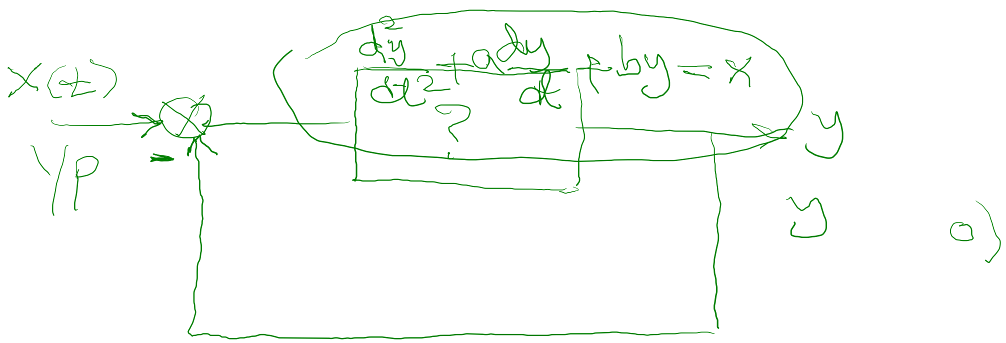
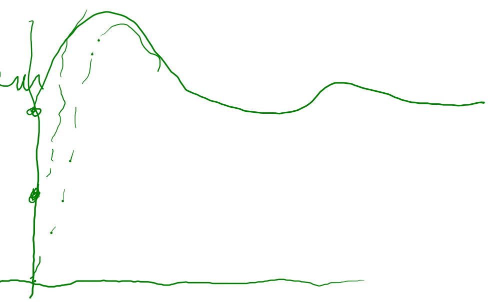
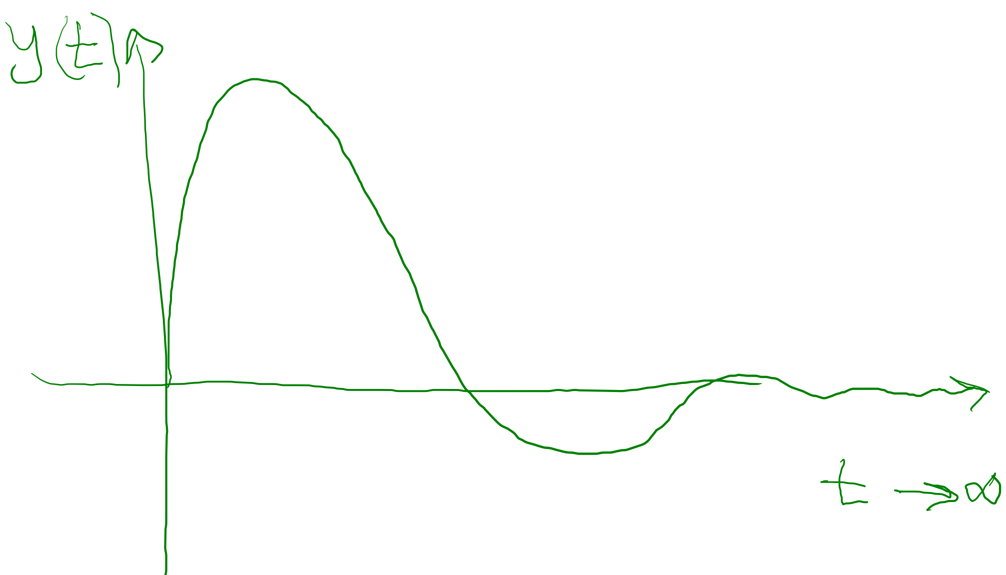


Initial Value Theorem

نظرية القيمة الابتدائية



$$y(t=0) = \lim_{s \to \infty} s \cdot Y(s)$$



Example

Find the o/p y(t) at (t=0) and (t=∞) for the following dynamic system?

$$\frac{d^2y}{dt^2}+10\frac{dy}{dt}+7y(t)=x(t)$$

$x(t)=8 \leftarrow$ i/p [step Function]

$\mathcal{L}\left[\frac{d^2y}{dt^2}\right]=s^2Y(s)$; $\mathcal{L}\left[10\frac{dy}{dt}\right]=10.s\,Y(s)$

$\mathcal{L}[7y(t)]=7Y(s)$; $\mathcal{L}[8]=\frac{8}{s}$

$$\boxed{s^2Y(s)+10.s\,Y(s)+7Y(s)=\frac{8}{s}}$$

Laplace transform of $\left[\frac{d^2y}{dt^2}+10\frac{dy}{dt}+7y=8\right]$

$$Y(s)\left[s^2+10s+7\right]=\frac{8}{s}$$

$$Y(s)=\frac{8}{s\left[s^2+10s+7\right]}$$

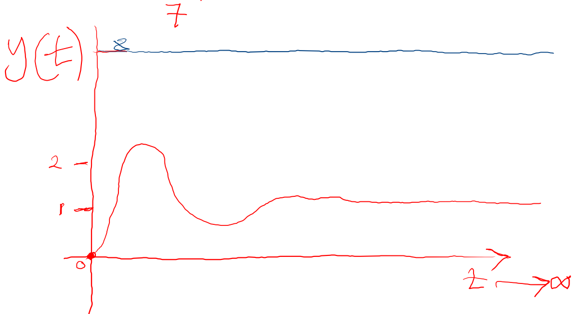
$$y(t=0)=\lim_{s\to\infty} s.Y(s)$$

$$=\lim_{s\to\infty}\cancel{s}.\frac{8}{\cancel{s}\left[s^2+10s+7\right]}$$

$$=\lim_{s\to\infty}\frac{8}{s^2+10s+7}=\frac{8}{\infty}=\phi$$

$$y(t=\infty)=\lim_{s\to\phi} s.Y(s) \quad \cancel{\lim_{s\to 0}\frac{s(8)}{s(s^2+10s+7)}}$$

$$=\frac{8}{7}$$



o/p at steady state $=\frac{8}{7}\approx 1.15$

i/p $x(t)=8$

steady state Error $=$ i/p $-$ o/p$[\infty]$ (y(∞)) $=8-1.15=6.95$

$x(t)=\cos(3t)$

$$\mathcal{L}[\cos 3t]=\frac{s}{s^2+3^2}$$

$$\left[s^2+10s+7\right]Y(s)=\frac{s}{s^2+9}$$

$$Y(s)=\frac{s}{(s^2+10s+7)(s^2+9)}$$

i/p = Unit Impulse $=\delta(t)$

$$\mathcal{L}[\delta(t)]=1$$

$$\left[s^2+10s+7\right]Y(s)=1$$

$$\boxed{Y(s)=\frac{1}{s^2+10s+7}}$$