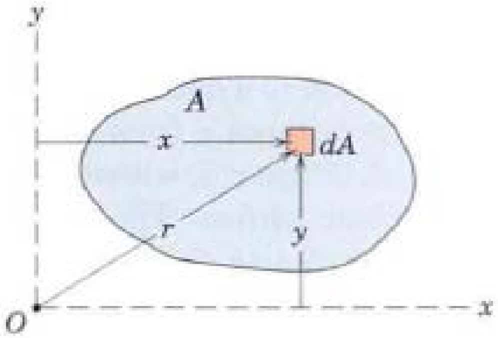
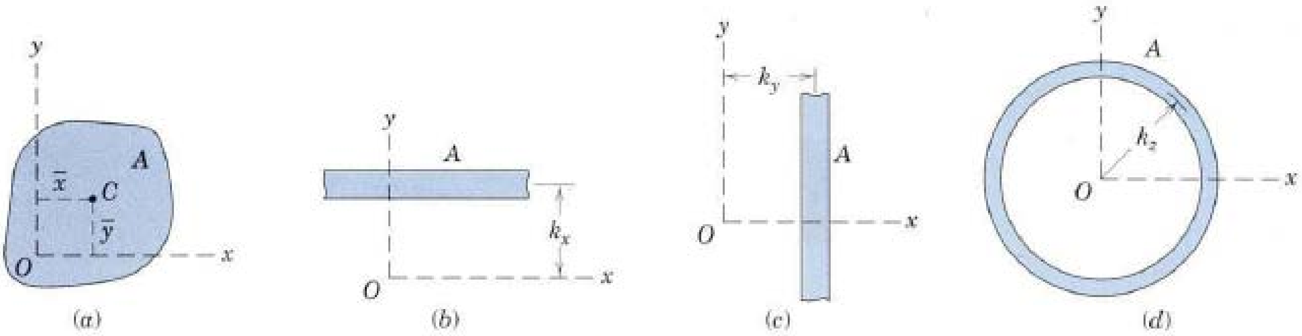
MOMENTS OF INERTIA



$$I_x = \int y^2 \, dA$$

$$I_y = \int x^2 \, dA$$



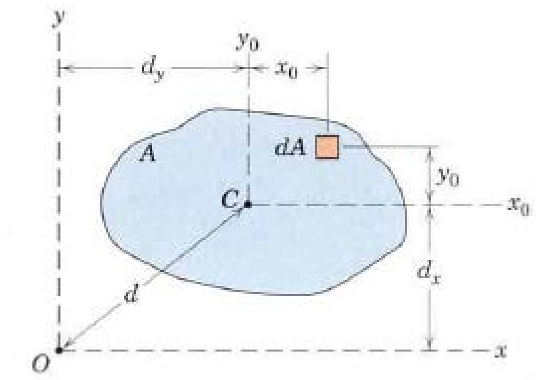
$$I_x = k_x^2 A \qquad I_y = k_y^2 A \qquad I_z = k_z^2 A$$

or

$$k_x = \sqrt{I_x/A} \qquad k_y = \sqrt{I_y/A} \qquad k_z = \sqrt{I_z/A}$$

$$k_z^2 = k_x^2 + k_y^2$$

Transfer of Axes



$$dI_x = (y_0 + d_x)^2\, dA$$

$$I_x = \int y_0^2\, dA + 2d_x \int y_0\, dA + d_x^2 \int dA$$

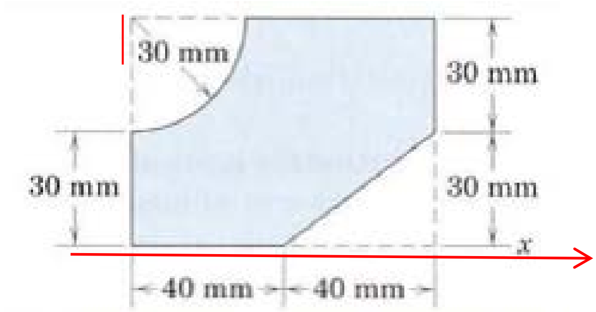
$$I_x = \bar{I}_x + Ad_x^2$$
$$I_y = \bar{I}_y + Ad_y^2$$

COMPOSITE AREAS

$$I_x = \Sigma \bar{I}_x + \Sigma A d_x^2$$
$$I_y = \Sigma \bar{I}_y + \Sigma A d_y^2$$

Problem 1

Calculate the moment of inertia and radius of gyration about the x -axis for the shaded area shown.



$$I_x = \frac{1}{3}bh^3 \ , I_x = \frac{1}{3}80 * 60^3 = 5.76(10^6)mm^4$$

the moment of inertia of the negative quarter-circu lar area about its base axis x' is

$$I_{x'} = -\frac{1}{4}\left(\frac{\pi r^4}{4}\right) = -\frac{\pi}{16}(30^4) = -0.159(10^6)\ mm^4$$

We now transfer this result through the distance $\bar{r} = \frac{4r}{3\pi} = \frac{4(30)}{3*3.14} = 12.73\ mm$ by th e transfer-or-axis theorem to get the centroidal moment of inertia of part (2)

$$\bar{I} = I - Ad^2 , \bar{I_x} = -0.159(10^6) - \left[-\frac{\pi(30^2)}{4}(12.73^2)\right] = -0.0445(10^6)\ mm^4$$

The moment of inertia of t he quarter-circular part about the .x-axis is now

$$\bar{I} = I + Ad^2 \ , I_x = -0.0445(10^6) + \left[-\frac{\pi(30^2)}{4}(60-12.73)^2\right] = -1.624(10^6)\ mm^4$$

Finally, th e momen t of inertia of the negative triangular area (3) about its base

$$I_x = -\frac{1}{12}bh^3 \ , I_x = -\frac{1}{12}40 * 30^3 = -0.09(10^6)mm^4$$

The total moment of inertia about the z-axi s of t he composite area is,

consequently,

$$I_x = 5.76(10^6) - 1.624(10^6) - 0.09(10^6) = 4.05(10^6)mm^4$$

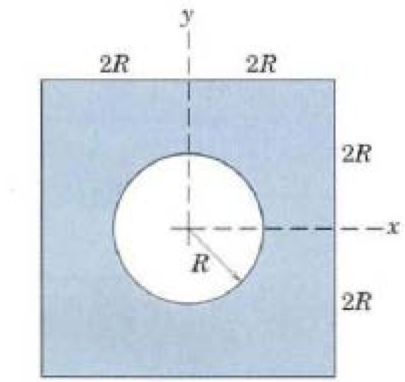
$$A = 60(80) - \frac{1}{4}\pi 30^2 - \frac{1}{2}40(30) = 3490\ mm^2$$

the radius of gyration about t he x-axis is

$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{4.05(10^6)}{3490}} = 34\ mm$$

Problem 2

Determine the moment of inertia about the .x-axis of the square area with out and with the central circular hole.



Without hole

$$I_x = \frac{1}{12}bh^3 = \frac{1}{12}(4R)(4R)^3 = \frac{64}{3}R^4$$

With hole

$$I_x = \frac{1}{12}bh^3 - \frac{1}{4}\pi r^4 = \frac{1}{12}(4R)(4R)^3 - \frac{1}{4}\pi(R^4) = \frac{64}{3}R^4 - \frac{1}{4}\pi(R^4) = 20.6\ R^4$$