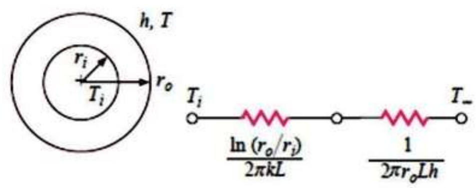
CRITICAL THICKNESS OF INSULATION

* لدينا اسطوانة (circular pipe) بنصف قطر (r_i) ودرجة حرارة (T_i) ، تم إضافة مادة عازلة على الإسطوانة بنصف قطر (r_o) وتم أيضاً تعريض السطح الخارجي إلى مائع (fluid) عند درجة حرارة T_∞.

$$q = \frac{\Delta T}{\sum R_{th}} = \frac{T_i - T_\infty}{\frac{\ln(r_o/r_i)}{2\pi k L} + \frac{1}{h A_o}}$$

Figure 2-7 | Critical insulation thickness.



$$\boxed{A_o = 2\pi r_o L}$$

$$q = \frac{T_i - T_\infty}{\frac{\ln(r_o/r_i)}{2\pi k L} + \frac{1}{2\pi r_o L h}} \Rightarrow q = \frac{2\pi L (T_i - T_\infty)}{\frac{\ln(r_o/r_i)}{k} + \frac{1}{r_o h}}$$

* r_o هو نصف القطر الذي يحصل عنده أقصى إنتقال للحرارة .

لإيجاد r_o نشتق المعادلة بالنسبة له ونكون المشتقة تساوي صفر عند أقصى إنتقال حرارة .

$$\frac{dq}{dr_o} = 0 \quad @ \text{ maximum Heat Transfer.}$$

$$\frac{dq}{dr_o} = \frac{\left(\frac{\ln(r_o/r_i)}{k} + \frac{1}{h r_o}\right)(0) - 2\pi L (T_i - T_\infty)\left[\frac{\frac{1}{r_i}/\frac{r_o}{r_i}}{k} + \left[\frac{-h}{r_o^2 h^2}\right]\right]}{\left[\frac{\ln(r_o/r_i)}{k} + \frac{1}{r_o h}\right]} = 0$$

0.0

$$-2\pi L (T_i - T_\infty)\left[\frac{1}{r_o k} - \frac{1}{r_o^2 h}\right] = 0$$

$$\therefore \frac{1}{r_o k} - \frac{1}{r_o^2 h} = 0 \Rightarrow r_o k = r_o^2 h$$

$$\boxed{r_o = \frac{k}{h}}$$

The Equation expresses the critical-radius-of-insulation concept. If the outer radius is less than the value given by this equation, then the heat transfer will be *increased* by adding more insulation. For outer radii greater than the critical value an increase in insulation thickness will cause a decrease in heat transfer. The central concept is that for sufficiently small values of h the convection heat loss may actually increase with the addition of insulation because of increased surface area.

EXAMPLE 2-6

Calculate the critical radius of insulation for asbestos [$k = 0.17$ W/m · °C] surrounding a pipe and exposed to room air at 20°C with $h = 3.0$ W/m² · °C. Calculate the heat loss from a 200°C, 5.0-cm-diameter pipe when covered with the critical radius of insulation and without insulation.

Solu:– $K_{(asbestos)} = 0.17$, $T_\infty = 20°C$, $h = 3\ W/m^2.°C$, $T_i = 200°C$

$D = 5\ cm$, $r_i = 2.5\ cm$.

Find ① $\frac{q}{L}$ with insulation, ② $\frac{q}{L}$ without insulation

① $r_o = \frac{K}{h} \Rightarrow r_o = \frac{0.17}{3} \Rightarrow \boxed{r_o = 5.67\ cm = 0.0567\ m}$

$$q = \frac{2\pi L (T_i - T_\infty)}{\frac{Ln(r_o/r_i)}{K} + \frac{1}{r_o h}} \Rightarrow \frac{q}{L} = \frac{2\pi (T_i - T_\infty)}{\frac{Ln(r_o/r_i)}{K} + \frac{1}{r_o h}}$$

$$\frac{q}{L} = \frac{2\pi(200 - 20)}{\frac{Ln(5.67/2.5)}{0.17} + \frac{1}{0.0567 * 3}} \Rightarrow \boxed{\frac{q}{L} = 105.7\ W/m}$$

② ما يكون عزل مع المحيط الخارجي وال pipe

$$q = h A (T_w - T_\infty) \Rightarrow q = h (2\pi r L)(T_w - T_\infty)$$

(A ← المساحة الجانبية للإسطوانة.)

$$\therefore \frac{q}{L} = h(2\pi r)(T_w - T_\infty) \Rightarrow \frac{q}{L} = 3 * 2 * \pi * 0.025 * (200 - 20) \Rightarrow \boxed{\frac{q}{L} = 84.8\ W/m}$$

نتيجة:

$r < r_{(critical)} \Rightarrow r\uparrow\ q\uparrow$

$r > r_{(critical)} \Rightarrow r\uparrow\ q\downarrow$