4} + \frac{3}{s+7} + \frac{6}{s}$$