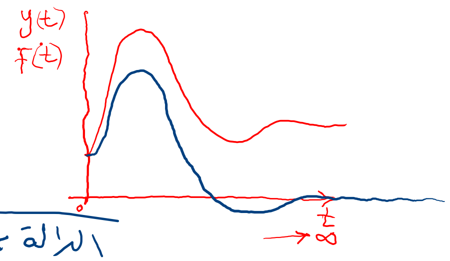
الدالة بالتحويل لابلاس

$F(s)$

$Y(s)$



$f(t)$ $y(t)$ الدالة بالزمن

$f(0), f(\infty)$?

Example

$$\boxed{f(t) = 4\cos 3t + 2e^{-3t} + 7}$$

Find $f(0)$, & $f(\infty)$ using Laplace Transform.

$F(s)$?

$$\mathcal{L}[\cos \omega t] = \frac{s}{s^2+\omega^2}$$

$$\boxed{\mathcal{L}[4\cos 3t] = \frac{4s}{s^2+9}}$$

$$\mathcal{L}[2e^{-3t}] = \frac{2}{s+3}$$

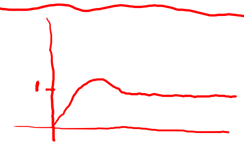
$$\mathcal{L}[7] = \frac{7}{s}$$

$$F(s) = \frac{7}{s} + \frac{2}{s+3} + \frac{4s}{s^2+9}$$

$$= \frac{7(s+3)(s^2+9) + 2s(s^2+9) + 4s^2(s+3)}{s(s+3)(s^2+9)}$$

$$f(0) = \lim_{s\to\infty} s.$$

$$f(\infty) = \lim_{s\to 0} s.F(s) = \frac{21}{27}$$



Example

$$\mathcal{L}\left[\frac{d^2y}{dt^2} + 12\frac{dy}{dt} = e^{-2t}\right]$$

driving Function

input

Find $y(0)$ and $y(\infty)$

$$\mathcal{L}[e^{-2t}]$$

$$s^2Y(s) + 12sY(s) = \frac{1}{s+2}$$

$$\boxed{s^2Y(s) + 12sY(s) = \frac{1}{s+2}}$$

$$Y(s)[s^2 + 12s] = \frac{1}{s+2}$$

$$\boxed{Y(s) = \frac{1}{s(s+12)(s+2)}}$$

$y(0)$, $y(\infty)$

$$y(0) = \lim_{s\to\infty} \cancel{s}.\frac{1}{\cancel{s}(s+12)(s+2)} = 0$$

$$y(\infty) = \lim_{s\to 0} \frac{\cancel{s}\cdot 1}{\cancel{s}(s+12)(s+2)} = \frac{1}{24}$$