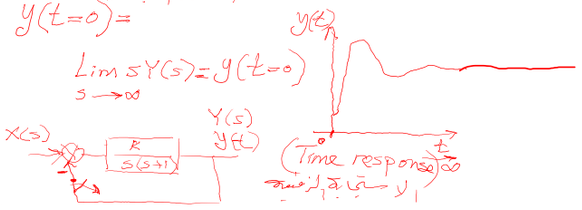
Initial Value Theorem

$$y(t=0) = \lim_{s\to\infty} sY(s) = y(t=0)$$



$$Y(s) = \frac{s+2}{(s+1)^2+5^2}$$

Final Value Theorem

$$y(t=\infty) = \lim_{s\to 0} sY(s)$$

(at steady state)

$$y(t=\infty) = \lim_{s\to 0} s\,\frac{(s+2)}{(s+1)^2+5^2}$$

Example

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 12y(t) = 3(t)$$

3(t): step function

Find ① $y(t=0)$

② $y(t=\infty)$

$$\mathcal{L}\left[\frac{d^2y}{dt^2}\right] = s^2Y(s) \quad ; \quad \mathcal{L}\left[5\frac{dy}{dt}\right] = 5sY(s)$$

$$\mathcal{L}\left[12y(t)\right] = 12Y(s) \quad ;$$

$$\mathcal{L}\left[3\right] = \frac{3}{s}$$

$$s^2Y(s) + 5sY(s) + 12Y(s) = \frac{3}{s}$$

$$(s^2+5s+12)\,Y(s) = \frac{3}{s}$$

$$Y(s) = \frac{3}{s(s^2+5s+12)}$$

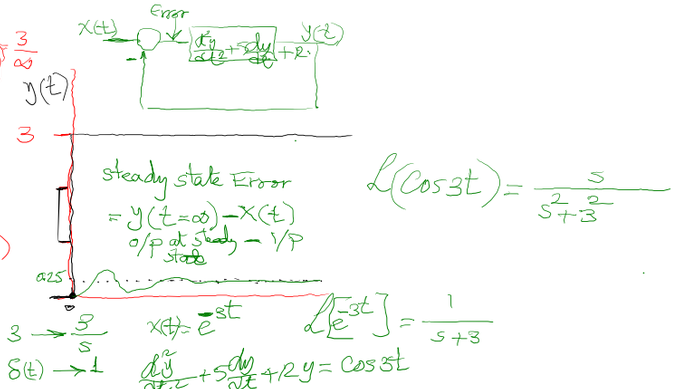
$y(t=0)$ ⟸ initial Value Theorem

$$y(t=0) = \lim_{s\to\infty} sY(s) = \lim_{s\to\infty} s\,\frac{3}{s(s^2+5s+12)} = \frac{3}{\infty}$$

$$y(t=0) = \emptyset$$

$y(t=\infty)$ = y at steady state

$$y(t=\infty) = \lim_{s\to 0} sY(s) = \lim_{s\to\emptyset} s\,\frac{3}{s(s^2+5s+12)} = \frac{3}{12} = 0.25$$



$$\mathcal{L}(\cos 3t) = \frac{s}{s^2+3^2}$$

$$\mathcal{L}\left[e^{3t}\right] = \frac{1}{s+3}$$

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 12y = \cos 3t$$